Effective field theories for dark matter pairs in the early universe

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Cosmology from Home

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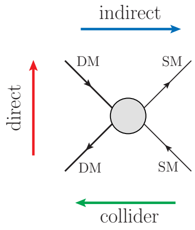
Particle interpretation of DM and freeze-out

- DM from many compelling (gravitational) observations

- DM as a particle: many candidates (Bertone and Hooper [1605.04909])
- Any model has to comply with

$$\Omega_{\mathrm{DM}} h^2(M_{\mathrm{DM}}, M_{\mathrm{DM'}}, \alpha_{\mathrm{DM}}, \alpha_{\mathrm{SM}}) = 0.1200 \pm 0.0012$$

◇ from CMB anysotropies with ΛCDM *Planck Collab. Results 2018*



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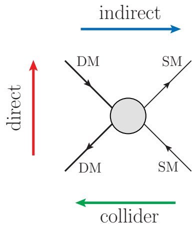
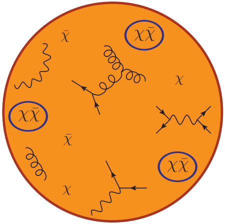




Thermal freeze-out Gondolo and Gelmini (1991)

- Boltzmann equation for DM (χ)

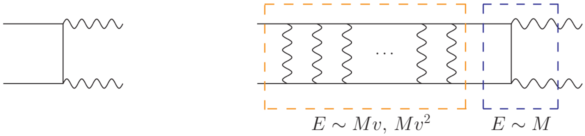
$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma v \rangle (n_\chi^2 - n_{\chi,\rm eq}^2)$$

- relevant processes $\chi\chi \leftrightarrow \mathrm{SM\,SM}$, $\chi\chi \leftrightarrow \chi'\chi'$
- decoupling from $H \sim n_{\rm eq} \langle \sigma_{\rm ann} v_{\rm rel} \rangle$

$$H \simeq \frac{T^2}{M_{\rm Pl}}\,, \quad \langle \sigma_{\rm ann} v_{\rm rel} \rangle \simeq \frac{\alpha^2}{M^2}\,, \quad \frac{T}{M} \approx \frac{1}{25}$$

Going towards a realistic picture

- DM and/or coannihilating partners interact with gauge bosons and scalars



- repeated soft interactions: Sommerfeld enhancement and bound states Hisano, Matsumoto, Nojiri [hep-ph/0212022], [hep-ph/0307216]; B. von Harling and K. Petraki [1407.7874]; Beneke, Hellmann, Ruiz-Femenia [1411.6924] ...

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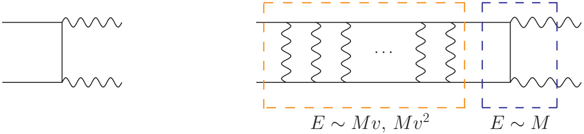

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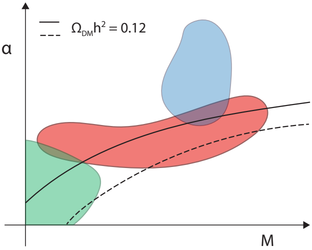
$$\frac{dn_X}{dt} + 3Hn_X = -\langle \sigma_{\rm eff}\, v_{\rm rel} \rangle (n_X^2 - n_{X,\rm eq}^2) \,,$$

$$\langle \sigma_{\rm eff}\, v_{\rm rel} \rangle = \langle \sigma_{\rm ann}\, v_{\rm rel} \rangle + \sum_n \langle \sigma^n_{\rm bsf}\, v_{\rm rel} \rangle \frac{\Gamma^n_{\rm ann}}{\Gamma^n_{\rm ann} + \Gamma^n_{\rm bsd}}$$

J. Ellis, F. Luo, and K. A. Olive [1503.07142], M.Garny and J.Heisig [2112.01499]



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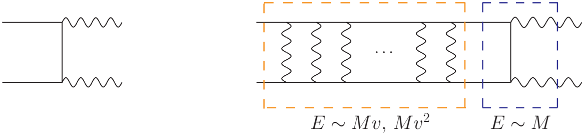

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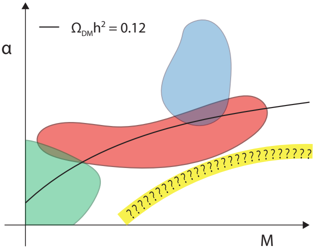
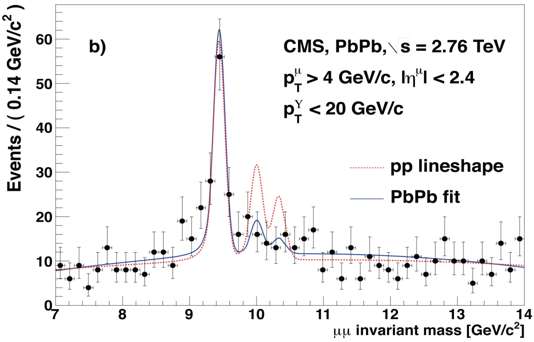
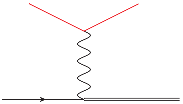


Lessons from heavy quarkonium...

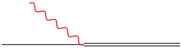
- many bound states may appear in the spectrum
- their existence depends on the temperature
  →**dissociation** and **recombination** processes
- bound-states calculations can be performed in NREFT/pNREFTs

Matsui and Satz (1986); Laine, Philipsen, Romatschke and Tassler [hep-ph/0611300]; Brambilla, Ghiglieri, Petreczky, Vairo [0804.0993]





$(\chi\chi)_{\rm bound} + f \to (\chi\chi)_{\rm open} + f$



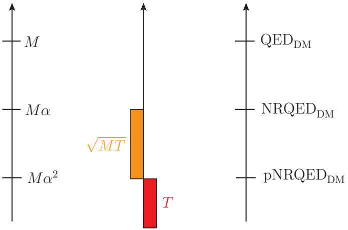
$(\chi\chi)_{\rm bound} + A^{\mu} \to (\chi\chi)_{\rm open}$

Abelian DM model

- Dark matter model Lagrangian

$$\mathcal{L} = \bar{X}(i\not{D} - M)X - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \mathcal{L}_{\rm portal}$$

- $M \gg M\alpha \gg M\alpha^2 \gtrsim T$



- Non-relativistic $\mathrm{QED_{DM}}$: integrate out the scale M

$$\begin{aligned}\mathcal{L}_{\mathrm{NRQED_{DM}}} &= \psi^\dagger\left(iD_0 - M + \frac{\mathbf{D}^2}{2M} + c_{\rm F}\frac{\boldsymbol{\sigma}\cdot g\mathbf{B}}{2M} + c_{\rm D}\frac{\boldsymbol{\nabla}\cdot g\mathbf{E}}{8M^2} + ic_{\rm S}\frac{\boldsymbol{\sigma}\cdot(\mathbf{D}\times g\mathbf{E} - g\mathbf{E}\times\mathbf{D})}{8M^2}\right)\psi \\ &+ \chi^\dagger\left(iD_0 + M - \frac{\mathbf{D}^2}{2M} - c_{\rm F}\frac{\boldsymbol{\sigma}\cdot g\mathbf{B}}{2M} + c_{\rm D}\frac{\boldsymbol{\nabla}\cdot g\mathbf{E}}{8M^2} + ic_{\rm S}\frac{\boldsymbol{\sigma}\cdot(\mathbf{D}\times g\mathbf{E} - g\mathbf{E}\times\mathbf{D})}{8M^2}\right)\chi \\ &- \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{d_2}{M^2}F^{\mu\nu}\mathbf{D}^2F_{\mu\nu} + \frac{d_s}{M^2}\psi^\dagger\chi\,\chi^\dagger\psi + \frac{d_v}{M^2}\psi^\dagger\boldsymbol{\sigma}\chi\cdot\chi^\dagger\boldsymbol{\sigma}\psi \\ &+ \mathcal{L}_{\rm portal}\,,\end{aligned}$$

pNRQED$_{\rm DM}$

- integrating out the scale $M\alpha$: EFT for DM pairs and ultrasoft dark photons

$$\mathcal{L}_{\rm pNRQED_{DM}} = \int d^3\boldsymbol{r}\, \phi^\dagger(t,\boldsymbol{r},\boldsymbol{R})\, [i\partial_0 - H(\boldsymbol{r},\boldsymbol{p},\boldsymbol{P},\boldsymbol{S}_1,\boldsymbol{S}_2) + g\,\boldsymbol{r}\cdot\boldsymbol{E}(t,\boldsymbol{R})]\, \phi(t,\boldsymbol{r},\boldsymbol{R}) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

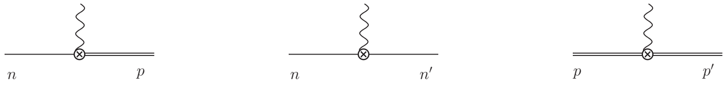
- Hamitonian and potential

$$H(\boldsymbol{r},\boldsymbol{p},\boldsymbol{P},\boldsymbol{S}_1,\boldsymbol{S}_2) = 2M + \frac{\boldsymbol{p}^2}{M} + \frac{\boldsymbol{P}^2}{4M} - \frac{\boldsymbol{p}^4}{4M^3} + V(\boldsymbol{r},\boldsymbol{p},\boldsymbol{P},\boldsymbol{S}_1,\boldsymbol{S}_2) + \dots$$

$$V(\boldsymbol{r},\boldsymbol{p},\boldsymbol{P},\boldsymbol{S}_1,\boldsymbol{S}_2) = V^{(0)} + \frac{V^{(1)}}{M} + \frac{V^{(2)}}{M^2} + \dots$$

- account for above-threshold and below-threshold states (for heavy quakonium X. Yao and T. Mehen [1811.07027])

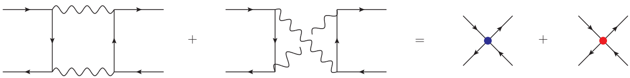
$$\phi_{ij}(t,\boldsymbol{r},\boldsymbol{R}) = \int \frac{d^3\boldsymbol{P}}{(2\pi)^3}\left[\sum_n e^{-iE_nt+i\boldsymbol{P}\cdot\boldsymbol{R}}\,\Psi_n(\boldsymbol{r})\,S_{ij}\,\phi_n(\boldsymbol{P}) + \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} e^{-iE_pt+i\boldsymbol{P}\cdot\boldsymbol{R}}\,\Psi_{\boldsymbol{p}}(\boldsymbol{r})\,S_{ij}\,\phi_{\boldsymbol{p}}(\boldsymbol{P})\right]$$



pNRQED$_{\rm DM}$ at work: annihilation $(X\bar{X})_p$

$$
\begin{aligned}
(\sigma_{\rm ann} v_{\rm rel})(\boldsymbol{p}) &= \frac{1}{2M^2}\langle\, \boldsymbol{p}, 0| \int d^3\boldsymbol{r}\, \phi^\dagger(\boldsymbol{r}, \boldsymbol{R}, t)\, \left[-{\rm Im}\delta {\rm V}^{\rm ann}(\boldsymbol{r})\right]\, \phi(\boldsymbol{r}, \boldsymbol{R}, t)\, |\boldsymbol{p}, 0\rangle \\
&= \frac{{\rm Im}(d_s) + 3{\rm Im}(d_v)}{M^2} |\Psi_{\boldsymbol{p}0}(0)|^2 = (\sigma^{\rm NR}_{\rm ann} v_{\rm rel})\, S_{\rm ann}(\zeta)
\end{aligned}
$$

- [hard scale M]: ${\rm Im}(d_s) = \pi\alpha^2 \left[1 + \frac{\alpha}{\pi}\left(\frac{\pi^2}{4} - 5\right)\right]$ and ${\rm Im}(d_v) = \frac{4}{9}(\pi^2 - 9)\alpha^3$



- [soft scale $M\alpha$]: $S_{\rm ann}(\zeta) = \frac{2\pi\zeta}{1 - e^{-2\pi\zeta}}$, $\zeta = \frac{1}{a_0 p} = \frac{\alpha}{v_{\rm rel}}$

$$
\sigma_{\rm ann} v_{\rm rel} = \frac{\pi\alpha^2}{M^2}\left[1 + \frac{\alpha}{\pi}\left(\frac{19}{12}\pi^2 - 17 + \frac{4}{3}\log 2\right)\right] S_{\rm ann}(\zeta)
$$

pNRQED$_{\rm DM}$ at work: decay $(X\bar{X})_n$

$$\Gamma^n_{\rm ann} = \frac{1}{M^2}\langle n,0| \int d^3\boldsymbol{r}\,\phi^\dagger(\boldsymbol{r},\boldsymbol{R},t)\,\left[-{\rm Im}\delta {\rm V}^{\rm ann}(\boldsymbol{r})\right]\,\phi(\boldsymbol{r},\boldsymbol{R},t)\,|n,0\rangle$$

- here we distinguish between para ($S=0$) and orthodarkonium ($S=1$)

$$\Gamma_{\rm para} = \frac{4{\rm Im}(d_s)}{M^2}\frac{|R_{n0}(0)|^2}{4\pi} = \frac{M\alpha^5}{2}\left[1+\frac{\alpha}{\pi}\left(\frac{\pi^2}{4}-5+\frac{4}{3}\log 2\right)\right]$$

$$\Gamma_{\rm ortho} = \frac{4{\rm Im}(d_v)}{M^2}\frac{|R_{n0}(0)|^2}{4\pi} = \frac{2(\pi^2-9)M\alpha^6}{9\pi}$$

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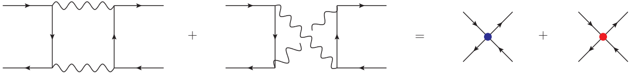
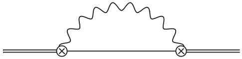


- [soft scale $M\alpha$]: $|R_{n0}(0)|^2 = 4/a_0^3$

pNRQED$_{\rm DM}$ at work: bound-state formation

- $(X\bar{X})_p \to \gamma + (X\bar{X})_n$



$$\Sigma_p^{11} = -ig^2 \frac{d-2}{d-1}\mu^{4-d} r^i \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{i}{p^0 - k^0 - H + i\epsilon} \times k_0^2 \left[\frac{i}{k_0^2 - |\boldsymbol{k}|^2 + i\epsilon} + 2\pi\delta(k_0^2 - |\boldsymbol{k}|^2) n_B(|k_0|) \right] r^i$$

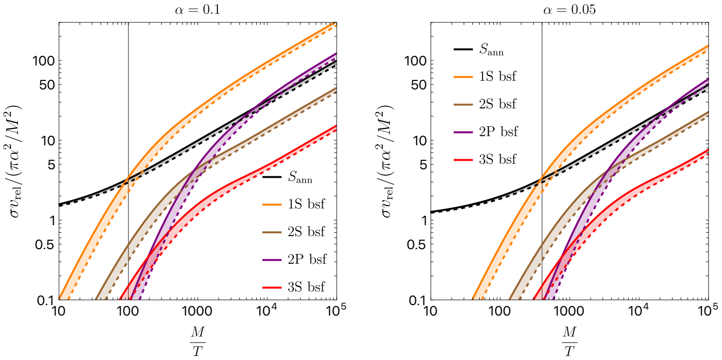
$$(\sigma_{\rm bsf}\, v_{\rm rel})(\boldsymbol{p}) \equiv \sum_n (\sigma_{(X\bar{X})_p \to \gamma + (X\bar{X})_n} v_{\rm rel})_n = \frac{g^2}{3\pi} \sum_n \left[1 + n_{\rm B}(\Delta E_n^p)\right] |\langle n|\boldsymbol{r}|\boldsymbol{p}\rangle|^2 (\Delta E_n^p)^3$$

- Our work: matrix elements in a closed form for any bound-state see also M.Garny and J.Heisig [2112.01499]

$$(\sigma_{1\rm S\, bsf} v_{\rm rel})(\boldsymbol{p}) = \frac{\alpha^2 \pi^2\, 2^{10}\, \zeta^5}{3\, M^2\, (1+\zeta^2)^2} \frac{e^{-4\zeta \,{\rm arccot}\, \zeta}}{1 - e^{-2\pi\zeta}} \left[1 + n_{\rm B}(\Delta E_{1\rm S}^{p1})\right] , \quad \Delta E_{1\rm S}^{p1} = \frac{M v_{\rm rel}^2}{4}\left(1 + \frac{\alpha^2}{v_{\rm rel}^2}\right)$$

agreement with B. von Harling and K. Petraki [1407.7874], T. Binder, B. Blobel, J. Harz and K. Mukaida [2002.07145]

Thermal average and bound-state dissociation



- $\gamma + (X\bar{X})_n \to (X\bar{X})_p$

$$\Gamma^n_{\rm bsd} = 2 \int_{|\boldsymbol{k}| \geq |E^b_n|} \frac{d^3k}{(2\pi)^3}\, n_{\rm B}(|\boldsymbol{k}|)\, \sigma^n_{\rm ion}(|\boldsymbol{k}|)$$

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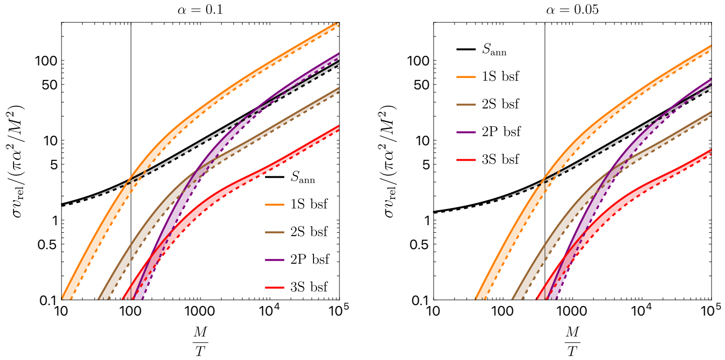

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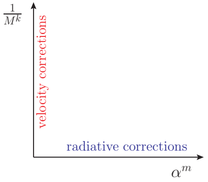
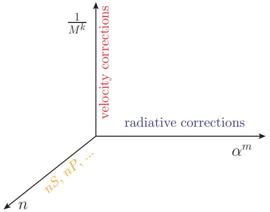
- All ingredients are ready for

$$\frac{dn_X}{dt} + 3Hn_X = -\langle\sigma_{\rm eff}\, v_{\rm rel}\rangle(n_X^2 - n_{X,\rm eq}^2)\,,$$

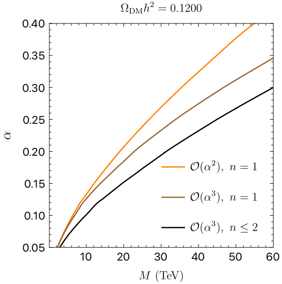
$$\langle\sigma_{\rm eff}\, v_{\rm rel}\rangle = \langle\sigma_{\rm ann}\, v_{\rm rel}\rangle + \sum_n \langle\sigma^n_{\rm bsf}\, v_{\rm rel}\rangle\, \frac{\Gamma^n_{\rm ann}}{\Gamma^n_{\rm ann} + \Gamma^n_{\rm bsd}}$$

Dark matter energy density





- Result for the cosmologically viable parameter space with Sommerfeld and BSF



- maximal effects with combination of NLO and excited states
- draw experimental constraints with $\mathcal{L}_{\rm portal}$

Non-abelian model

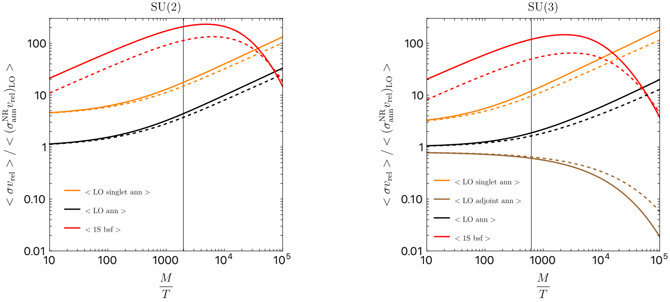
- Non-abelian SU(N) model, rich dark sectors (phase transitions/GWs)

$$\mathcal{L} = \bar{X}(i\not{D} - M)X - \frac{1}{4}G^a_{\mu\nu}G^{a\,\mu\nu} + \mathcal{L}_{\rm portal}$$

- the corresponding construction brings to pNQCD-like theories, $V_s^{(0)} = -C_F\frac{\alpha}{r}$ and $V_o^{(0)} = \frac{\alpha}{2Nr}$

$$\mathcal{L}_{{\rm pNREFT}_{\rm DM}} = \int d^3r \left\{ {\rm Tr}\left[{\rm S}^\dagger \left(i\partial_0 - H_s\right){\rm S} + {\rm O}^\dagger \left(iD_0 - H_o\right){\rm O}\right] \right.$$

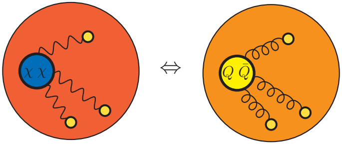
$$\left. + {\rm Tr}\left[V_A(r) g({\rm S}^\dagger \boldsymbol{r}\cdot\boldsymbol{E}{\rm O} + {\rm O}^\dagger \boldsymbol{r}\cdot\boldsymbol{E}{\rm S}) + \frac{V_B(r)}{2} g({\rm O}^\dagger \boldsymbol{r}\cdot\boldsymbol{E}{\rm O} + {\rm O}^\dagger {\rm O}\boldsymbol{r}\cdot\boldsymbol{E})\right]\right\} - \frac{1}{4}G^a_{\mu\nu}G^{a\,\mu\nu}$$



Conclusions

- DM particles and thermal freeze-out in the early universe:
  non-relativistic particles in a thermal environment
- Many models feature vector or scalar force carriers between DM particles

  $\Rightarrow$ Similarities with heavy quarkonium at finite temperature



- Adapted NREFTs and pNREFT techniques for determining dark matter energy density
- Bound-state formation under the roof of pNREFTs

  $\Rightarrow$ Large effects on the model parameter space

Outlook

- Inspect the thermal scale $\sqrt{MT}$ and Debye mass
- our framework also highlights inconsistencies and offer a pathway for improvements
- $T \simeq M\alpha$ needed to cover a broader range around the freeze-out