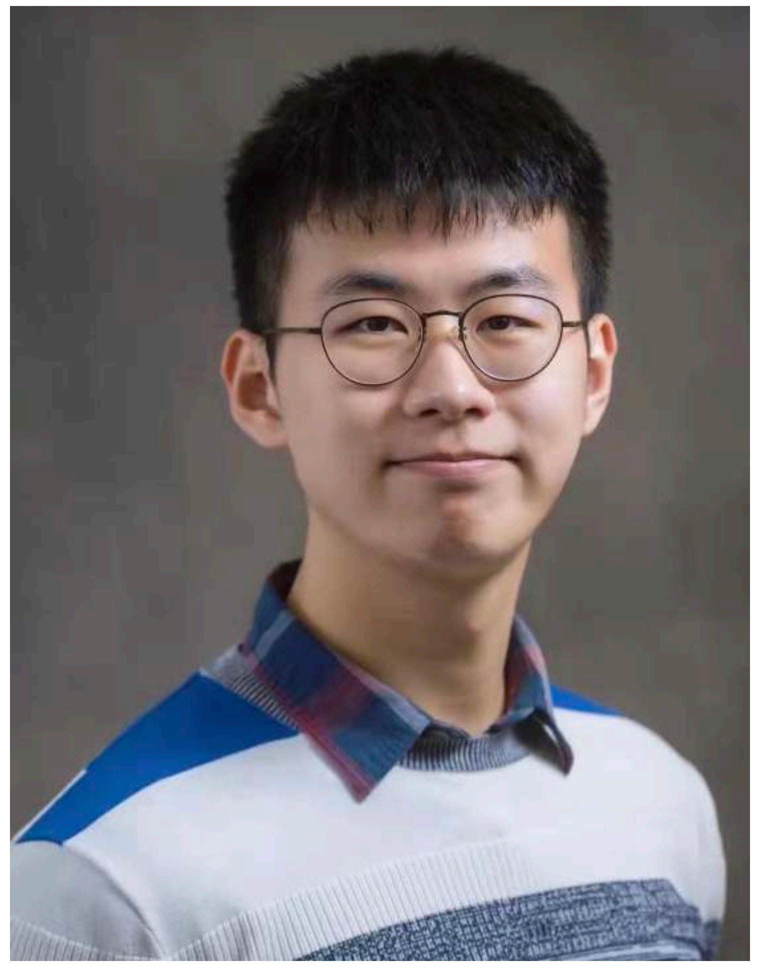
Perturbation Theory **Remixed:**

Improved Nonlinearity Modeling beyond Standard Perturbation Theory

arXiv:2209.00033
DOI: 10.1103/PhysRevD.107.103534

Zhenyuan Wang
The Pennsylvania State University, US

Cosmology From Home 2023



Zhenyuan Wang
Penn State

Donghui Jeong
Penn State & KIAS

Atsushi Taruya
Kyoto U & IPMU

Takahiro Nishimichi
Kyoto Sangyo U & IPMU

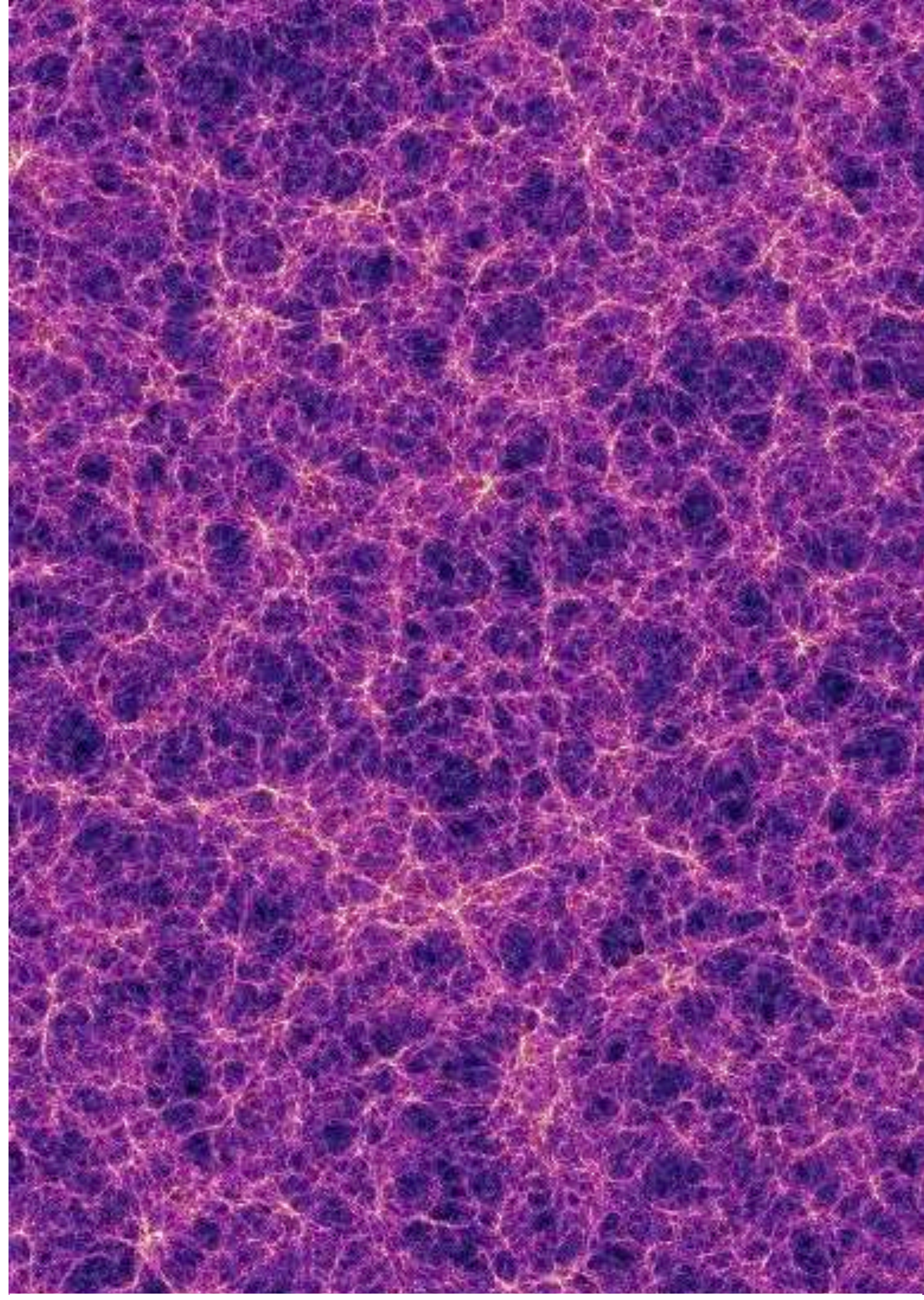
Ken Osato
Chiba University

PHYSICAL REVIEW D **107,** 103534 (2023)

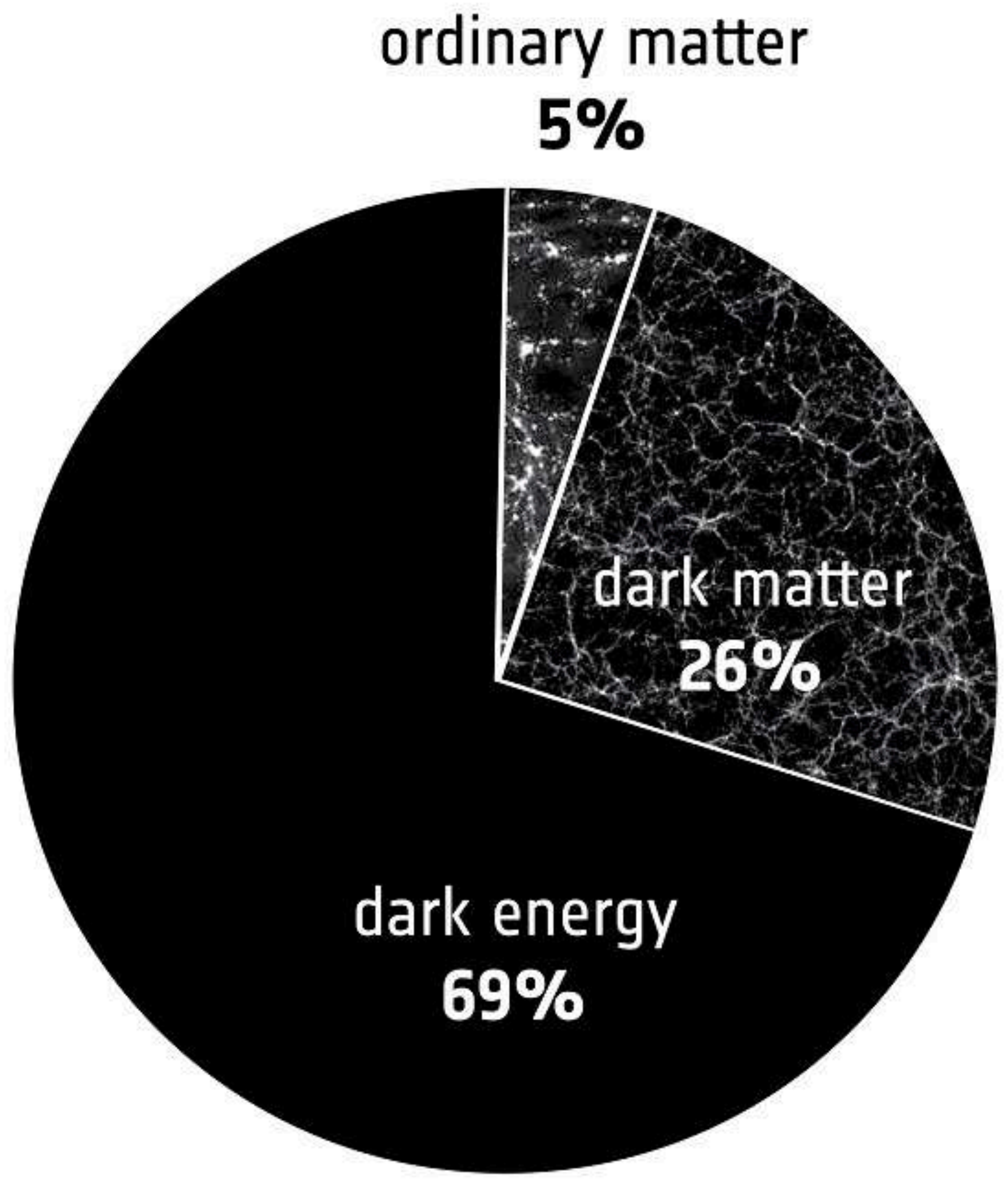
Perturbation theory remixed: Improved nonlinearity modeling beyond standard perturbation theory

Zhenyuan Wang (王震远),[1] Donghui Jeong,[1,2] Atsushi Taruya,[3,4] Takahiro Nishimichi,[3,4,5] and Ken Osato[6,7,3]
[1]*Department of Astronomy and Astrophysics and Institute for Gravitation and the Cosmos, The Pennsylvania State University, University Park, Pennsylvania 16802, USA*
[2]*School of Physics, Korea Institute for Advanced Study, Seoul, South Korea*

Large-scale structure (**LSS**) in a nutshell



- Galaxies (matter) are not distributed randomly
- **LSS** can help understand fundamental questions in cosmology!
  - **Inflation**
    How does the universe begin?
  - **Dark matter**
    Structure growth
  - **Dark energy**
    Accelerating Expansion
  - **Gravity**
    Is General Relativity valid on cosmological scales?



Two-point correlation function (Power Spectrum)

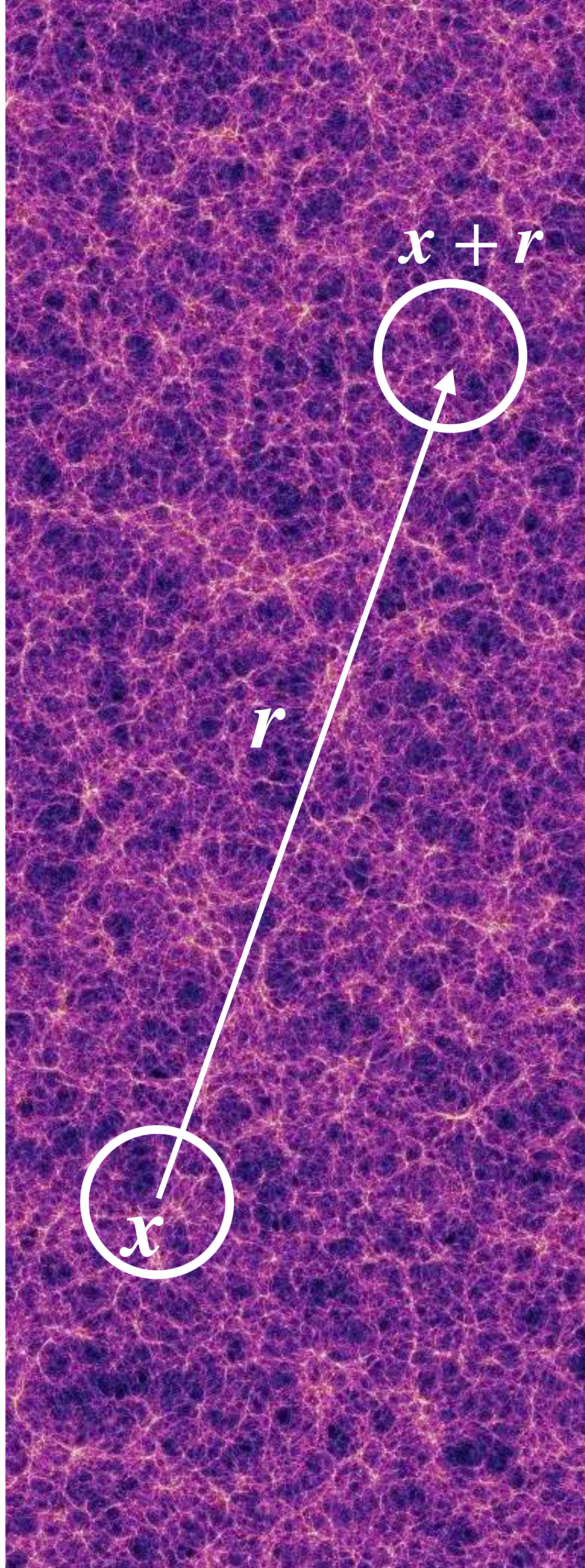
- Main observable: density contrast $\delta(\boldsymbol{x}) \equiv \rho(\boldsymbol{x})/\bar{\rho} - 1$

- Two-point correlation function

  - $\xi(r) = \langle \delta(\boldsymbol{x})\delta(\boldsymbol{x}+\boldsymbol{r}) \rangle_{\boldsymbol{x}}$ (in configuration space)

  - $P(k) = \frac{1}{V} \langle \delta(\boldsymbol{k})\delta(-\boldsymbol{k}) \rangle_{k}$ (in Fourier space)

- Two ways to model power spectrum
  N-**body simulation** or **Cosmological Perturbation Theory**



Standard (Eulerian) **Perturbation Theory**

Mass conservation Law $\dot{\delta} + \nabla \cdot [(1+\delta)\boldsymbol{v}] = 0$

Euler's equation $\dot{\boldsymbol{v}} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} + \frac{\dot{a}}{a}\boldsymbol{v} = -\nabla \phi$

Poisson's equation $\nabla^2 \phi = 4\pi G \bar{\rho}_{\rm m} a^2 \delta$

Solve the equations **perturbatively**

$$\delta(\boldsymbol{k}, z) = \sum_n \delta_n(k) D^n(z) = \delta_1(\boldsymbol{k}) D(z) + \delta_2(\boldsymbol{k}) D^2(z) + \delta_3(\boldsymbol{k}) D^3(z) + \cdots$$

$$\delta_n(k) = \int_{\boldsymbol{k}_1, \cdots, \boldsymbol{k}_n} (2\pi)^3 \delta^{\rm D}(\boldsymbol{k}_1 + \cdots + \boldsymbol{k}_n - \boldsymbol{k})\ F_n(\boldsymbol{k}_1, \cdots, \boldsymbol{k}_n)\ \delta_1(\boldsymbol{k}_1)\delta_1(\boldsymbol{k}_2)\cdots\delta_1(\boldsymbol{k}_n)$$

What is nEPT (n-th order Eulerian Perturbation Theory)?

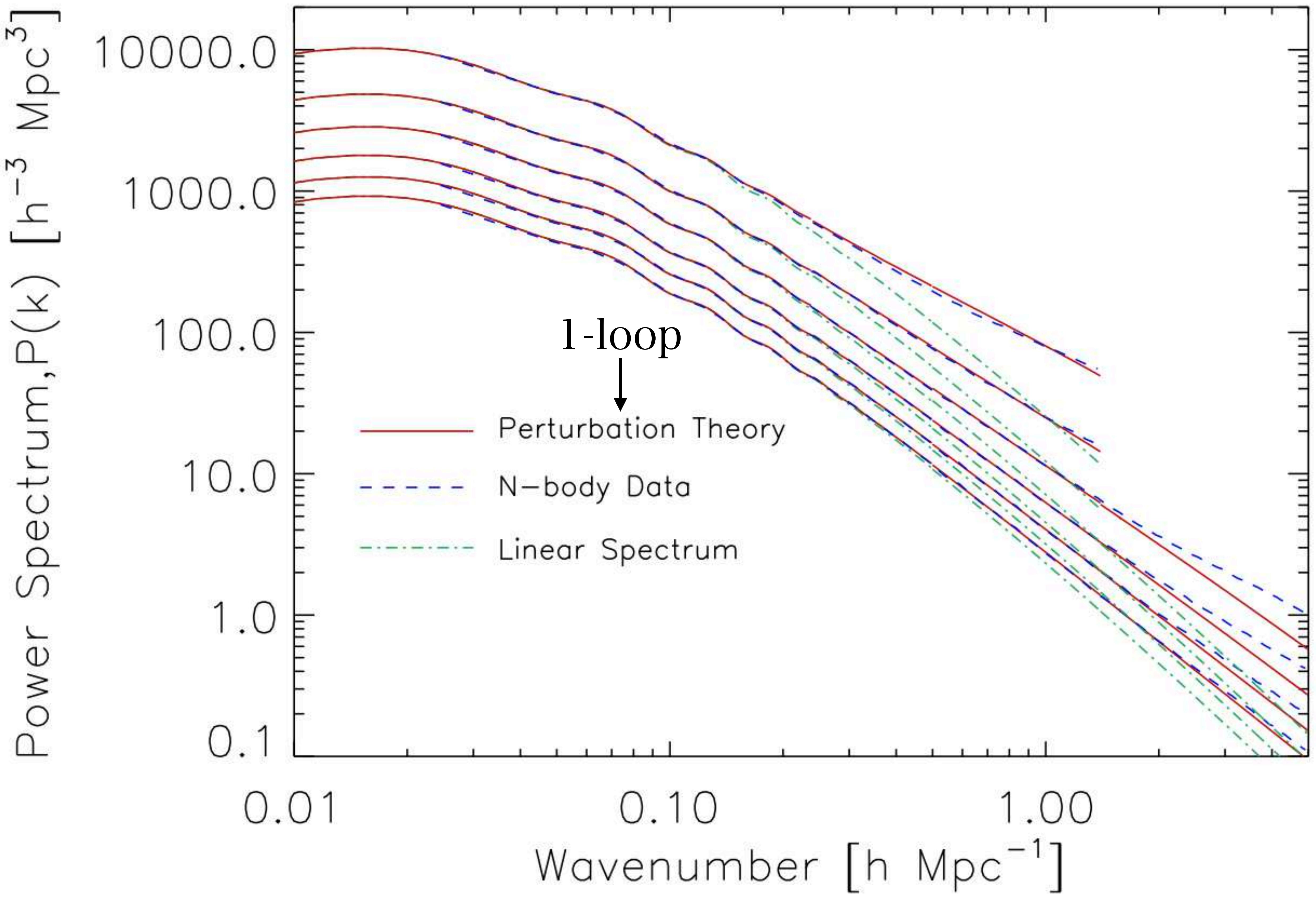
$$P_{nm}(k) = \left\langle \delta^{(n)}(\boldsymbol{k})\delta^{(m)}(-\boldsymbol{k}) \right\rangle$$

The Standard Perturbation Theory (SPT)
(order-by-order calculation of power spectrum):

$$P_{\rm Linear} = P_{11}$$

$$P_{1-\rm loop} = P_{11} + (P_{22} + 2P_{13})$$

$$P_{2-\rm loop} = P_{11} + (P_{22} + 2P_{13}) + (2P_{15} + 2P_{24} + P_{33})$$



Credit: Jeong & Komatsu (2006)

What is nEPT (n-th order Eulerian Perturbation Theory)?

$$P_{nm}(k) = \left\langle \delta^{(n)}(\boldsymbol{k})\delta^{(m)}\left(-\boldsymbol{k}\right)\right\rangle$$

The Standard Perturbation Theory (SPT)
(order-by-order calculation of power spectrum):

$$P_{\rm Linear} = P_{11}$$

$$P_{1-\rm loop} = P_{11} + (P_{22} + 2P_{13})$$

$$P_{2-\rm loop} = P_{11} + (P_{22} + 2P_{13}) + (2P_{15} + 2P_{24} + P_{33})$$

The new way: ***n*EPT**

First add the non-linear density perturbation to order n

$$\delta_{NL} = \delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \cdots + \delta_n$$

Then measure its power spectrum

$$P_{n\rm EPT} = \langle \delta_{NL}(\boldsymbol{k})\delta_{NL}(-\boldsymbol{k})\rangle'$$

$$\begin{aligned} P_{5\rm EPT} = {} & P_{11} \\ & + 2P_{12} + P_{22} \\ & + 2P_{13} + 2P_{23} + P_{33} \\ & + 2P_{14} + 2P_{24} + 2P_{34} + P_{44} \\ & + 2P_{15} + 2P_{25} + 2P_{35} + 2P_{45} + P_{55} \end{aligned}$$

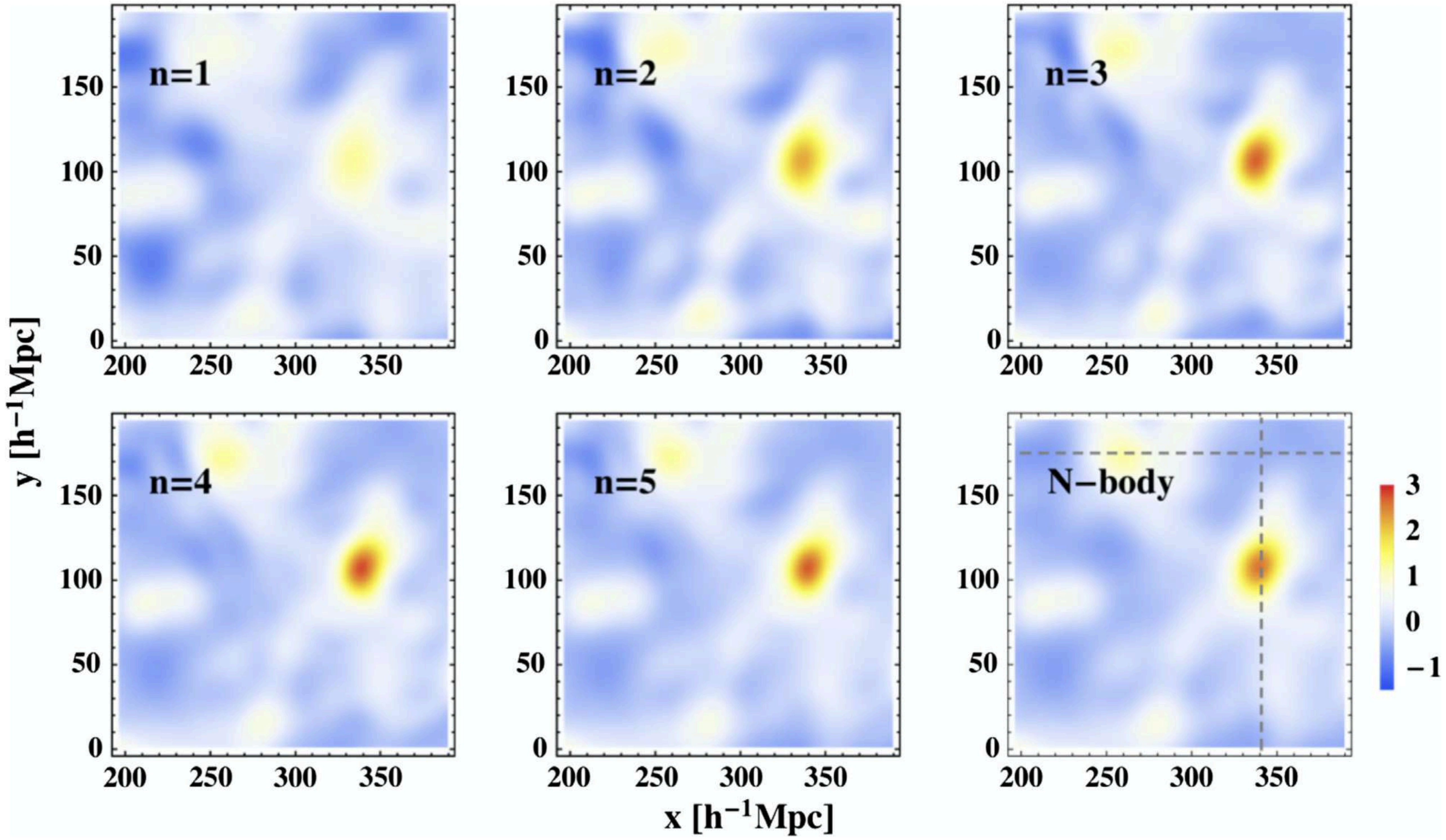
Taruya, Nishimichi, Jeong (2018)

Grid-based calculation of **S**tandard **P**erturbation **T**heory (GridSPT)

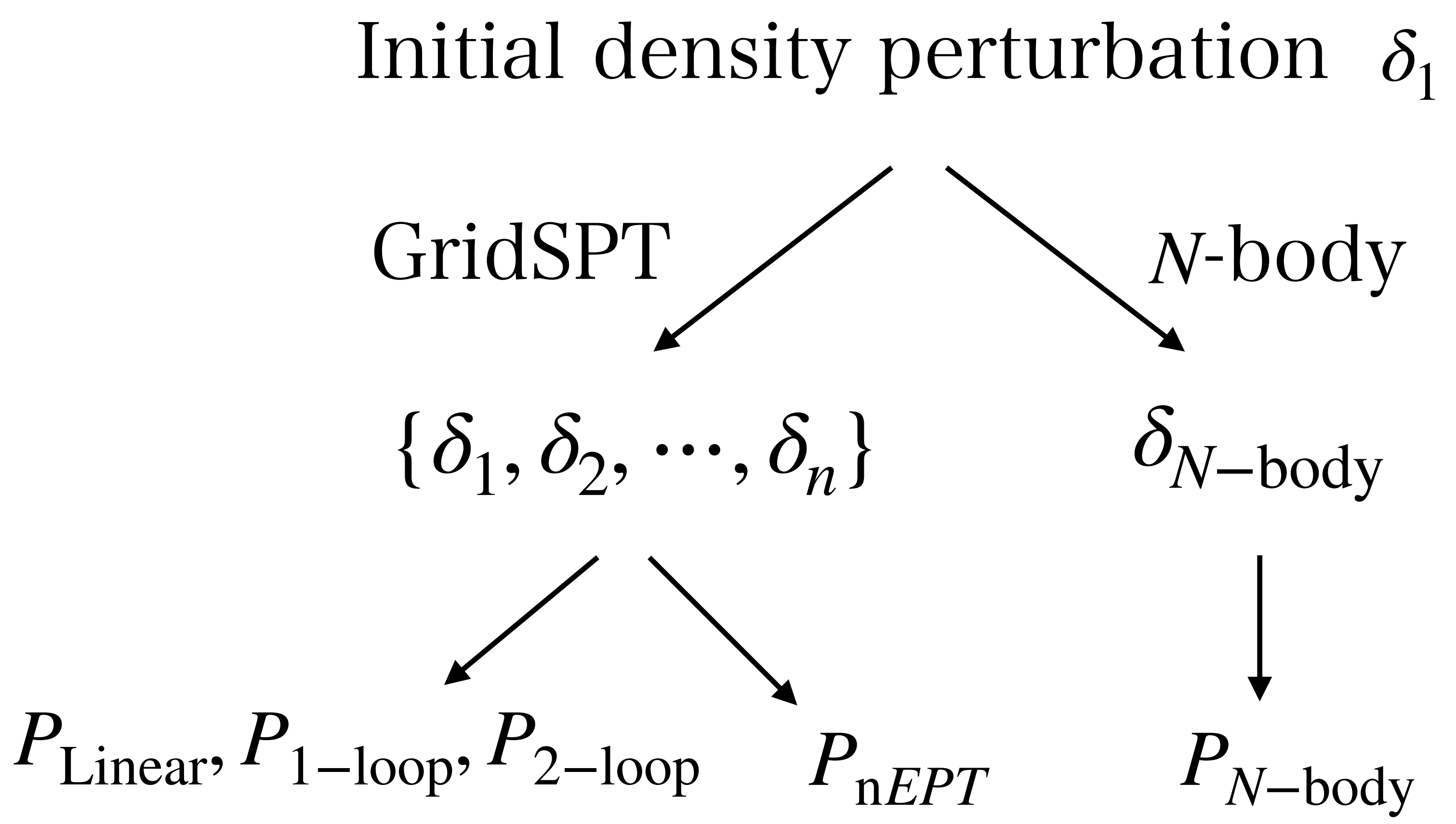
- The recursion relation for the n-th order density perturbation and velocity

∇ is $i\boldsymbol{k}$ in Fourier space!

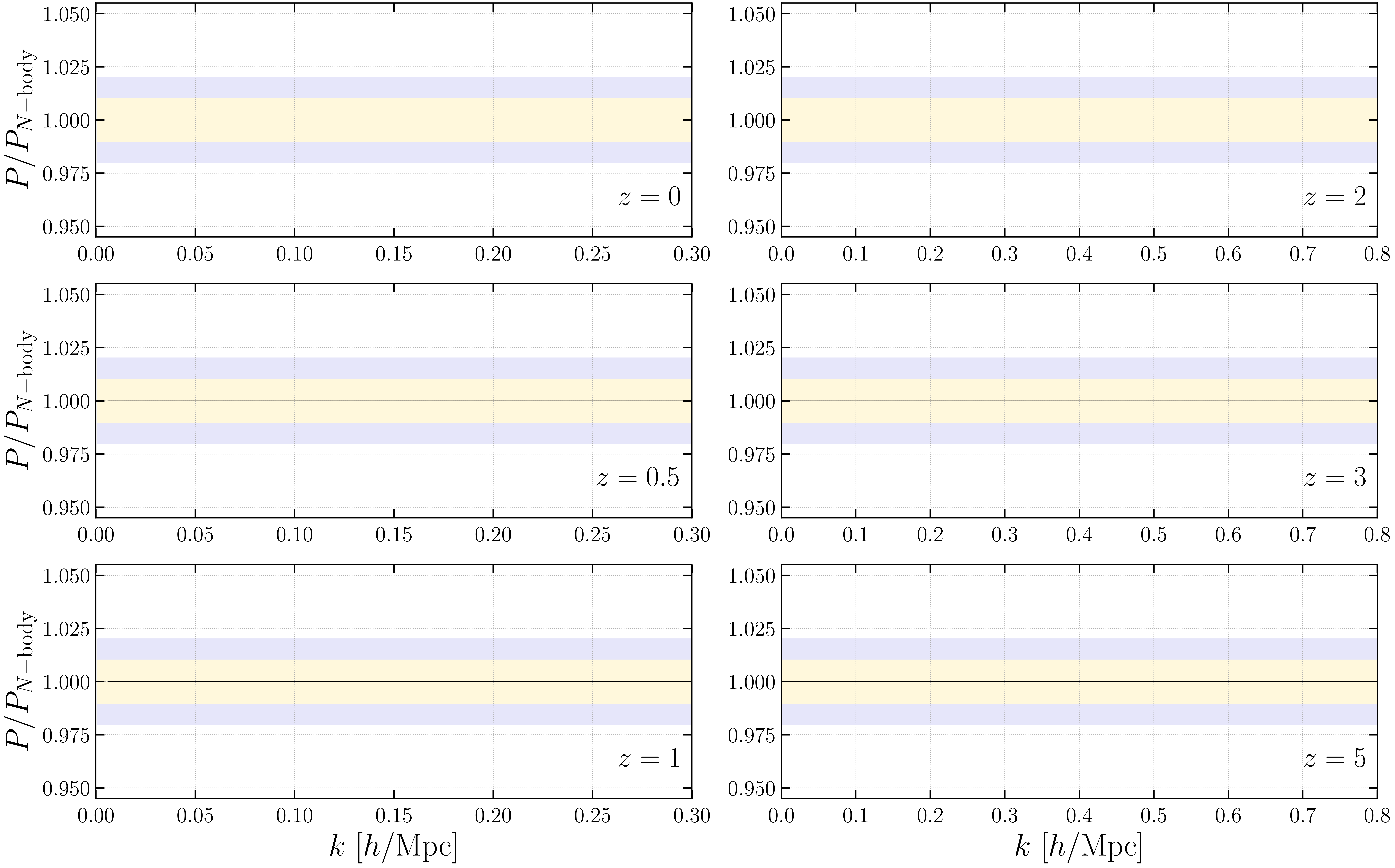
$$\begin{pmatrix} \delta_n(\boldsymbol{x}) \\ \theta_n(\boldsymbol{x}) \end{pmatrix} = \frac{2}{(2n+3)(n-1)} \begin{pmatrix} n+\frac{1}{2} & 1 \\ \frac{3}{2} & n \end{pmatrix} \sum_{m=1}^{n-1} \begin{pmatrix} (\nabla\delta_m)\cdot\boldsymbol{u}_{n-m} + \delta_m\theta_{n-m} \\ [\partial_j(\boldsymbol{u}_m)_k][\partial_k(\boldsymbol{u}_{n-m})_j] + \boldsymbol{u}_m\cdot(\nabla\theta_{n-m}) \end{pmatrix}$$



$\delta \equiv \rho/\bar{\rho} - 1$	density contrast
u	reduced velocity
$\theta \equiv \nabla\cdot\boldsymbol{u}$	divergence of velocity

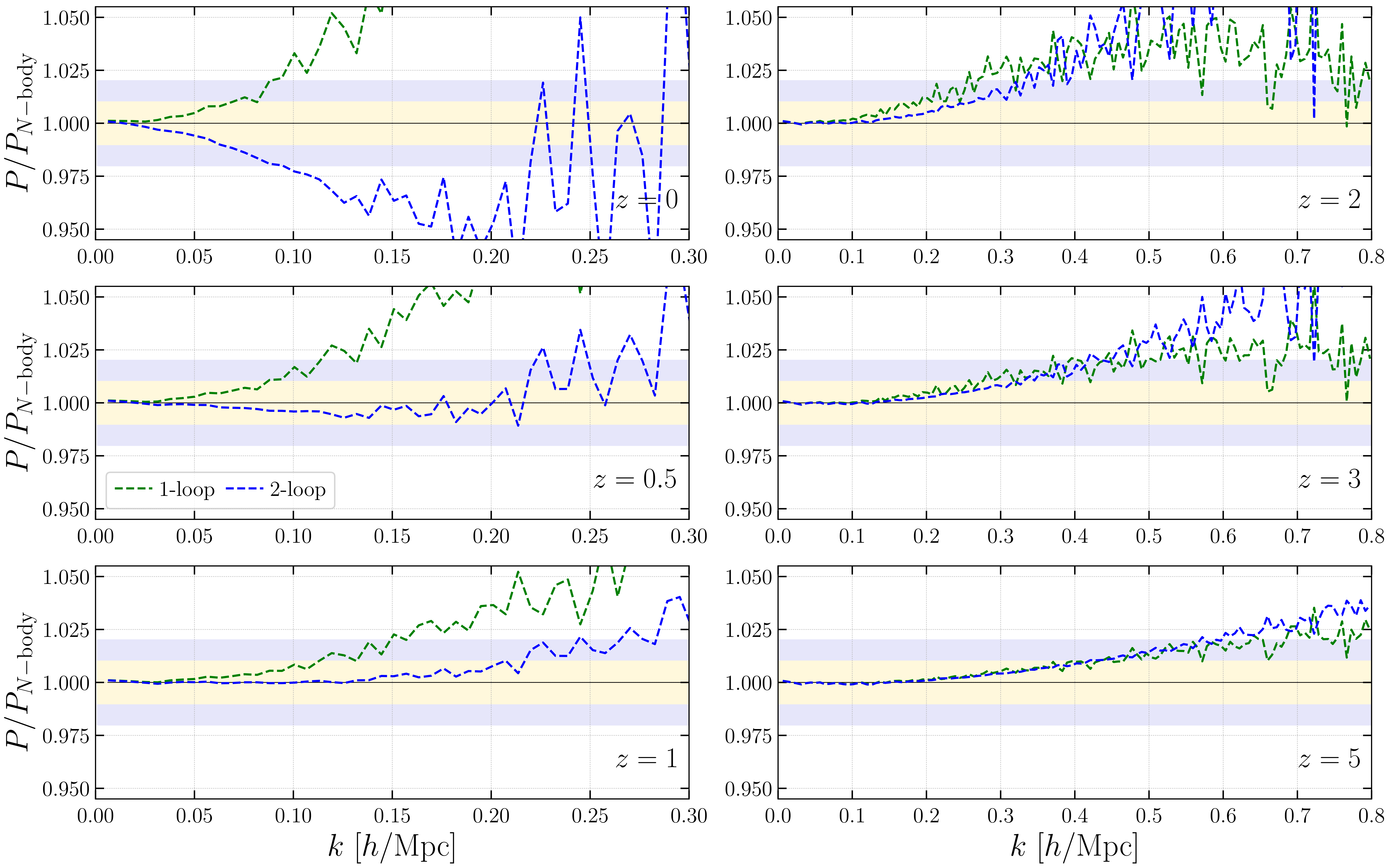


Result I: Matter Power Spectrum (WMAP cosmology)



We run GridSPT and N-body simulation from the same random initial condition.

Result I: Matter Power Spectrum (WMAP cosmology)



We run GridSPT and N-body simulation from the same random initial condition.

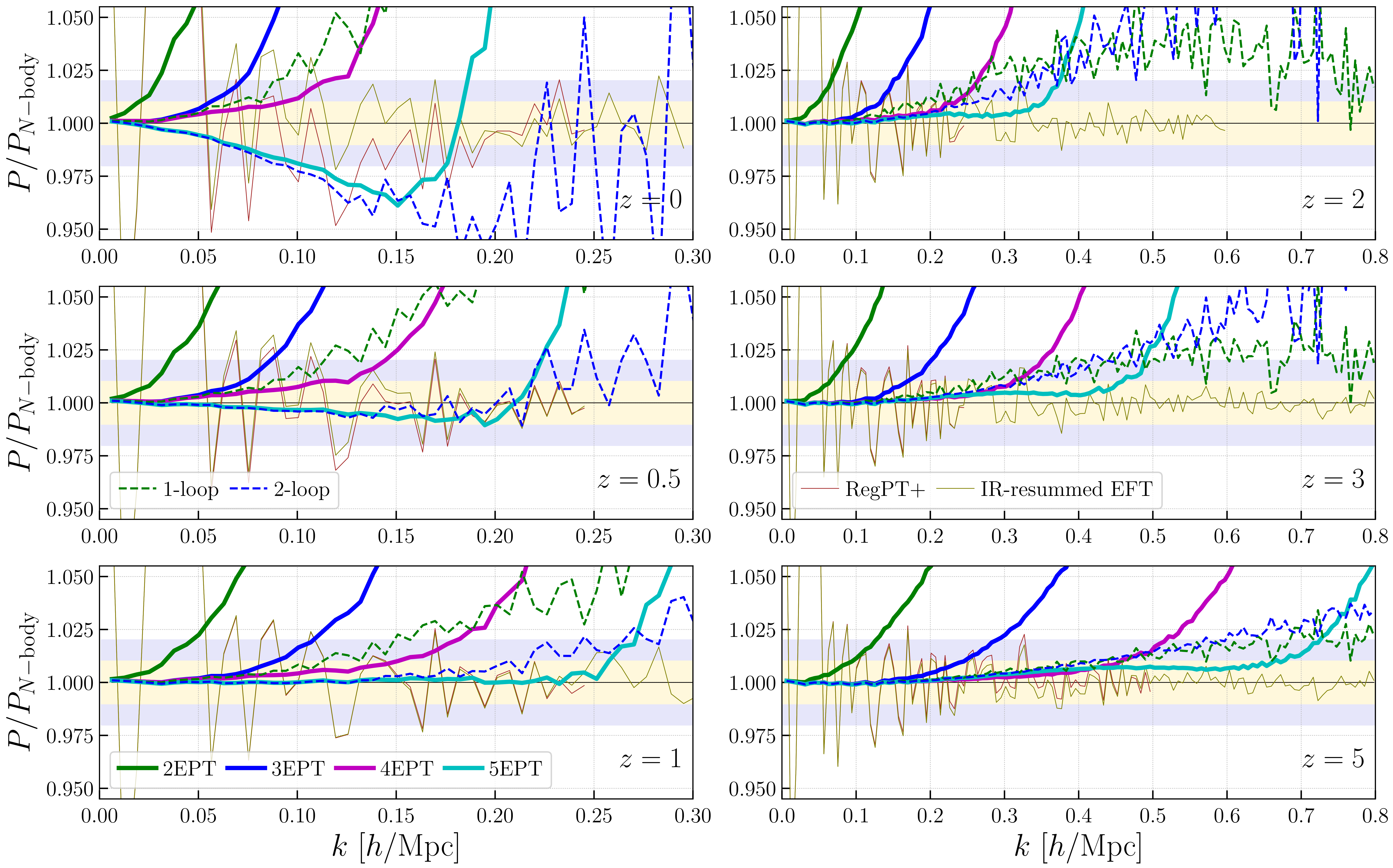
The Standard way (order-by-order):

$P_{\mathrm{Linear}} = P_{11}$

$P_{1-\mathrm{loop}} = P_{11} + (P_{22} + 2P_{13})$

$P_{2-\mathrm{loop}} = P_{11} + (P_{22} + 2P_{13}) + (2P_{15} + 2P_{24} + P_{33})$

Result I: Matter Power Spectrum (WMAP cosmology)



We run GridSPT and N-body simulation from the same random initial condition.

The Standard way (order-by-order):

$P_{\mathrm{Linear}} = P_{11}$

$P_{1-\mathrm{loop}} = P_{11} + (P_{22} + 2P_{13})$

$P_{2-\mathrm{loop}} = P_{11} + (P_{22} + 2P_{13}) + (2P_{15} + 2P_{24} + P_{33})$

The new way: *n*EPT

First add the non-linear density to order n

$\delta_{NL} = \delta_1 + \delta_2 + \delta_3 + \cdots + \delta_n$

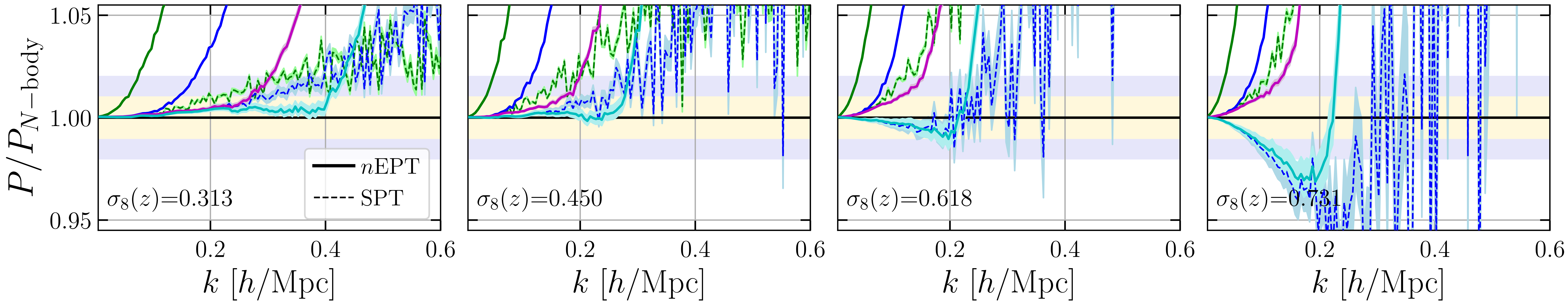
Then measure its power spectrum

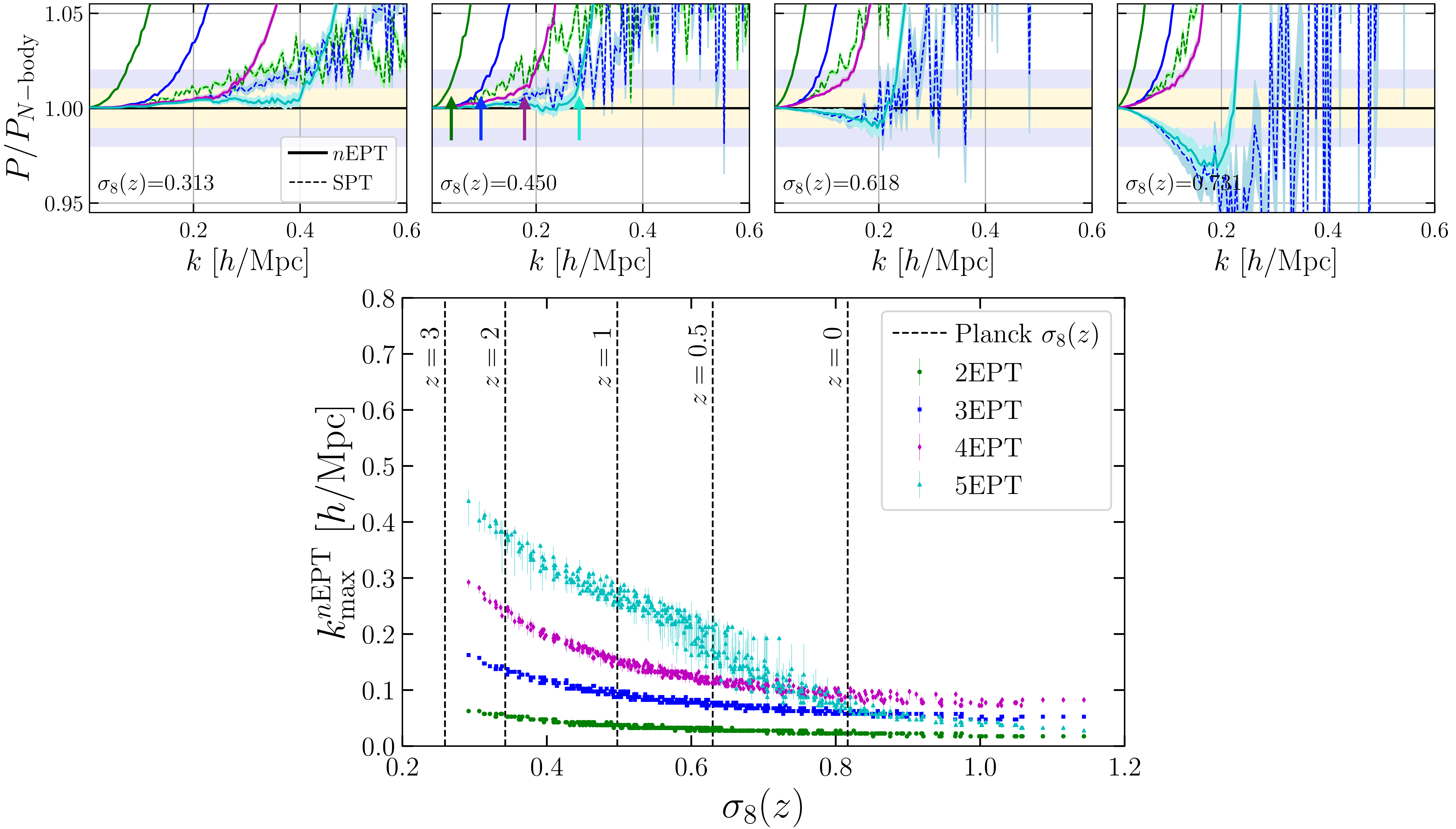
$P_{n\mathrm{EPT}} = \langle \delta_{NL}(\boldsymbol{k})\delta_{NL}(-\boldsymbol{k})\rangle'$

*n*EPT needs **NO** free parameters!

Result II: Matter Power Spectrum (In wCDM cosmology)

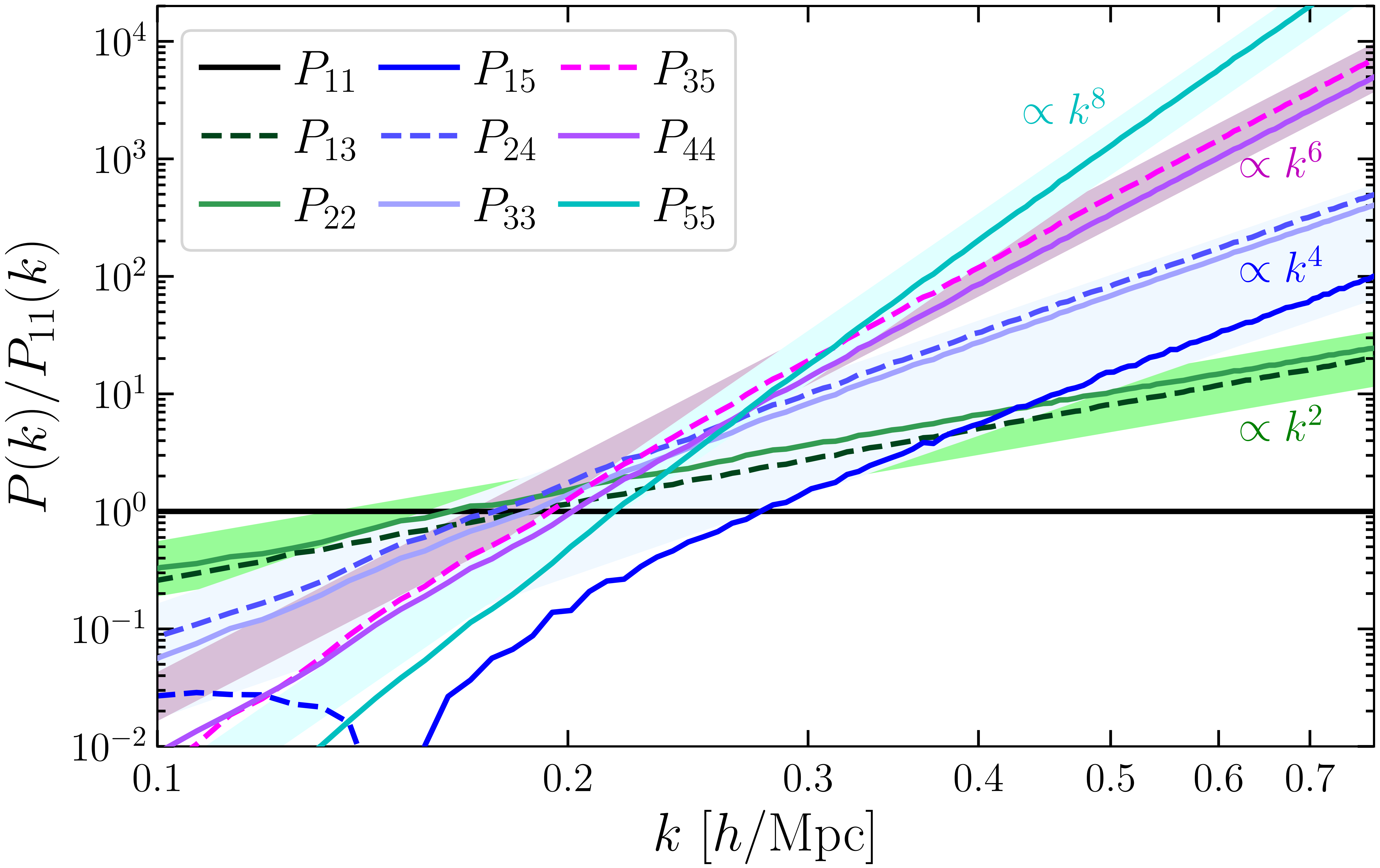
- nEPT also outperform SPT in general wCDM cosmologies!





$P/P_{N-\mathrm{body}}$
1.05
1.00
0.95
k [h/Mpc]
0.2
0.4
0.6
nEPT
SPT
$\sigma_8(z)$=0.313
$\sigma_8(z)$=0.450
$\sigma_8(z)$=0.618
$\sigma_8(z)$=0.731
$k_{\max}^{n\mathrm{EPT}}$ [h/Mpc]
$\sigma_8(z)$
$z = 3$
$z = 2$
$z = 1$
$z = 0.5$
$z = 0$
Planck $\sigma_8(z)$
2EPT
3EPT
4EPT
5EPT

Result III: EFT correction absorbs the UV dependence

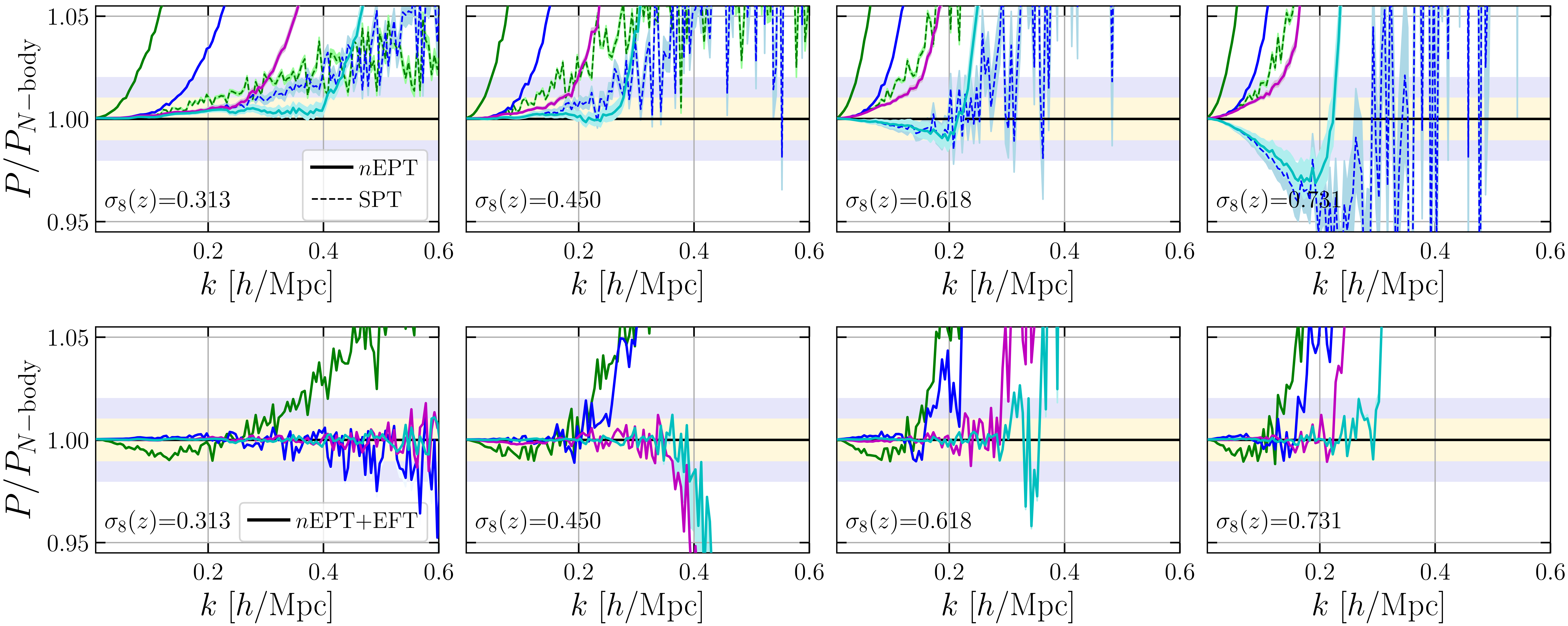


Free parameters

$$P_{n\mathrm{EPT+EFT}}(k) = P_{n\mathrm{EPT}}(k) - \sum_{i=1}^{n-1} \alpha_i k^{2i} P_{11}(k)$$

EFT-like counter terms

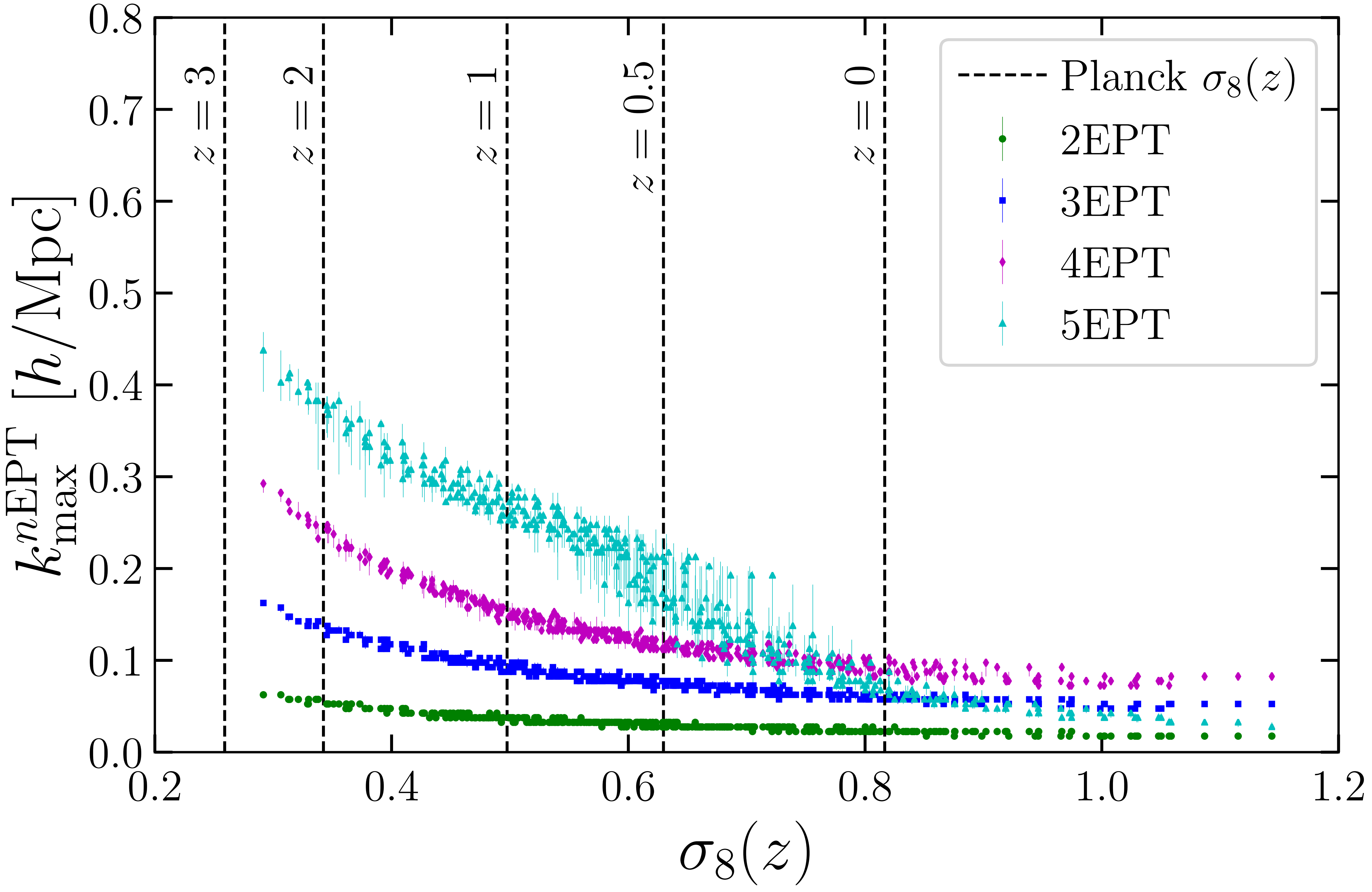
Result III: EFT correction absorbs the UV dependence

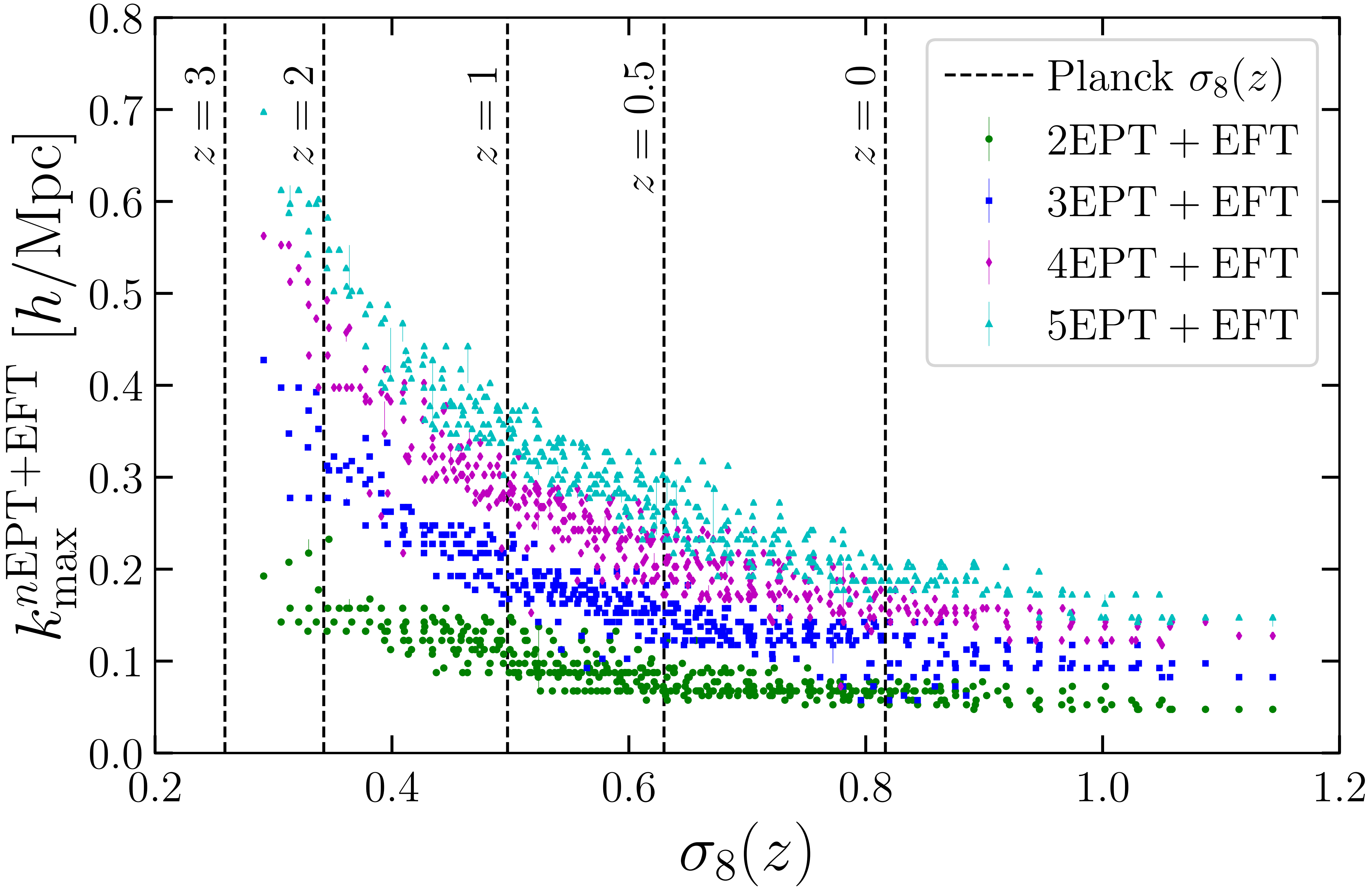


Result III: EFT correction absorbs the UV dependence

Free parameters

$$P_{n\mathrm{EPT+EFT}}(k) = P_{n\mathrm{EPT}}(k) - \sum_{i=1}^{n-1} \alpha_i k^{2i} P_{11}(k)$$





Summary

- nEPT can model matter power spectrum better than 1% accuracy to $k \approx 0.4\,h/\mathrm{Mpc}$ between $z = 2$ and $z = 3$(HETDEX mean redshift $\bar{z} \approx 2.7$)
- The UV-dependence of nEPT can be absorbed into counter terms, at the price of a few additional free parameters
- Future work
  - Bispectrum
  - Galaxy Bias
  - Redshift-space distortion