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# Interacting dark sectors from a dynamical system perspective


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# State of the Art

- Our universe is not only expanding but it is also accelerating!!
- $\Lambda$CDM model has been constrained with unprecedented accuracy.
- With the improvement in our ability to constrain the cosmological parameters, a few statistically significant tensions has emerged.
- It seems that the late time cosmological data and early time cosmological data are in tension.
- We need to extent our imagination beyond standard $\Lambda$CDM.

# Hubble Tension

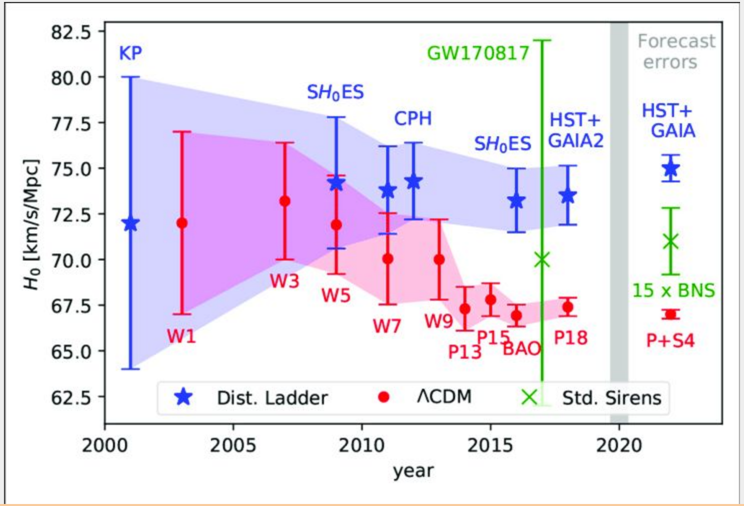



Fig 1(a)

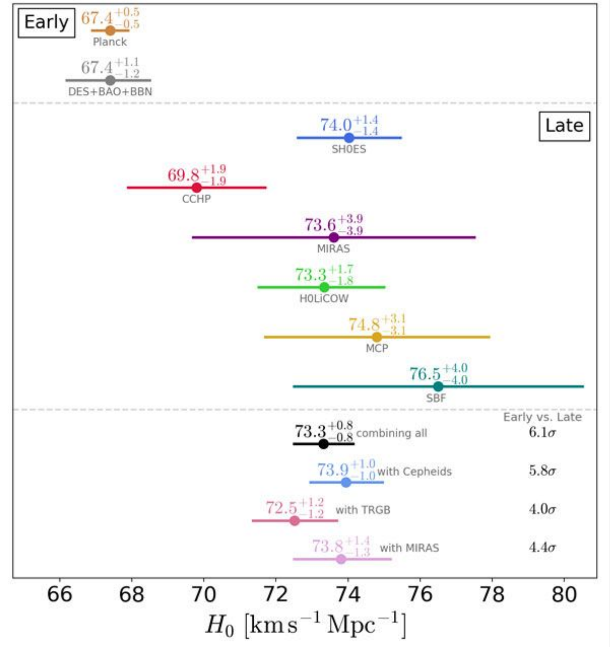
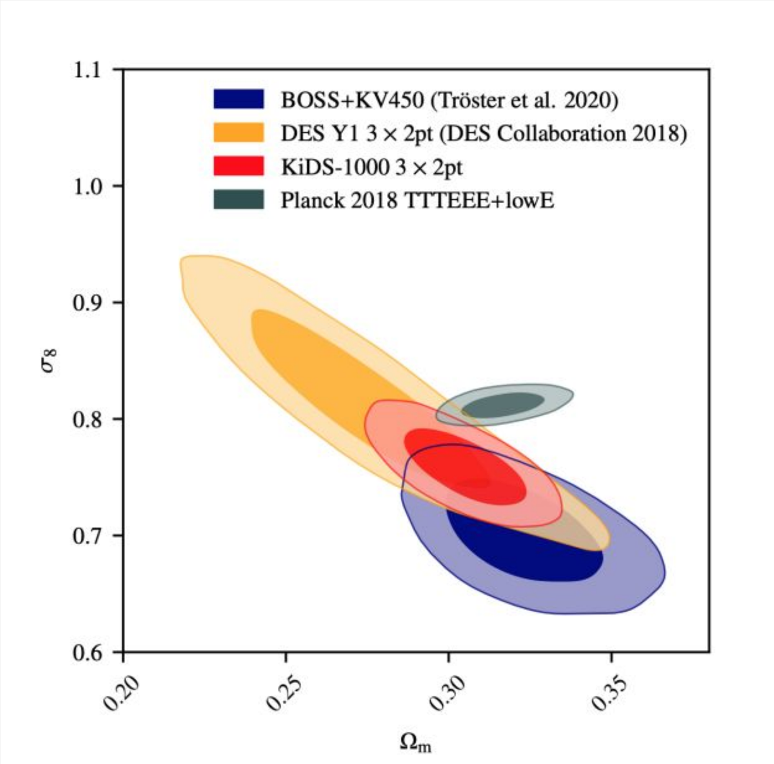


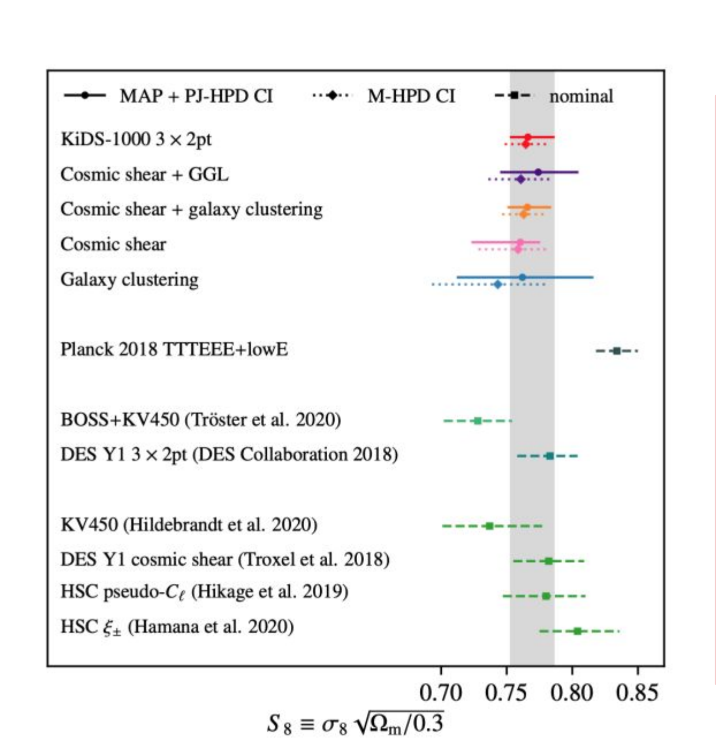

Fig 1(b)

CMB Planck data together with BAO, BBN, and DES have constraint the Hubble parameter to be H0 ~ (67.0 – 68.5)km/s/Mpc. On the other hand, cosmic distance ladder and time delay measurement like those reported by SH0ES and H0LiCOW collaborations have reported H0 = (74.03 ± 1.42) km/s/Mpc and H0 = (73.3 +1.7 −1.8 ) km/s/Mpc respectively by observing the local Universe.

arXiv: 2008.11284

# $\sigma_8$ Tension




Apart from the Hubble tension, another tension between the Planck data with the weak lensing and the redshift surveys has been reported.

Fig 2

arXiv:2008.11285

# Scalar Field as Dark energy

- The ΛCDM model happens to be most consistent with the observations but it suffers from problems arising from both theoretical and observational aspects.
- From the theoretical side it has to deal with the cosmological constant problem, coincidence problem and the fine tuning problem.
- From the observational side it is unable to explain the tension between the early time (Planck, BAO) and late time observations (SH0ES).
- There could be new physics involved and we should think beyond ΛCDM model.
- Scalar fields models are considered as one of the best alternatives to the cosmological constant.

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa^2}R + \mathcal{L}_\phi\right] + S_M$$

$$\mathcal{L}_\phi = -\epsilon\frac{1}{2}g^{\mu v}\partial_\mu\phi\partial_\nu\phi - V(\phi)$$

# Scalar Field Dynamics

For a spatially flat, homogeneous and isotropic universe filled with matter and nonminimally coupled scalar field components

$$3H^2 = \rho_m + \rho_\phi = \rho_m + \frac{1}{2}\epsilon\dot{\phi}^2 + V(\phi)$$

$$2\dot{H} + 3H^2 = -p_\phi = -\frac{1}{2}\epsilon\dot{\phi}^2 + V(\phi) \qquad 1)$$

$$\epsilon\ddot{\phi} + 3\epsilon H\dot{\phi} + \frac{dV}{d\phi} = 0$$

$\epsilon$ → Switch parameter

$$\epsilon = \begin{cases} +1, & \text{Quintessence} \\ -1, & \text{Phantom} \end{cases}$$

$\rho_m$ → Matter energy density

$\rho_\phi = \frac{1}{2}\epsilon\dot{\phi}^2 + V(\phi)$ → Scalar field energy density

$p_\phi = \frac{1}{2}\epsilon\dot{\phi}^2 - V(\phi)$ → Pressure component

# Interacting Dark Sectors

If we take into account the dark energy scenario featuring a modification to the regular conservation equations, with the addition of an interaction to the dark sectors of the universe.

Interaction term

$$\dot{\rho}_m + 3H\rho_m = -Q$$
$$\dot{\rho}_\phi + 3H\left(\rho_\phi + P_\phi\right) = Q \quad (2)$$

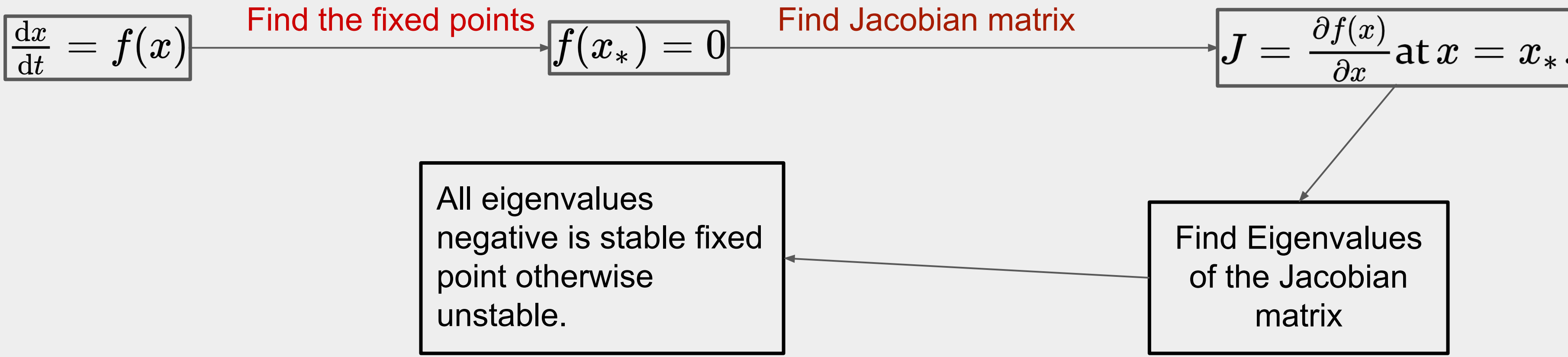
Continuity equation of the dark matter and dark energy.

The K.G equation of the scalar field.

$$\ddot{\phi} + 3H\dot{\phi} + \epsilon\frac{dV}{d\phi} = \epsilon\frac{Q}{\dot{\phi}}, \quad (3)$$

# Dynamical Systems Analysis

- A dynamical system can be considered to be a model describing the temporal evolution of a system.
- Dynamical system is mathematical tool to understand the qualitative behaviour of the linear or nonlinear systems where finding exact solution to the system is not trivial.
- Dynamical system analysis has been used extensively in many areas of science and engineering.
- Dynamical system analysis has been also widely used in cosmology.



# Construction of the autonomous system

$$x=\frac{\kappa\dot{\phi}}{\sqrt{6}H}, y^2=\frac{\kappa^2 V(\phi)}{3H^2}, \lambda=-\frac{1}{V(\phi)}\frac{dV(\phi)}{d\phi}. \quad (5)$$

$$\Omega_\phi=\epsilon x^2+y^2,$$
$$w_\phi=\frac{\epsilon x^2-y^2}{\epsilon x^2+y^2},$$
$$q=-1+\frac{3}{2}(1+\epsilon x^2-y^2).$$

Cosmological parameters in terms of dynamic variables.

$$x'=-3x+\sqrt{3/2}\epsilon\lambda y^2 + \frac{3}{2}x\left(1+\epsilon x^2-y^2\right)+\epsilon f(x,y),$$
$$y'=-\sqrt{3/2}\lambda xy + \frac{3}{2}y\left(1+\epsilon x^2-y^2\right),$$
$$\lambda'=-\sqrt{6}\lambda^2(\Gamma-1)x, \quad (6)$$

$$Q=\sqrt{6}\dot{\phi}H^2 f(x,y)$$

Interaction term

$$\Gamma=\frac{V(\phi)\frac{\partial^2 V(\phi)}{\partial\phi^2}}{\left(\frac{\partial V(\phi)}{\partial\phi}\right)^2}$$

Potential

# Classification of Potential

Depending in the choice of $\Gamma = \frac{V(\phi)\frac{\partial^2 V(\phi)}{\partial \phi^2}}{\left(\frac{\partial V(\phi)}{\partial \phi}\right)^2}$ potential can be classified in two classes.

$$\lambda' = -\sqrt{6}\lambda^2(\Gamma - 1)x,$$

- $\Gamma = 1$ → Exponential Potential
- $\Gamma \neq 1$ → Non-exponential Class of Potentials

# Exponential Potential

For the exponential potential $\Gamma = 1$ so that $\lambda$ is constant.

The system reduces to a 2D system

$$x' = -3x + \sqrt{3/2}\epsilon\lambda y^2 + \frac{3}{2}x\left(1 + \epsilon x^2 - y^2\right) + \epsilon f(x,y),$$
$$y' = -\sqrt{3/2}\lambda xy + \frac{3}{2}y\left(1 + \epsilon x^2 - y^2\right), \qquad (7)$$

# Fixed Points

List of the fixed points for the exponential potential

| Fixed Points | $x$ | $y$ | $f(x,y)$ |
|---|---|---|---|
| $Q_1$ | 0 | 0 | 0 |
| $Q_{2i}$ | $\frac{3}{2}x(1+\epsilon x^2)$ <br> $-3x+\epsilon f(x,0)=0$ | 0 | $f(x,y)=0$ for $x=0, x\pm 1$ |
| $Q_{3i}$ | $\frac{1}{2}\left(\epsilon\left(2f(x,y)+\sqrt{6}\lambda\right)+2\sqrt{6}\lambda x^2-2x\left(\lambda^2\epsilon+3\right)\right)=0$ | $\pm\frac{\sqrt{3x^2\epsilon-\sqrt{6}\lambda x+3}}{\sqrt{3}}$ | $f(x,y)$ |

$Q_1$ is completely matter dominated and unstable in nature.

$$\left[\frac{3}{2}, \frac{1}{2}(2\epsilon\partial_x f-3)\right]$$

$Q_2$ Depending on the choice of the form of interaction $f(x,y)$ it could be either unstable or saddle.

$$\left[\frac{1}{2}\left(3x^2\epsilon-\sqrt{6}\lambda x+3\right), \frac{1}{2}\left(2\epsilon\partial_x f+9x^2\epsilon-3\right)\right]$$

$Q_3$ Depends on the choice of the interaction $f(x,y)$.

$$T_{Q3}=-6+\epsilon\partial_x f$$

$$D_{Q3}=-3\epsilon\partial_x f+\sqrt{\frac{3}{2}}\lambda\epsilon\partial_y f+3\epsilon\lambda^2+9$$

For $x=0, y=\pm 1$.

# Non-exponential class of potentials

For this case the system is 3D

$$x' = -3x + \sqrt{3/2}\epsilon\lambda y^2 + \frac{3}{2}x\left(1 + \epsilon x^2 - y^2\right) + \epsilon f(x, y),$$
$$y' = -\sqrt{3/2}\lambda xy + \frac{3}{2}y\left(1 + \epsilon x^2 - y^2\right),$$
$$\lambda' = -\sqrt{6}\lambda^2(\Gamma - 1)x \qquad (8)$$

| Fixed Points | $x$ | $y$ | $\lambda$ | $f(x,y)$ |
|---|---|---|---|---|
| $P_1$ | 0 | 0 | $\lambda$ | 0 |
| $P_{2i}$ | $\frac{3}{2}x(1-\epsilon x^2)$ -$\epsilon f(x,0) = 0$ | 0 | 0 | $f(x,y) = 0$ for $x = 0, x \pm 1$ |
| $P_{3\pm}$ | 0 | $\pm 1$ | $\lambda = -\sqrt{\frac{2}{3}}f(x,y)$ | $f(x,y)$ |
| $P_{4\pm}$ | $3x - \epsilon f(x,y) = 0$ | $y^2 = (1 + \epsilon x^2)$ | 0 | $f(x,y)$ |

# Stability

- $P_1$ is a completely matter dominated unstable fixed point.
- $P_2$ does not exist for the Phantom field and also unstable in nature. It could be an mixed state of both the matter and dark energy.
- $P_3$ is completely dark energy dominated fixed point.
- $P_4$ can have both dark energy and dark matter contribution and its stability depends on the choice of the interaction.

**$P_1$** $\left[\frac{3}{2}, 0, \frac{1}{2}(2\epsilon\partial_x f - 3)\right]$

**$P_2$** $\left[0, \frac{3}{2}\left(x^2\epsilon + 1\right), \frac{1}{2}\left(2\epsilon\partial_x f + 9x^2\epsilon - 3\right)\right]$

**All are non-hyperbolic fixed points**

**$P_3$** $\left[-3, 0, -3 + \epsilon\partial_x f\right]$

**$P_4$** $\left\{0, \frac{1}{6}(-A + 3\epsilon\partial_x f - 18), \frac{1}{6}(A + 3\epsilon\partial_x f - 18)\right\}$

*where*

$$A = \sqrt{3}\sqrt{4f^2(\partial_x f - 3\epsilon) + 4\epsilon f\sqrt{9 + \epsilon f^2}\partial_y f + 3(\partial_x f)^2}.$$

# Example

Let us consider a form of the f(x,y) and check our general analysis;

$$f(x,y) = \alpha(1 - \epsilon x^2 - y^2)^m x^\gamma \qquad (9)$$

Depending on the choice of the it could incorporate a large class of interaction.

# Exponential Potential

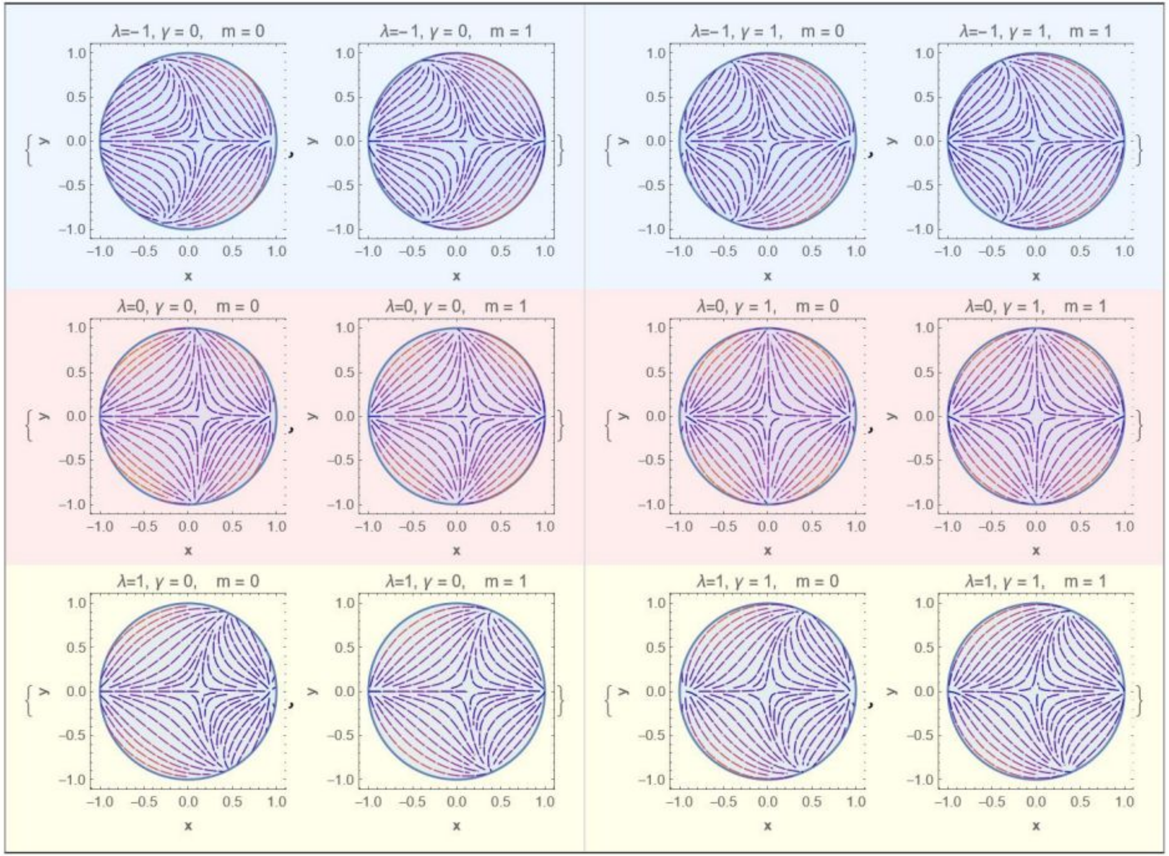



Fig.(3). Phase plot of the quintessence scalar field with exponential potential.

$$0 \leq \Omega_\phi \leq 1;$$

$$0 \leq x^2 + y^2 \leq 1.$$

# Exponential Potential

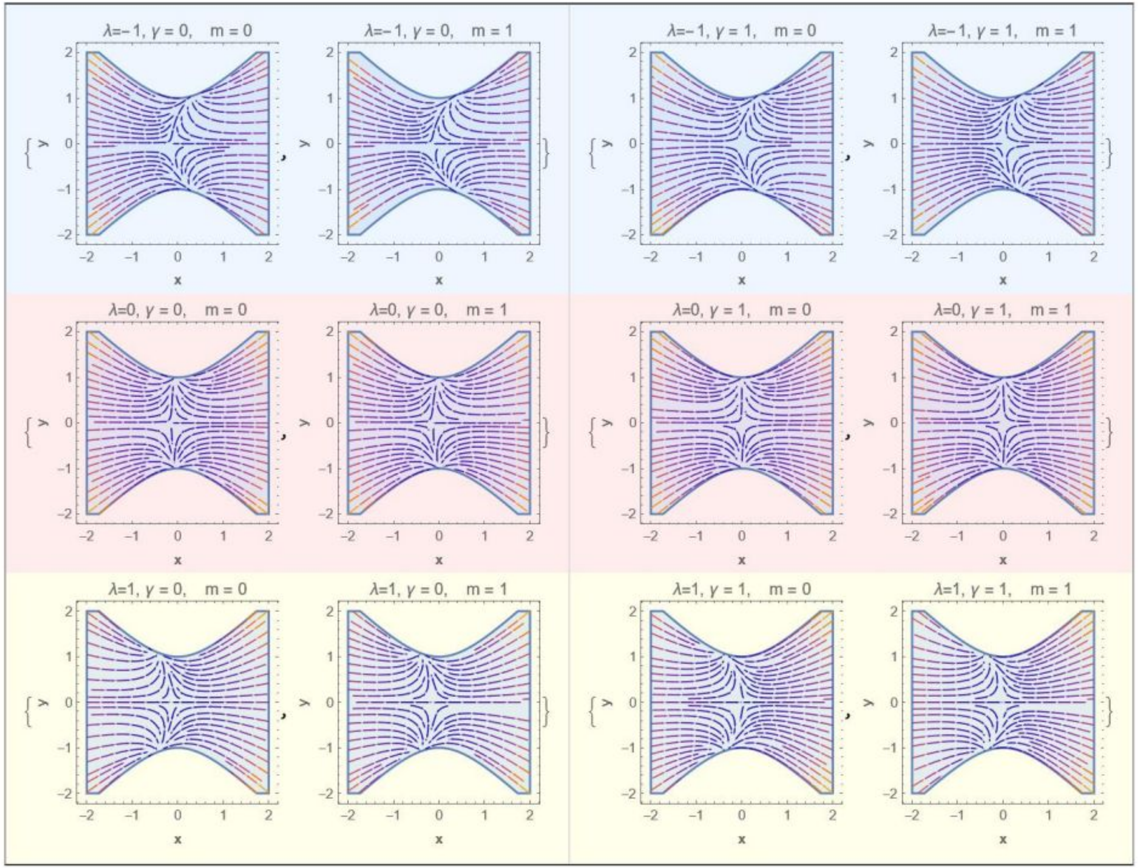



Fig.(4). Phase plot of the phantom scalar field with exponential potential.

$$0 \leq \Omega_\phi \leq 1;$$

$$0 \leq -x^2 + y^2 \leq 1.$$

# Exponential Potential

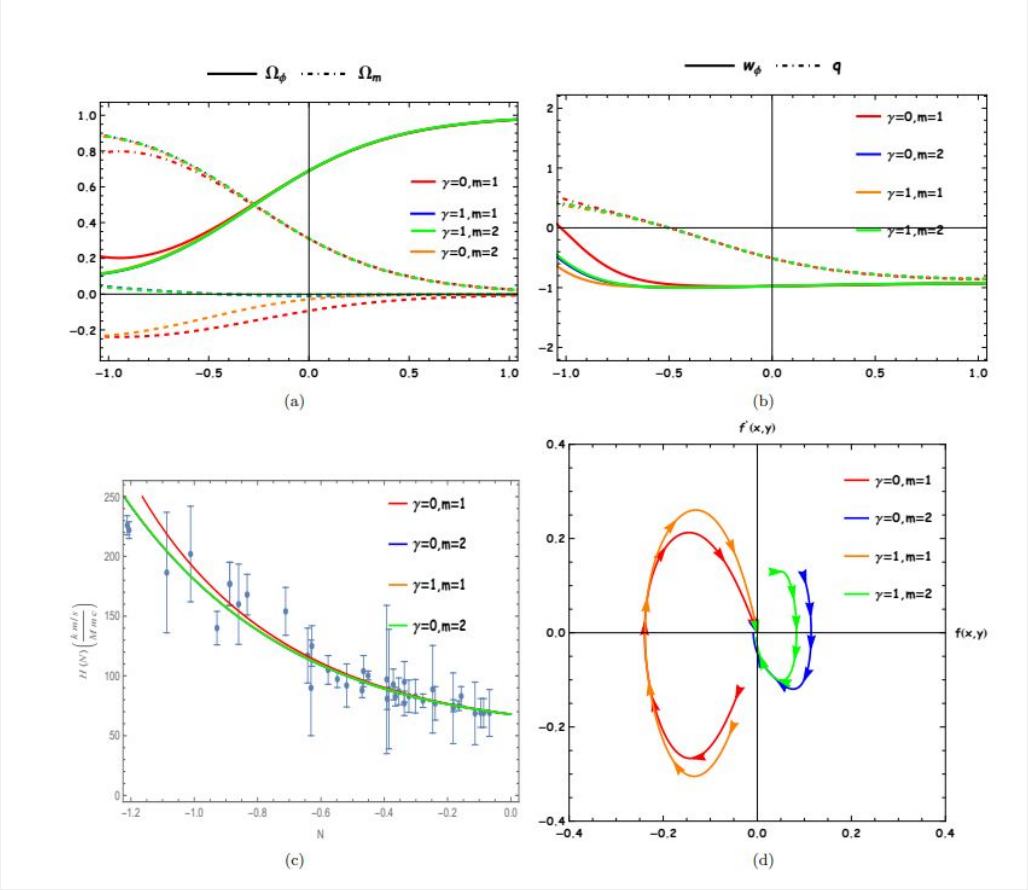



Fig.5, Plots of different cosmological parameters for the quintessence field for the exponential potential.

# Exponential Potential

- We have estimated the current values of the $x,y$ from the observation by solving,

$$\Omega_{\phi 0} = \epsilon x_0^2 + y_0^2$$

$$q_0 = -1 + \frac{3}{2}(1 + \epsilon x_0^2 - y_0^2)$$

- For quintessence field we estimate $x_0 = 0.09$, $y_0=0.825$ and for the phantom field $x_0 = 0.01$, $y_0=0.824$.

$$\Omega_{\phi 0} = 0.68$$

$$q_0 = -0.51$$


1. N. Aghanim et al. Planck 2018 results. VI. Cosmological parameters. 2018
2. Adam G. Riess et al. Astrophys. J. Lett., 934(1):L7, 2022

# Exponential Potential

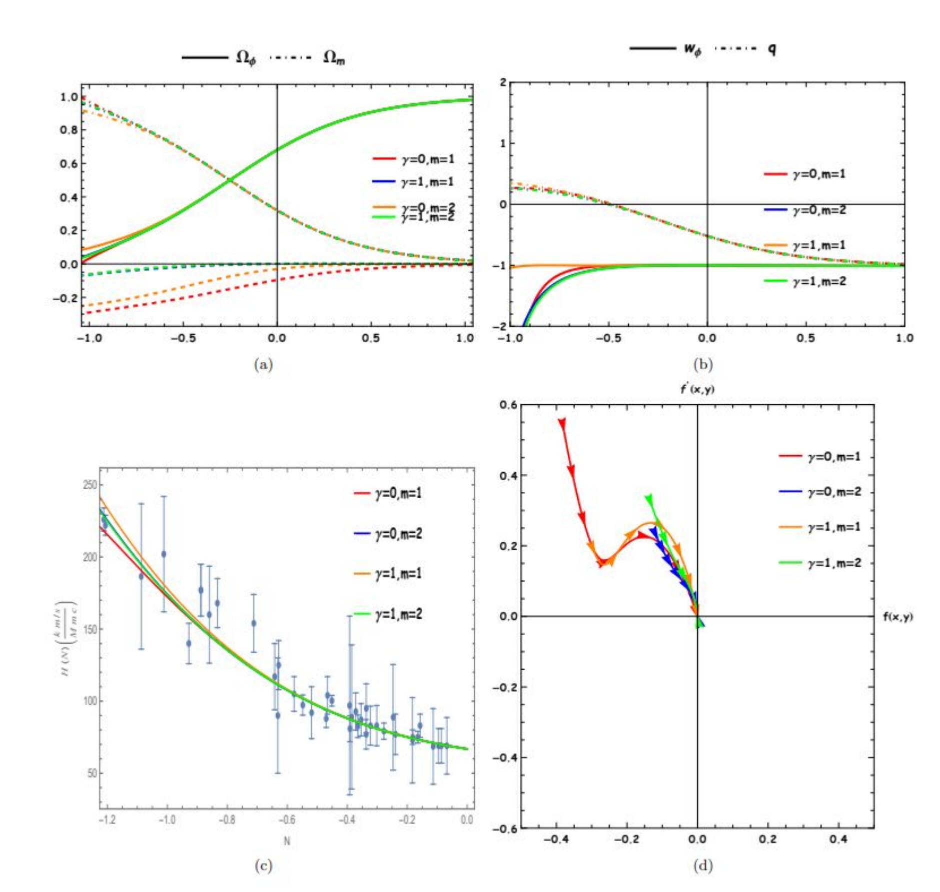



Fig.6, Plots of different cosmological parameters for the phantom field for the exponential potential.

# Non-Exponential Potential

$\Gamma$ could be a function of λ. Here we considered two different choices.

1. $\Gamma = 1/2$ for which the potential becomes to be $V(\phi) = (A + B\phi)^2$.
2. $\Gamma = \frac{1}{2}(\frac{1}{\lambda^2} - 1) + 1$, $V(\phi) = \cosh(\xi\phi) - 1$ where we chose $\xi = 1$.

# Γ = 1/2

Fig.7, quintessence case

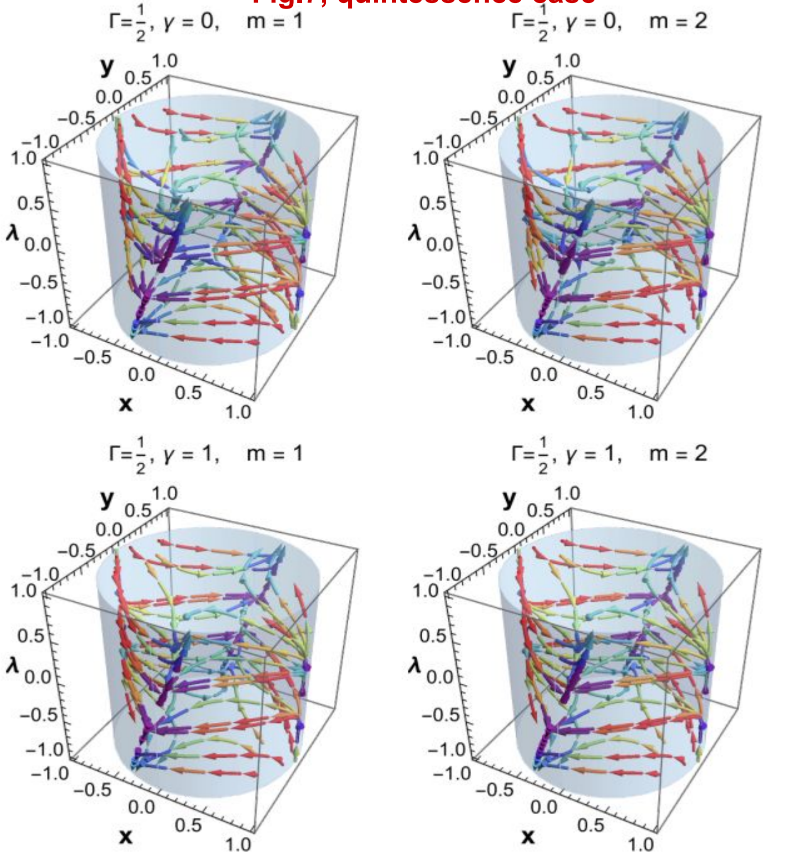
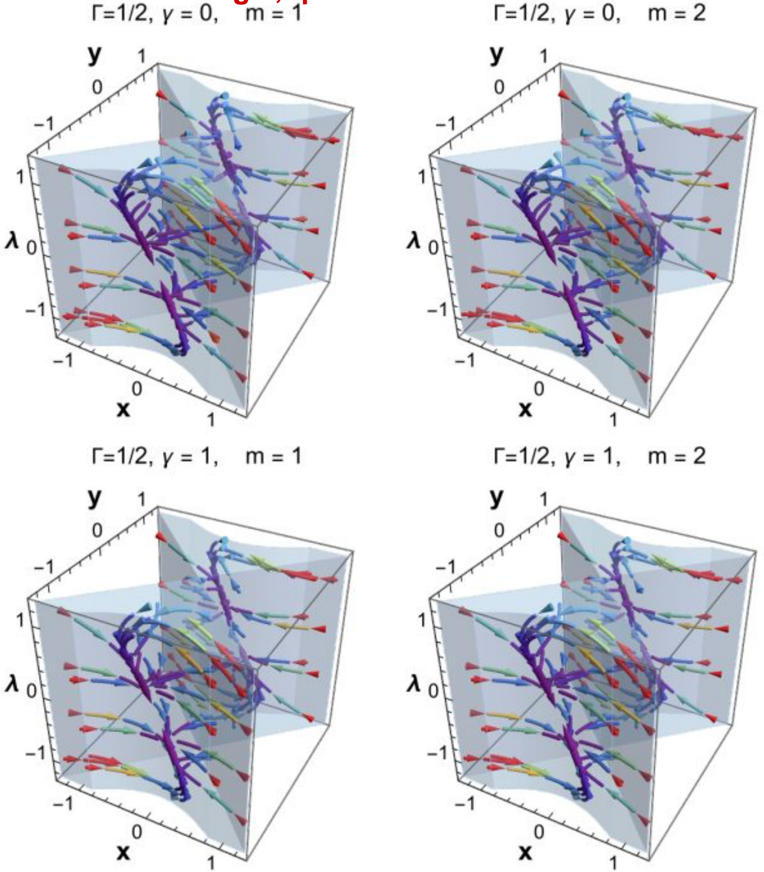



Fig.8, quintessence case



# Γ = 1/2

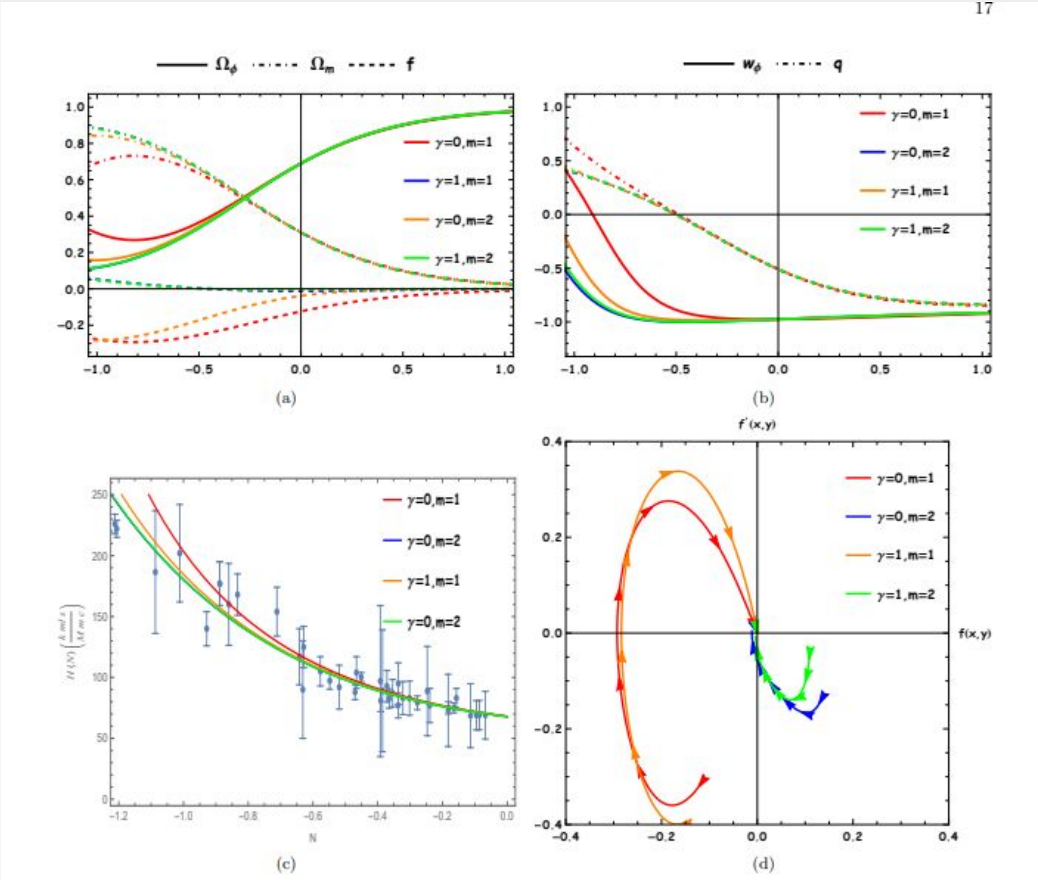



Fig.9, Plots of different cosmological parameters for the quintessence field for the exponential potential.

# Γ = 1/2

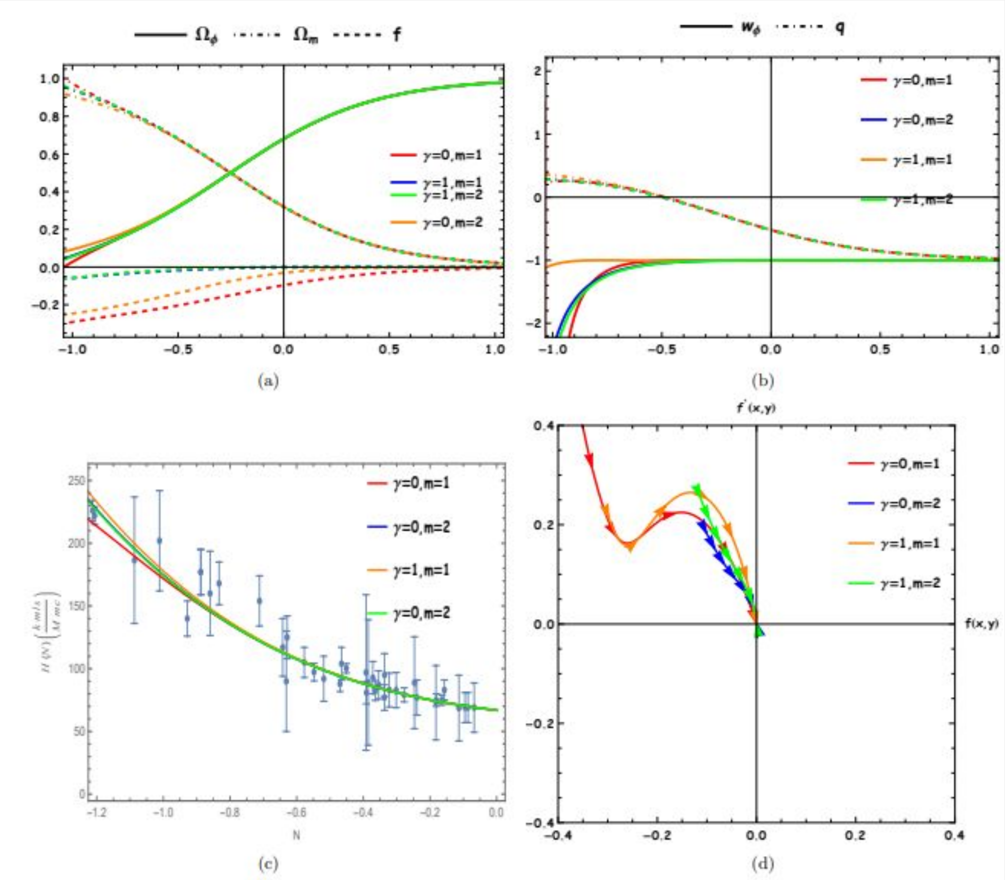
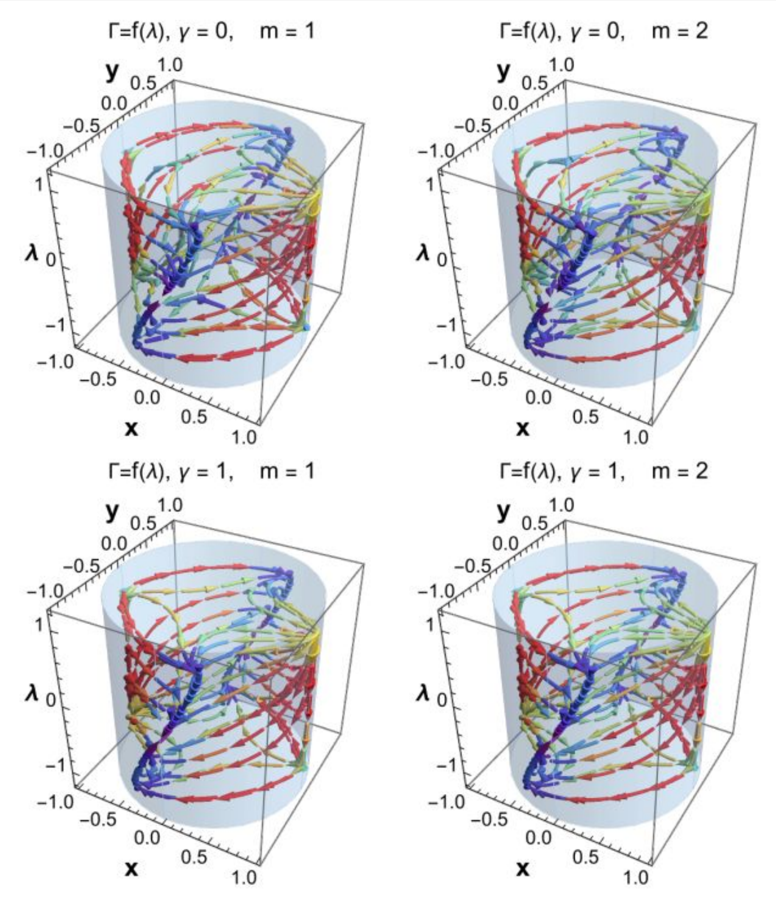
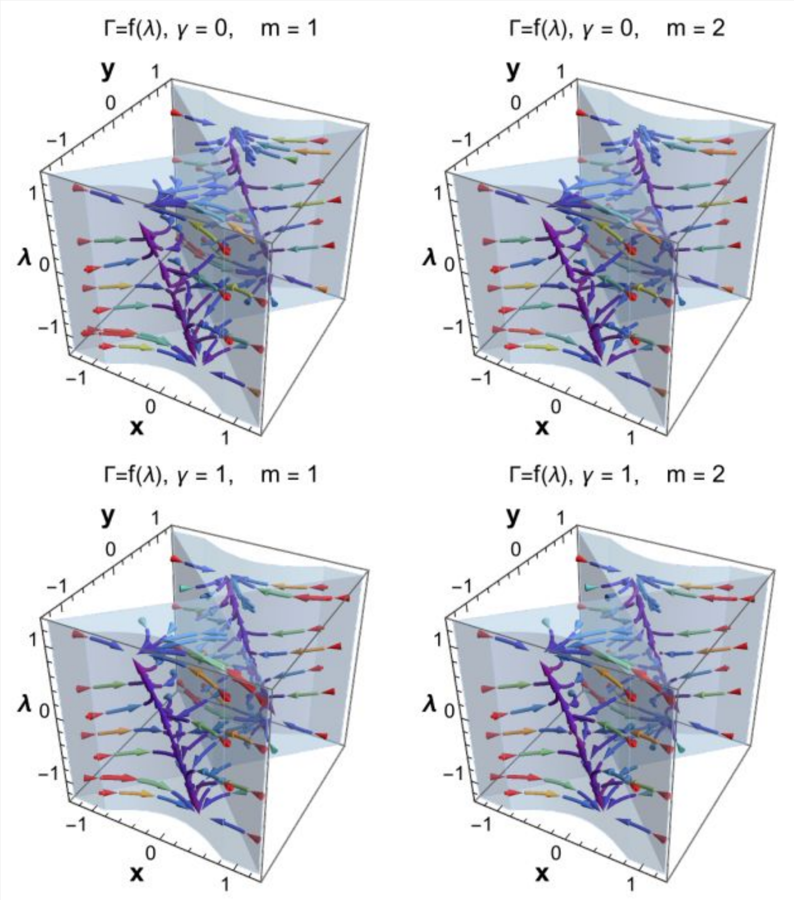



Fig.10, Plots of different cosmological parameters for the phantom field for the exponential potential.

$$\Gamma = \frac{1}{2}\left(\frac{1}{\lambda^2} - 1\right) + 1$$



$$\Gamma = \frac{1}{2}\left(\frac{1}{\lambda^2} - 1\right) + 1$$

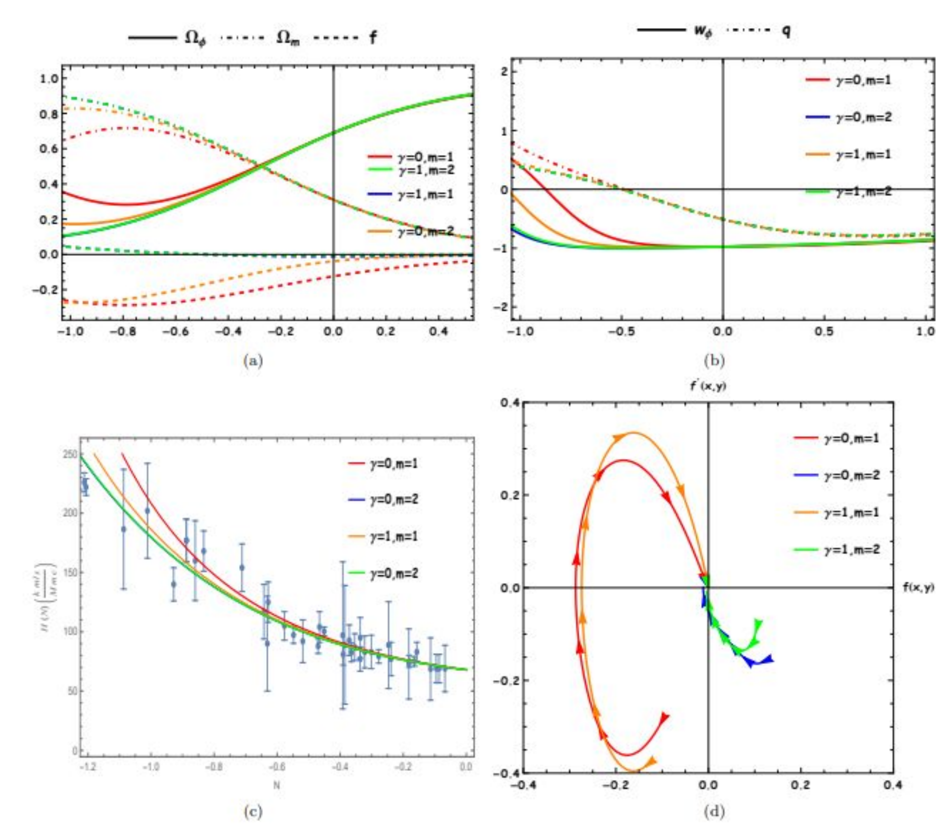



Fig.11, Plots of different cosmological parameters for the quintessence field for the non-exponential potential.

$$\Gamma = \frac{1}{2}\left(\frac{1}{\lambda^2} - 1\right) + 1$$

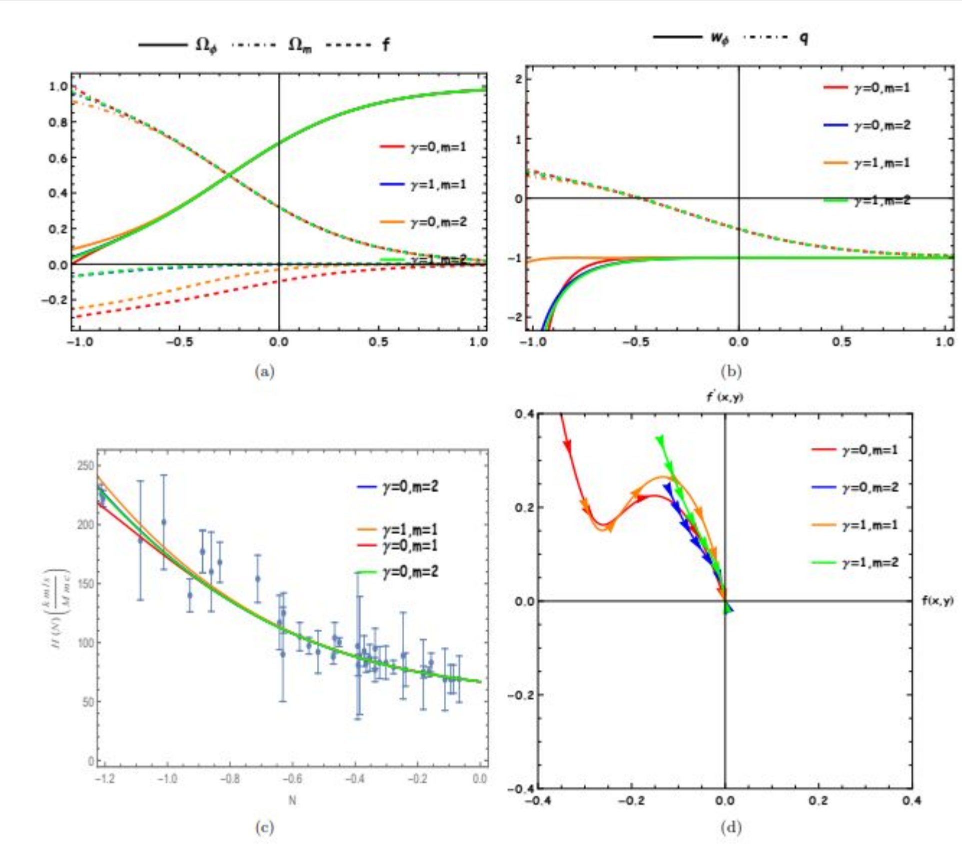



Fig.12, Plots of different cosmological parameters for the phantom field for the non-exponential potential.

# Conclusion

- We have studied interacting scalar field model dark energy models using dynamical system analysis.
- In the first part the analysis is done considering a general form of the potential. Our setup includes both the quintessence and the phantom field.
- In the second part both we have considered a particular form of the interaction and a detail phase space behaviour of the system has been studied.
- Our analysis suggests that there could be interaction at present between the dark sectors and in future the universe is completely dominated by the dark energy.

# Thank You