Primordial Black Holes sourced by a non-Gaussian curvaton component

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Motivation

Literature

$$\zeta = F(\zeta_G)$$

Power series expansion

$$\zeta = \zeta_G + \frac{3}{5} f_{NL} \zeta_G^2 + \frac{9}{25} g_{NL} \zeta_G^3 + \dots$$

Curvaton

$$\zeta = \log[\chi(\zeta_G)]$$

Ultra slow-roll

$$\zeta = \left(\frac{6}{5} f_{NL}\right)^{-1} \log\left(1 - \frac{6}{5} f_{NL} \zeta_G\right)$$

$+ \dots$

(Gow et al 2022; Ferrante et al 2022)

→ Light spectators with a vanishing mean can produce significant non-gaussian $P(\zeta)$

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Primordial Black Holes can be formed when

Gravitational collapse of large over-densities in the primordial density contrast field

$$\delta = \frac{\delta\rho}{\rho} \rightarrow \text{“compaction function” } C(r);$$

If $C > C_c$ at horizon entry;

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Hints on early universe physics

The Curvaton Model

We consider a light, $m^2 \ll H^2$, spectator χ with a quadratic potential

$$V(\chi) = \frac{1}{2}m^2\chi^2 \quad (1)$$

In the de Sitter equilibrium, χ is gaussian and (Starobinsky and Yokoyama, 1994)

$$\langle\chi\rangle = 0 \quad \text{and} \quad \langle\chi^2\rangle = \frac{3H^4}{8\pi^2 m^2} \quad (2)$$

However, fluctuations in the energy density

$$\frac{\delta\rho_\chi(\mathbf{x})}{\langle\rho_\chi\rangle} \equiv \frac{\rho_\chi(\mathbf{x}) - \langle\rho_\chi\rangle}{\langle\rho_\chi\rangle} = \frac{\chi(\mathbf{x})^2}{\langle\chi^2\rangle} - 1 \quad (3)$$

obey a Gaussian squared distribution! (Herranen, Markkanen and Tranberg, 2014; Markkanen, Rajantie, Stopyra and Tenkanen, 2019)

The spectrum of ζ_χ

Regime where self-interactions are important, the usual perturbative expansion fails for light fields → Stochastic formalism

The joint two point distribution can be written as (Markkanen, Rajantie, Stopyra and Tenkanen, 2019)

$$\rho_{2\chi}(\chi, \mathbf{r}, t; \chi', \mathbf{r}', t) = \left(\frac{m}{H^2}\right)^2 \psi_0(x)\psi_0(x') \sum_{n=0}^{\infty} \psi_n(x)\psi_n(x') \left[a(t)H|\mathbf{r}-\mathbf{r}'|\right]^{-2\Lambda_n/H} \tag{4}$$

where

$$x = \frac{m\chi}{H}, \quad \Lambda_n = \frac{nm^2}{3H}, \quad \psi_n(x) = \frac{1}{\sqrt{n!2^n}}\left(\frac{4\pi}{3}\right)^{1/4} e^{-2\pi^2 x^2/3} H_n\left(\frac{2\pi x}{\sqrt{3}}\right)$$

$H_n(x) = (-1)^n e^{x^2/2} \frac{d^n}{dx^n} e^{-x^2/2}$, and the eigenfunctions satisfy the orthonormal condition

$$\int_{-\infty}^{\infty} dx\, \psi_n(x)\psi_m(x) = \delta_{nm}$$

The spectrum of ζ_χ

The two point function of ζ_χ is written as

$$\langle \hat{\zeta}_\chi(\mathrm{r})\hat{\zeta}_\chi(\mathrm{r}')\rangle = \int_{-\infty}^{\infty} d\chi \int_{-\infty}^{\infty} d\chi' \hat{\zeta}(\chi)\hat{\zeta}(\chi')\rho_{2\chi}(\chi, \mathbf{r}; \chi', \mathbf{r}')$$

$$= \sum_{n=1}^{\infty} \left(\int_{-\infty}^{\infty} dx \, \frac{1}{3} \ln x^2 \psi_0 \psi_n \right)^2 \left[a(t) H \,|\, \mathrm{r} - \mathrm{r}' \,|\, \right]^{-2nm^2/3H^2} \quad (5)$$

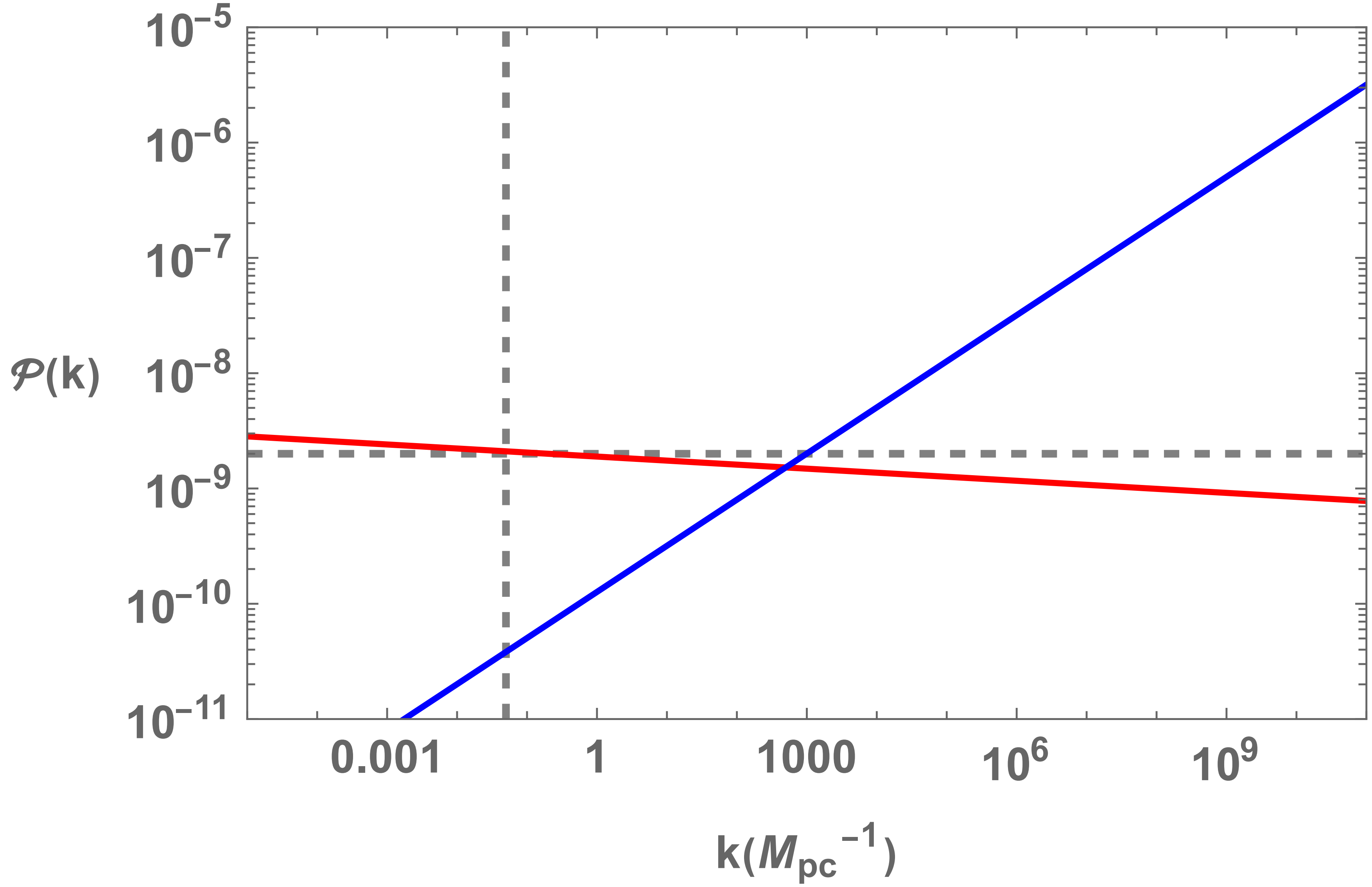
The power spectrum $\mathscr{P}_{\zeta_\chi}$ is the Fourier transform of equal time two point correlation function

$$\mathscr{P}_{\zeta_\chi} = \frac{k^3}{2\pi^2} \int d^3 r e^{-i\mathrm{k}\cdot\mathrm{r}} G(\Delta r) = \frac{2^{3-\frac{4m^2}{3H^2}}}{9\sqrt{\pi}} \frac{\Gamma\left(\frac{3}{2} - \frac{2m^2}{3H^2}\right)}{\Gamma\left(\frac{2m^2}{3H^2}\right)} \left(\frac{k}{aH}\right)^{\frac{4m^2}{3H^2}} \quad (6)$$

Requiring that $\mathscr{P}_{\zeta_\chi}(0.05\mathrm{Mpc}^{-1}) < 10^{-10}$, restrict the space of parameters of the curvaton model to

$$\frac{m^2}{H^2} \geqslant 0.28 \quad (7)$$

The spectrum of ζ_χ



Power spectrum for the curvaton (blue) with $m^2/H^2 = 0.3$ and a power law power spectrum sourced by the inflation field (red) with

$$\mathscr{P}_{\zeta_r}(k) = A_s \left(\frac{k}{0.05}\right)^{n_s - 1} , \quad A_s = 2.10 \times 10^{-9} , \quad n_s = 0.965$$

PDF of the density fluctuations

PBH formation criterion: compaction function C (Young, Musco, and Byrnes, 2019)

$$C(r) = C_l(r) - \frac{5+3w}{12(1+w)} C_l(r)^2 \quad (8)$$

where C_l is the linear part of the compaction function, which is determined by the curvature perturbation via

$$C_l(r) = -f(w) r \zeta'(r) \,, \quad f(w) = \frac{6(1+w)}{5+3w} \quad (9)$$

→ $P_{C_l}(C_l)$ to determine PBH properties !

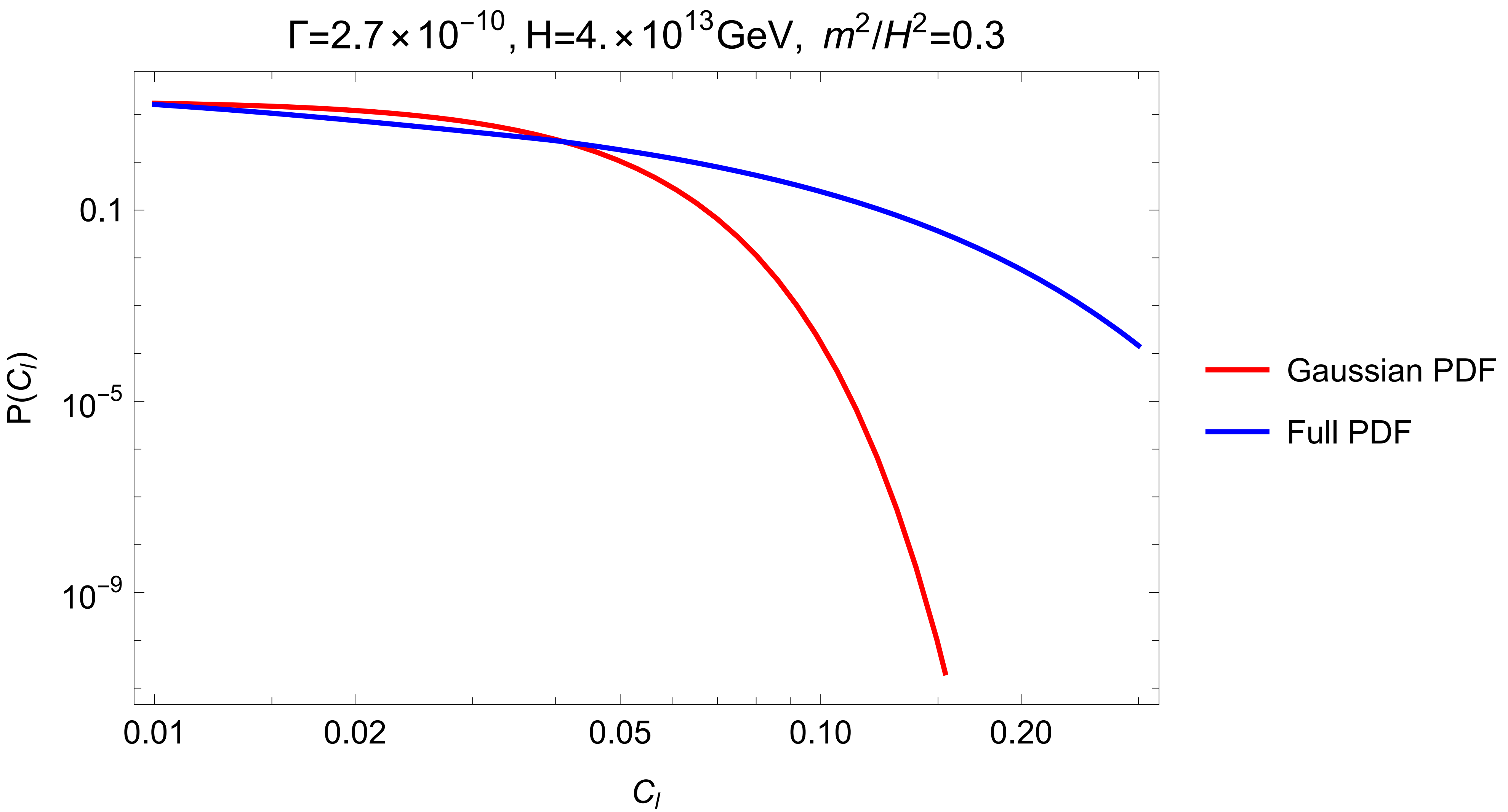
$$P_{C_l}(C_l) = \frac{1}{2\pi r f(w) |\Sigma(r)|^{1/2}} \times \int \frac{d\chi}{|\partial_\chi \zeta|} exp \left[\frac{1}{2|\Sigma(r)|} \left(\sigma^2_{\chi'\chi'}(r)\chi^2 + \frac{2\sigma^2_{\chi\chi'}\chi C_l(r)}{r f(w) \partial_\chi \zeta} + \frac{2\sigma^2_{\chi\chi}(r) C_l^2}{(r f(w) \partial_\chi \zeta)^2} \right) \right] \quad (10)$$

Recipe for computing the probability of forming PBHs:

- Use the power spectrum to compute $\sigma^2_{\chi\chi}(r)$, $\sigma^2_{\chi\chi'}(r)$ and $\sigma^2_{\chi'\chi'}(r)$;
- Perform the numerical integral to find $P_{C_l}(C_l, r)$;
- Obtain the contribution of PBHs to the total energy at the collapse time $\beta(r)$;
- Obtain the fraction of dark matter density constituted by PBHs at the present time f_{PBH};

Results

$$\mathscr{P}_\chi(k) = \left(\frac{H}{2\pi}\right)^2 \left(\frac{k}{k_e}\right)^{3-2\nu} \frac{2^{2\nu-1}\Gamma(\nu)^2}{\pi}, \qquad \nu = \sqrt{\frac{9}{4} - \frac{m^2}{H^2}} \quad (11)$$



The full probability distribution function $P_{C_l}(C_l, r_{\rm dec})$ and a Gaussian distribution with the same variance $\langle C_l^2 \rangle = 4.3 \times 10^{-4}$

Results

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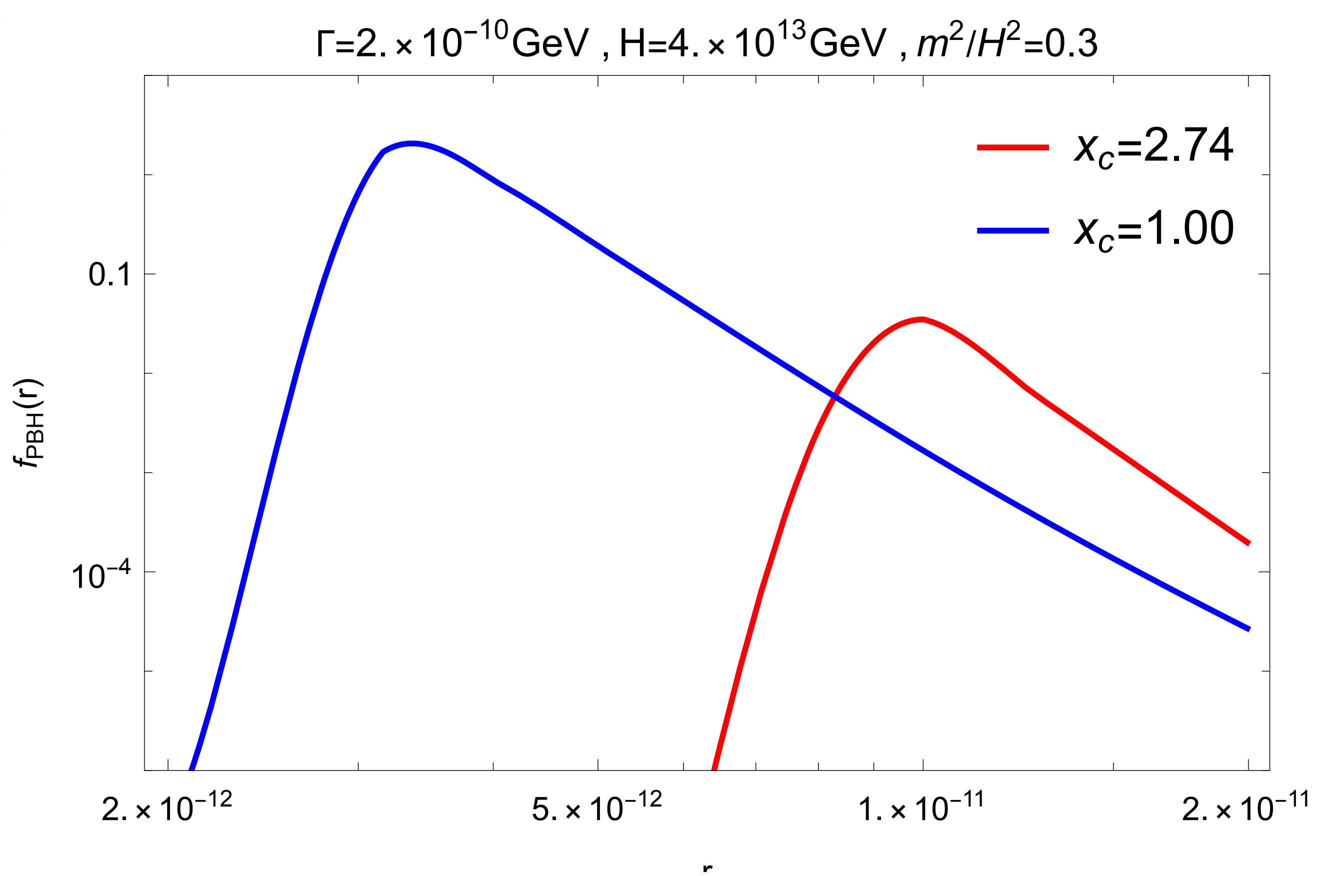
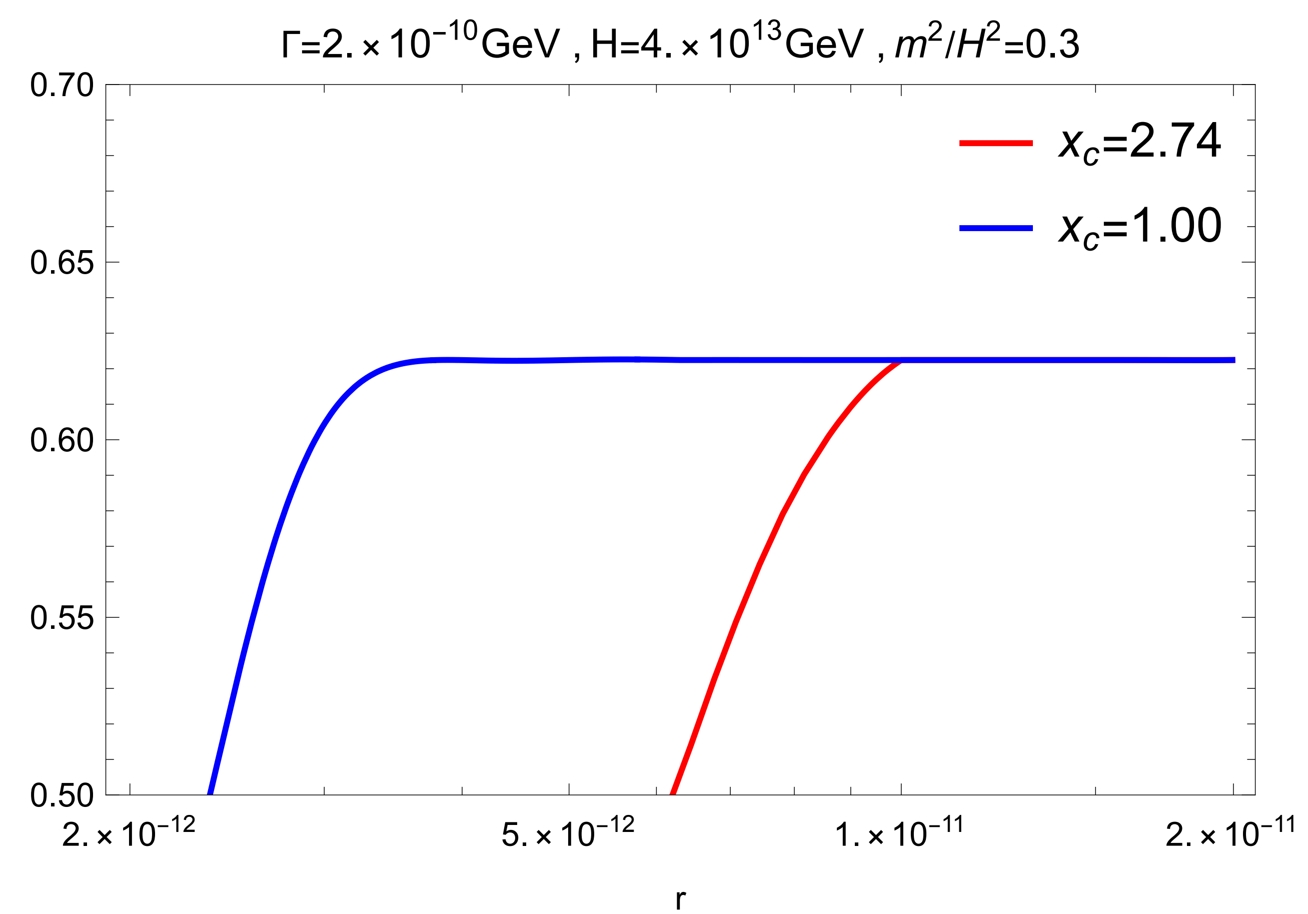
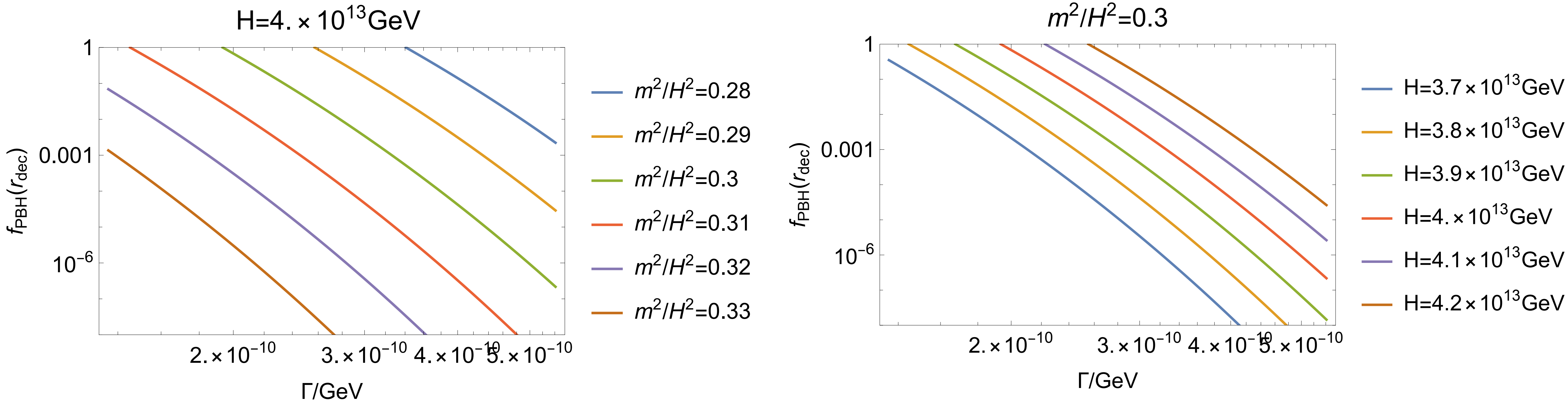


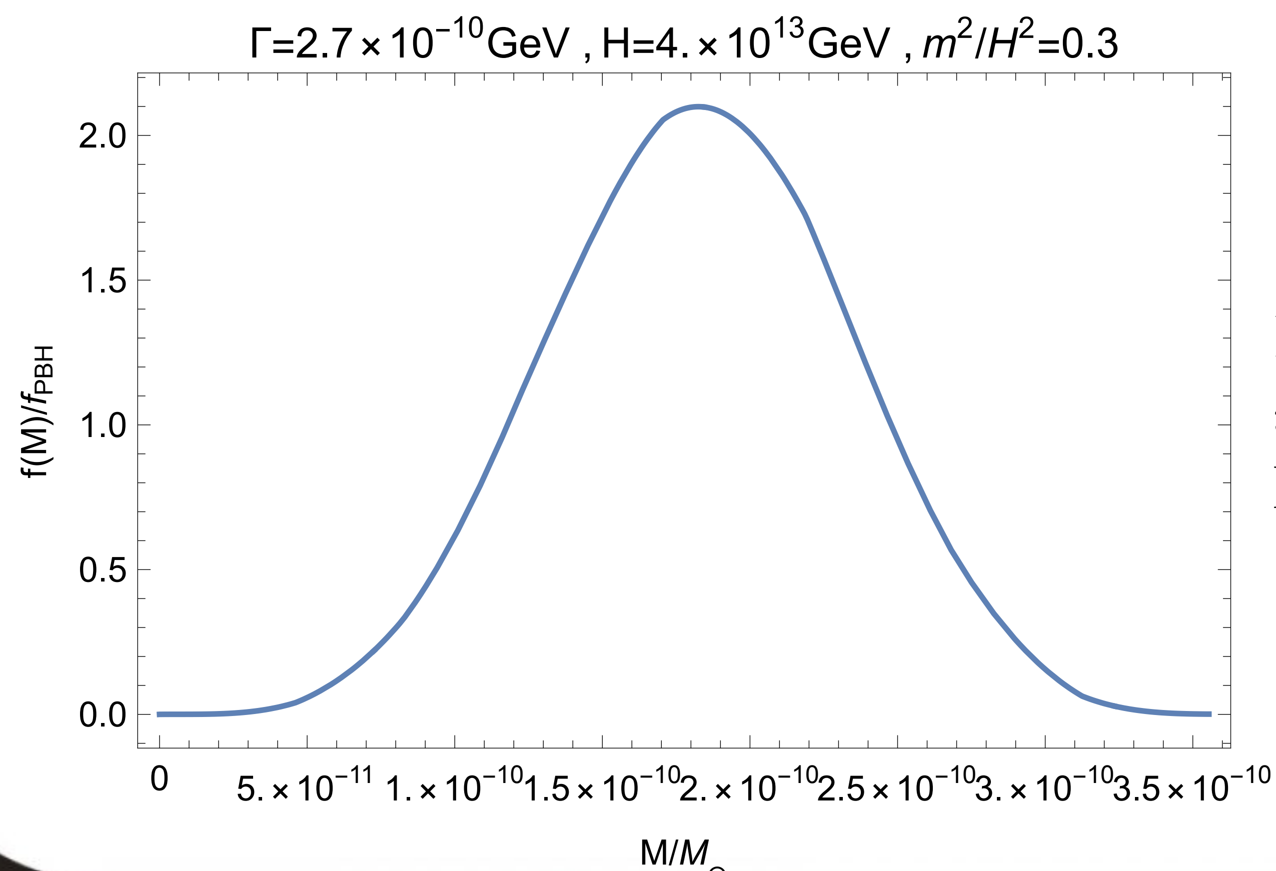


$f_{\rm PBH}$ and Ω_χ as function of the coarse-graining scale r for the power law curvaton spectrum with the Hubble scale at the end of inflation $H = 4 \times 10^{13}$ GeV, the curvaton mass $m^2 = 0.3H^2$, and instant curvaton decay at the temperature $T_{\rm dec} = 1.210^4$ GeV.

Results



$f_{\rm PBH}(r_{\rm dec})$ with $r_{\rm dec} = 2.74(aH)^{-1}_{\rm dec}$ as function of the curvaton decay rate Γ. In the left panel the curvaton mass m is varied and in the right plot the Hubble scale during inflation H is varied.



PBH mass distribution $f(M)$ obtained setting $r = r_{\rm dec}$ and choosing $m^2/H^2 = 0.3$, $H = 4.0 \times 10^{13}$ GeV, and $\Gamma = 2.7 \times 10^{-10}$ GeV.

Final Considerations

- We assumed a vanishing mean for the curvaton $\langle \chi \rangle = 0$ such that the curvature perturbation ζ_χ induced by χ has no leading Gaussian part;
- A phenomenological approach assuming the PBH mass generated by a spherical overdensity coarse-grained over radius r, and corresponding to the compatification value C;

For the maximum inflationary Hubble scale $H \sim 5 \times 10^{13}$ GeV

$\rightarrow m^2/H^2 \gtrsim 0.28$ $(0.28H^2 < m^2 < \frac{9}{4}H^2)$;

Two competitive effects:

ζ and Ω_χ grow until the curvaton decays. For t_r before decay, $\uparrow r$ makes $\zeta(t_r)$ larger;

$\uparrow r \rightarrow \sigma^2_{\chi\chi}(r) \downarrow$ which suppresses large $\chi(r)$ values;

The effect of curvature perturbations with a non-Gaussian leading term can change significantly PBHs abundance!

Final Considerations

1- LISA detection;

2- Full quantitative analysis of the PBH mass require extending the computation of $P_{C_l}(C_l)$ beyond the monopole truncation;

3- Fine tuning: $H \sim 10^{13}$ GeV; Instant decay time; $m^2/H^2 \gtrsim 0.28$;

Could the limit $f_{PBH} \leqslant 1$ put a constraint on the maximum value of Ω_χ?

Muito obrigada!