Modelling the emergence of cosmic anisotropy from non-linear structures

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Cosmology from Home, 2023

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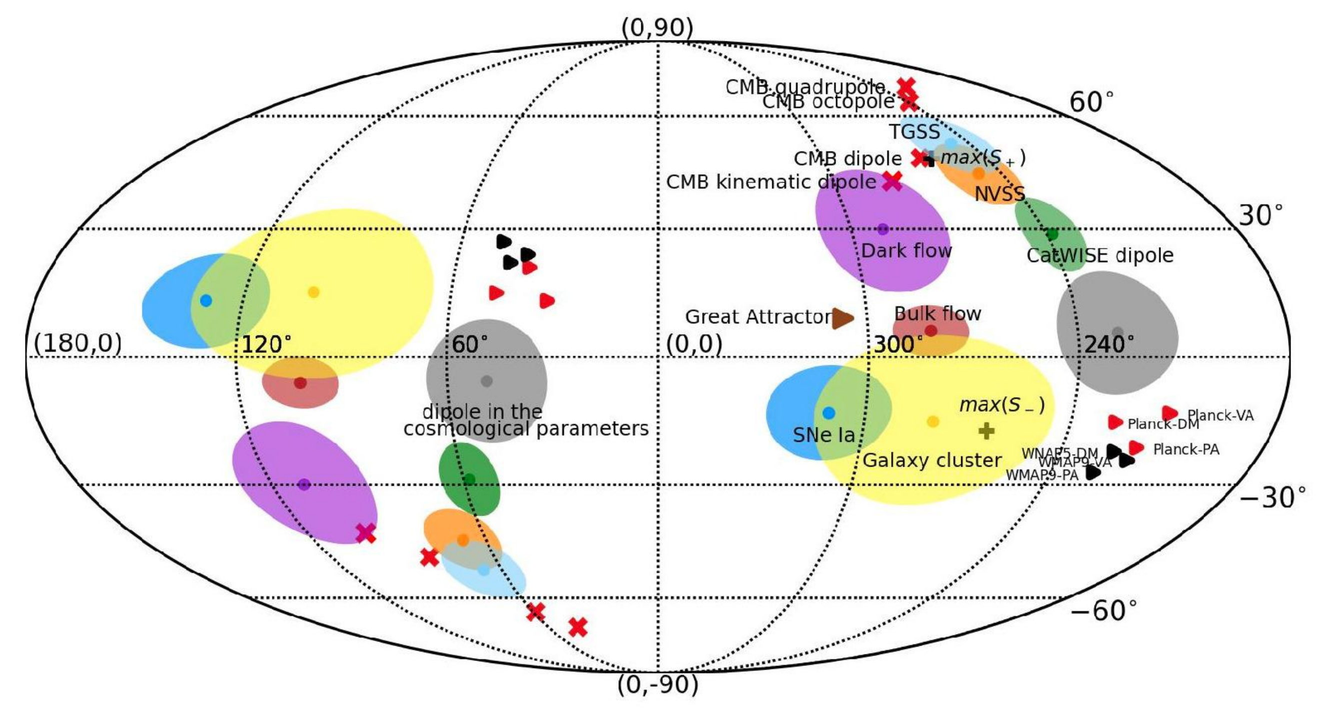
Based on arXiv:2302.05715

The (possibly) anisotropic universe?



Aluri et al. (2022)

How do you build a cosmological model?

Standard approach: fit to a model geometry

1. Assume a “background” cosmological model. Use observations (e.g. CMB, LSS, Type 1a supernovae) to fit the parameters of that model.
2. Add perturbations on top of it to describe inhomogeneous structure.

But..

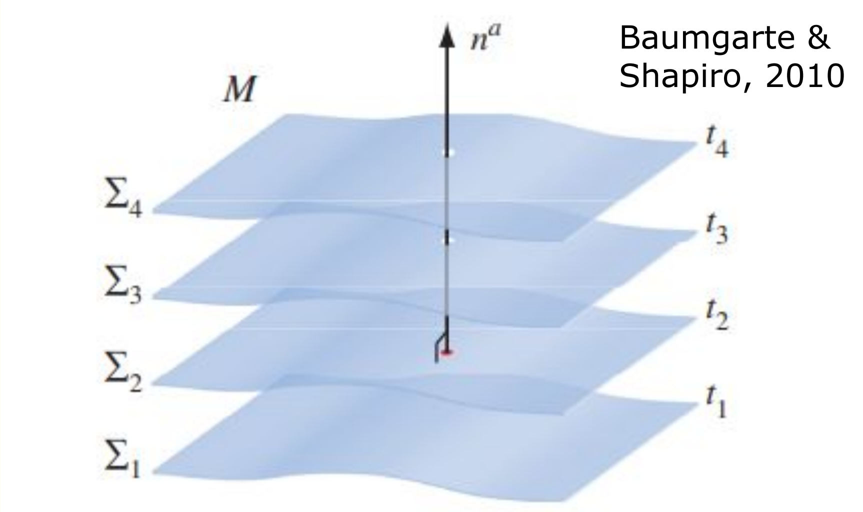
1. Hard to test the model if you assume it from the start!
2. The *evolution of the average* is **not** the same as the *average of the evolution*

$$G_{\mu\nu}\left(\left\langle g_{\alpha\beta}\right\rangle\right) \neq 8\pi G\left\langle T_{\mu\nu}\right\rangle$$

3. Standard anisotropic cosmologies (Bianchi, Kantowski-Sachs) tend to get more isotropic at late times.

Alternative approach: averaging

Average the Raychaudhuri equation, Hamiltonian and energy conservation equations over domains of spatial hypersurfaces (Buchert 1997+)



Baumgarte & Shapiro, 2010

$$3\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} = -\frac{1}{2}\left\langle \rho \right\rangle + \mathcal{Q}_{\mathcal{D}}$$

where

$$\mathcal{Q}_{\mathcal{D}} = \frac{2}{3}\left(\left\langle \Theta^2 \right\rangle - \left\langle \Theta \right\rangle^2\right) - 2\left\langle \sigma^2 \right\rangle$$

Isotropic expansion Shear

By construction, Buchert’s equations do not describe any anisotropic large-scale observables, such as a bulk flow. This is not a problem if we are satisfied that the universe is isotropic on large scales. But…

Buchert averaging scheme

1. Foliate the universe into spatial surfaces orthogonal to the worldlines of observers comoving with the matter flow.
2. Average scalars over domains of those surfaces.
3. Map those averaged scalars on to an homogeneous description.

$$\langle S \rangle := \frac{\int_{\mathcal{D}} \mathrm{d}^3x \sqrt{{}^{(3)}g}\, S}{\int_{\mathcal{D}} \mathrm{d}^3x \sqrt{{}^{(3)}g}}$$

FLRW model can be described by only 3 scalar quantities: isotropic expansion, energy density and spatial curvature.

So if you know how the averages of these evolve, you have an **emergent** cosmology that looks like an FLRW geometry…

But with an important caveat - there is a *back-reaction* effect.

Building an anisotropic model

We want to describe universes that are statistically homogeneous, but have a single direction of **large scale anisotropy**.

The cosmological models that are described by these conditions are the homogeneous, locally rotationally symmetric (LRS) cosmologies. (Ellis 1967, Stewart & Ellis 1968)

Homogeneous anisotropic cosmologies usually isotropise at late times. But we can exploit the inhomogeneity: apply Buchert’s averaging scheme.

Emergent anisotropy

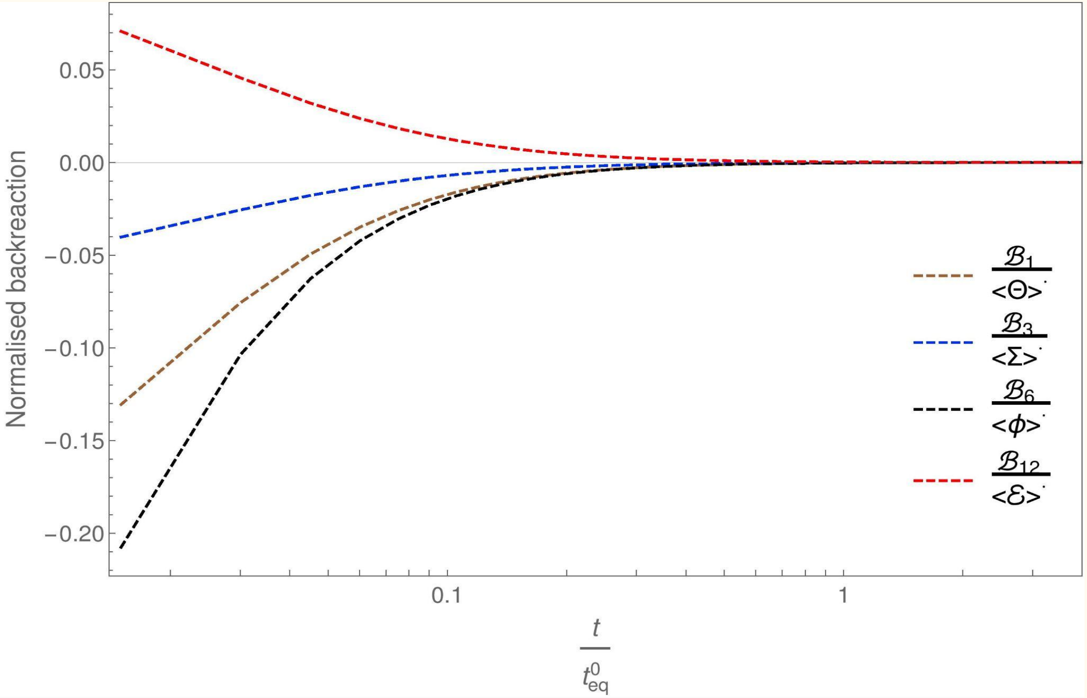
$$\langle Q\rangle^{\cdot} + \left(\frac{4}{3}\langle\Theta\rangle + \langle\Sigma\rangle\right)\langle Q\rangle - \langle\mathcal{A}\rangle\langle\mu + p + \Pi\rangle + \frac{3}{2}\langle\Pi\rangle\langle\phi\rangle = \mathcal{B}_{11}.$$

Inhomogeneity (on the RHS) may be able to **source** a large-scale bulk flow (LHS). Without a source on the RHS, the bulk flow decays at late times, roughly corresponding to the universe isotropising.

$$\begin{aligned}\mathcal{B}_{11} \;=\; & -\frac{1}{3}\mathrm{Cov}\left(\Theta, Q\right) - \left\langle m^{a} D_{a}\left(p + \Pi\right)\right\rangle - \left\langle M^{ab} D_{a}\Pi_{b}\right\rangle - \mathrm{Cov}\left(\frac{3}{2}\phi + \mathcal{A}, \Pi\right) \\ & -\mathrm{Cov}\left(\Sigma, Q\right) - \mathrm{Cov}\left(\mu + p, \mathcal{A}\right) - \left\langle\mathcal{A}_{a}\Pi^{a}\right\rangle + \left\langle\zeta_{ab}\Pi^{ab}\right\rangle + \left\langle\left(\alpha_{a} - \Sigma_{a}\right) Q^{a}\right\rangle + 2\left\langle a_{a}\Pi^{a}\right\rangle .\end{aligned}$$

Toy model

“Farnsworth cosmologies” (Farnsworth 1967). Exact, spatially homogeneous, solutions to Einstein’s equations, but with **bulk flow**. If we average in the matter rest spaces, we’ll see **back-reaction**, even though the spacetime has exact spatial homogeneity.



Summary

We have developed a formalism that describes the large-scale cosmological evolution as an homogeneous LRS model.

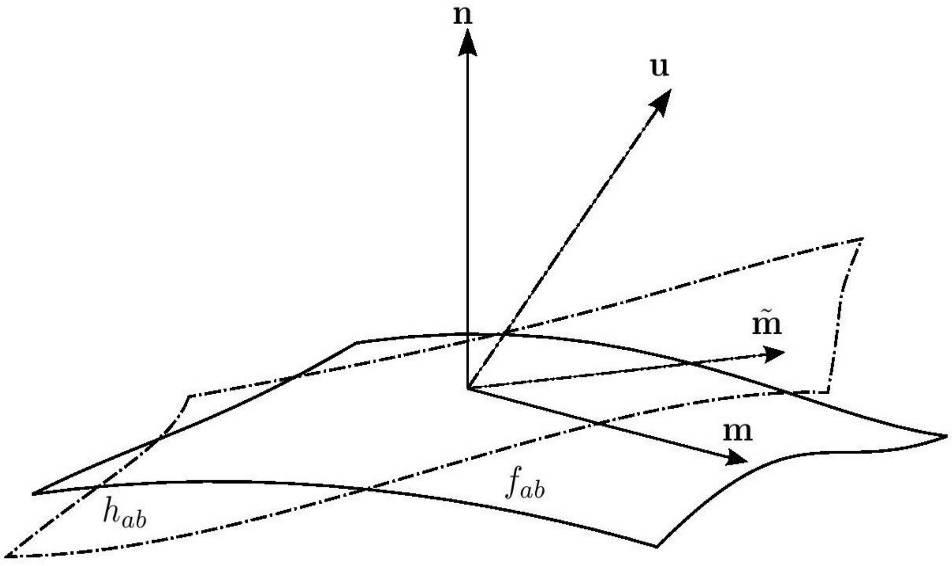
The equations contain a series of **back-reaction terms that can source anisotropy**, as well as the regular isotropic expansion.

More analysis is required…

Full details in paper (arXiv:2302.05715)

Back-up slides

Bulk flows



$$u_a = \gamma\left(n_a - v_a\right)$$

Change projection tensor

$$f_{ab} = g_{ab} + n_a n_b \longrightarrow h_{ab} = g_{ab} + u_a u_b$$

Project preferred spatial vector

$$m_a \longrightarrow \tilde{m}_a = k\, h_a{}^b m_b$$

Transform all scalars into new foliation

$$\tilde{\Theta} = \gamma\left[\Theta - \underset{\uparrow}{\kappa} - v_a\left(\dot{n}^a + \gamma^2 \underset{\uparrow}{\dot{v}^a}\right) + \gamma^2\left(\frac{1}{3}\kappa v^2 + \underset{\uparrow}{\beta_{ab}} v^a v^b\right)\right]$$

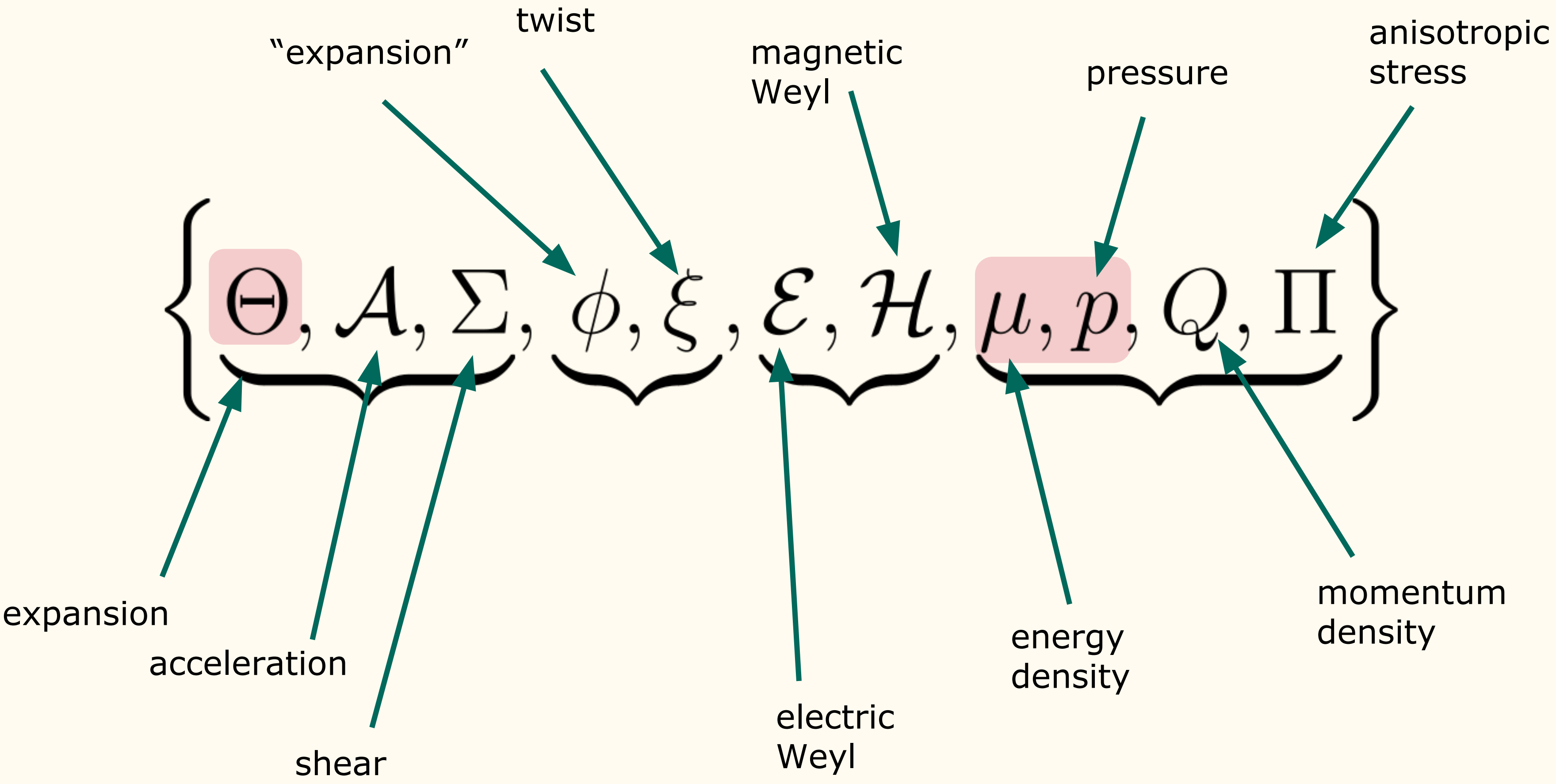
Expansion of **v**

Acceleration of **v**

Shear of **v**

Constructing an LRS model

1. Specify an irrotational timelike vector **n**. Not necessarily the 4-velocity of matter (bulk flows are allowed).
2. Specify a spacelike vector **m** (so that one can define scalars that describe large-scale anisotropy).
3. Project everything with respect to **n** and **m**: this is the called the 1+1+2 decomposition (Clarkson, arXiv: 0708.1398).
4. Average all the scalars according to the Buchert scheme.



$$\left\{\underbrace{\Theta,\mathcal{A},\Sigma},\underbrace{\phi,\xi},\underbrace{\mathcal{E},\mathcal{H}},\underbrace{\mu,p,Q,\Pi}\right\}$$

twist

“expansion”

magnetic
Weyl

pressure

anisotropic
stress

expansion

acceleration

shear

electric
Weyl

energy
density

momentum
density

Cosmological equations

$$\langle\Theta\rangle^{\cdot} - \langle\mathcal{A}\rangle\left(\langle\mathcal{A}\rangle + \langle\phi\rangle\right) + \frac{1}{3}\Theta^2 + \frac{3}{2}\Sigma^2 + \frac{1}{2}\left(\langle\mu\rangle + 3\langle p\rangle\right) - \Lambda = \mathcal{B}_1$$

where

$$\mathcal{B}_1 = \frac{2}{3}\mathrm{Var}\,\Theta - \frac{3}{2}\mathrm{Var}\,\Sigma + \mathrm{Var}\,\mathcal{A} + \mathrm{Cov}\left(\mathcal{A},\phi\right) - 2\left\langle\Sigma_a\Sigma^a\right\rangle - \left\langle\Sigma_{ab}\Sigma^{ab}\right\rangle + \left\langle M^{ab}D_a\mathcal{A}_b\right\rangle + \left\langle\left(\mathcal{A}_a - a_a\right)\mathcal{A}^a\right\rangle .$$

Source terms are modified from the exact case by the presence of *back-reaction.*

For the **isotropic expansion**, roughly the same story as Buchert’s equations (with more complicated-looking terms). What about anisotropic effects?

Farnsworth models

Exact cosmological solutions to Einstein’s equations, displaying:

1. Homogeneity
2. Anisotropy
3. Bulk flow: the homogeneous 3-spaces and the matter rest spaces are generally non-coincident.

$$ds^2 = -dt^2 + X^2(t + Cr)dr^2 + e^{-2r}Y^2(t + Cr)\left(dy^2 + dz^2\right)$$

$$\mathbf{u} = \partial_t$$

Matter 4-velocity

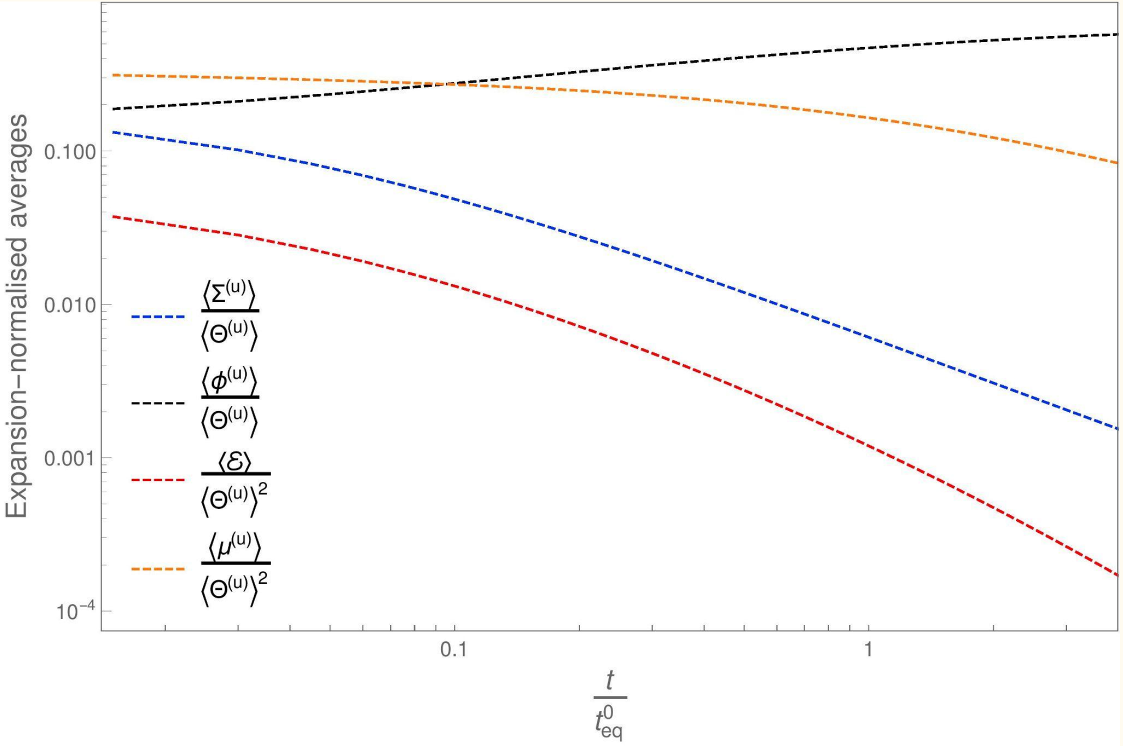
$$v = -\frac{C}{X}$$

Bulk flow

Surfaces of constant $t + Cr$ are homogeneous

For non-zero C, observers comoving with the matter will see apparent inhomogeneity

Back-reaction in the Farnsworth cosmology



Lapse function

We usually want our averaging surfaces to be level surfaces of some time coordinate.

This requires accounting for the lapse function N. We can do this by adapting our averaged scalars:

$$\langle\Theta\rangle \to \langle N\Theta\rangle \ , \quad \langle\mu\rangle \to \left\langle N^2\mu\right\rangle \ , \quad \langle\mathcal{E}\rangle \to \left\langle N^2\mathcal{E}\right\rangle \ .$$

N.B. If the matter source is presureless dust, then the lapse can be set to unity everywhere.