



# How to calculate Feynman rules in scalar-tensor theories and not collapse in the process

Based on arXiv:2211.14300

Sergio Sevilano





# FeynMG: a FeynRules extension for scalar-tensor theories of gravity

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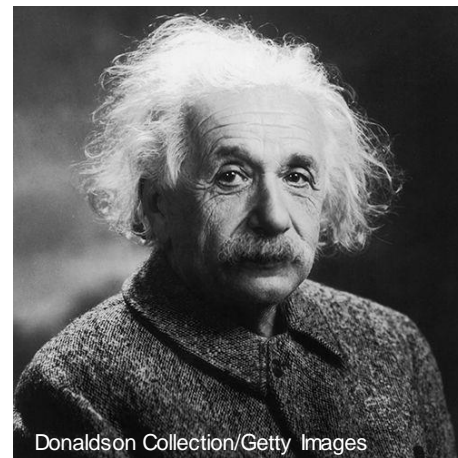
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# What are Scalar-Tensor theories

We all know and love Einstein's General Relativity, which is usually described by the Einstein-Hilbert action

$$S = \int d^4x \sqrt{-g} \left[ -\frac{M_{\text{Pl}}^2}{2} R + \mathcal{L}_m \{ \psi_i, \phi_i, g_{\mu\nu}, \dots \} \right]$$

Curvature                      Matter action



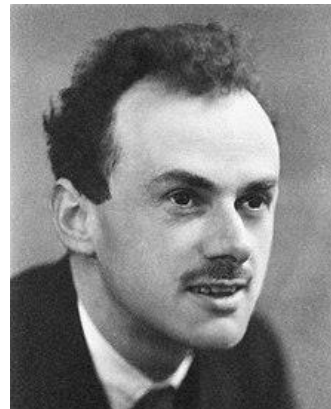
However, this might not be the whole story...

# Some history

One of the first motivations for modified gravity started with Dirac, and its **large number hypothesis**

He noticed that both dimensionless ratios were of the same order

$$\frac{ct}{r_e} \approx 3.47 \cdot 10^{41} \approx 10^{42}, \quad \frac{e^2}{4\pi\epsilon_0 G m_p m_e} \approx 10^{40}$$



From which he hypothesised that the strength of gravity might be changing with time!

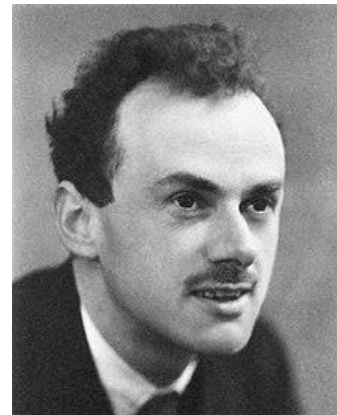
$$G \sim \frac{1}{t}$$

# Some history

Time-dependent gravitational constant? Quite crazy...  $G \sim \frac{1}{t}$

However, of course, we cannot just insert some function of time as:

$$S = \int d^4x \sqrt{-g} \left[ -\frac{M_{\text{pl}}(t)^2}{2} R + \mathcal{L}_{\text{m}}\{\psi_i, \phi_i, g_{\mu\nu}, \dots\} \right]$$



# Some history

However, Brans, Dicke and Jordan had already been thinking about something similar, but using a scalar field

$$S = \int d^4x \sqrt{-g} \left[ -\frac{F(X)}{2} R + \frac{1}{2} g^{\mu\nu} \partial_\mu X \partial_\nu X - U(X) + \mathcal{L}_m \{ \psi_i, \phi_i, g_{\mu\nu}, \dots \} \right]$$

Such that at late times, when the scalar stabilizes

$$F(v_X) \equiv M_{\text{pl}}^2$$

This is a consistent theory with changing gravitational force!



# What are Scalar-Tensor theories

## Modified gravity is not MOND

They live within General Relativity but have a non-canonical gravitational action

# What are Scalar-Tensor theories

Either from an **effective field theory** standpoint or more **fundamental theories** of gravity (such as compactifications of extra dimensions) it is natural to extend the gravitational action:

$$S_{\text{grav}} = \int d^4x \sqrt{-g} \left[ -aR - F(X)R - bR^2 - cR_{\mu\nu}R^{\mu\nu} + \frac{1}{2}g^{\mu\nu}\partial_\mu X\partial_\nu X + \dots \right]$$

Different functions of gravity

Non-minimal couplings to a scalar field

These extensions of gravity are called **Scalar-Tensor theories**



# What are Scalar-Tensor theories

$$S_{\text{grav}} = \int d^4x \sqrt{-g} \left[ -aR - F(X)R - bR^2 - cR_{\mu\nu}R^{\mu\nu} + \frac{1}{2}g^{\mu\nu}\partial_\mu X\partial_\nu X + \dots \right]$$

In most cases, they introduce new dynamics into the matter sector (**fifth forces**)

-Cosmological scales (Gravitational waves)

Bertotti et al. 2003

-Solar system scales (Cassini spacecraft)

Avilez et al. 2013

-Atomic scales (Atom interferometry)

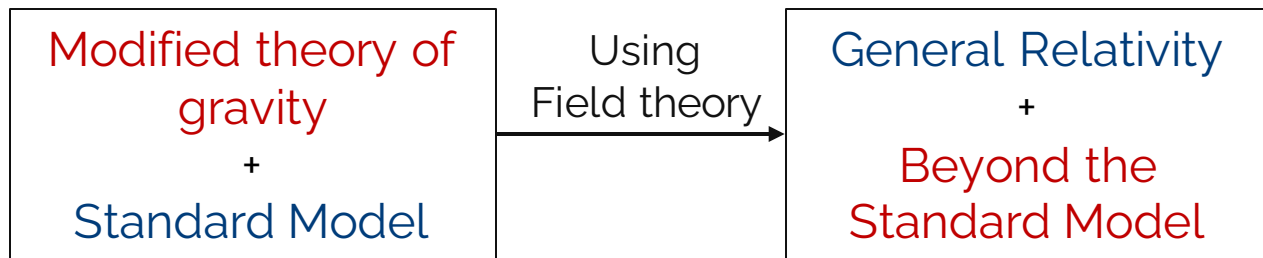
YT talk by Clare Burrage  
in “Cosmology Talks”

# Intro to Scalar-Tensor theories

However, we can also study them on the subatomic scale using particle theory, unveiling the effect that the modification of gravity has in the Standard Model

For that, instead of working with modified gravity, we will make the following transformation

## Our plan:



# Intro to Scalar-Tensor theories

As an example, let's study the simplest modified theory of gravity

Brans-Dicke Action:

$$S = \int d^4x \sqrt{-g} \left[ -\frac{F(X)}{2} R + \frac{1}{2} g^{\mu\nu} \partial_\mu X \partial_\nu X - U(X) + \mathcal{L}_m \{ \psi_i, \phi_i, g_{\mu\nu}, \dots \} \right]$$

Matter action  
↙

To satisfy basic constraints, we need at late times:  $F(v_X) \equiv M_{\text{pl}}^2$

# Intro to Scalar-Tensor theories

The common way of solving this called [Transforming to the Einstein frame](#)

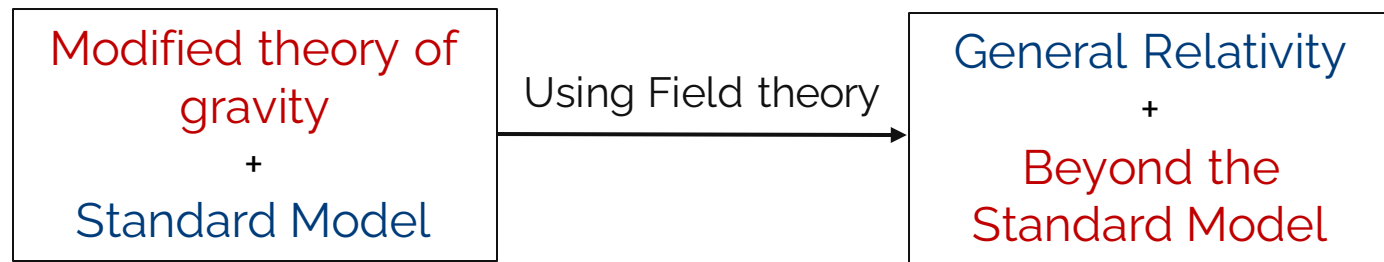
Make a conformal transformation such that gravity becomes canonical

$$g_{\mu\nu} \rightarrow \frac{\tilde{M}_{\text{Pl}}^2}{F(X)} \tilde{g}_{\mu\nu}, \quad g^{\mu\nu} \rightarrow \frac{F(X)}{\tilde{M}_{\text{Pl}}^2} \tilde{g}^{\mu\nu},$$

$$S_{\text{EF}} = \int d^4x \sqrt{-\tilde{g}} \left[ -\frac{\tilde{M}_{\text{pl}}^2}{2} \tilde{R} + \frac{\tilde{M}_{\text{pl}}^2}{2} \left[ \frac{1}{F(X)} + \frac{3F'(X)^2}{2F(X)^2} \right] \tilde{g}^{\mu\nu} \partial_\mu X \partial_\nu X \right. \\ \left. - \frac{\tilde{M}_{\text{pl}}^4}{F(X)^2} U(X) + \mathcal{L}_{\text{m}}\{\psi_i, \phi_i, X, \tilde{g}_{\mu\nu}, \dots\} \right]$$

# No Einstein frame?

If the gravitational action is very complicated, there is no such transformation of the metric that takes us to the Einstein frame, so we are stuck in the Jordan frame



In that case, we need to linearise gravity

# Jordan frame

If the gravitational action is very complicated, there is no transformation of the metric that takes us to the Einstein frame, then we are stuck in the Jordan frame

For the simplest Brans-Dicke case:

$$S = \int d^4x \sqrt{-g} \left[ -\frac{F(X)}{2} R + \frac{1}{2} g^{\mu\nu} \partial_\mu X \partial_\nu X - U(X) + \mathcal{L}_m\{\psi_i, \phi_i, g_{\mu\nu}, \dots\} \right]$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + \dots$$

$$g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu},$$

# Jordan frame

If the gravitational action is very complicated, there is no transformation of the metric that takes us to the Einstein frame, then we are stuck in the Jordan frame

For the simplest Brans-Dicke case:

$$\begin{aligned}
 \mathcal{L}_G = & \frac{F(X)}{4} \left( \frac{1}{4} \partial_\mu h \partial^\mu h - \frac{1}{2} \partial_\rho h_{\mu\nu} \partial^\rho h^{\mu\nu} \right) + \frac{F'(X)}{4} \partial_\mu X \partial^\mu h \\
 & + \frac{1}{2} \left( 1 + \frac{F'(X)^2}{2F(X)} \right) \partial_\mu X \partial^\mu X - U(X).
 \end{aligned}$$

Fierz-Pauli action

Kinetic mixing!

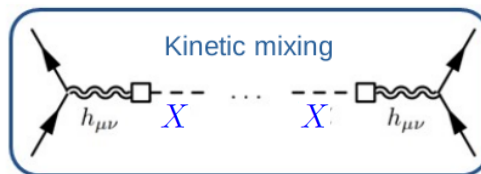
# Jordan frame

Fifth forces leak into the system via a kinetic mixing with gravity!

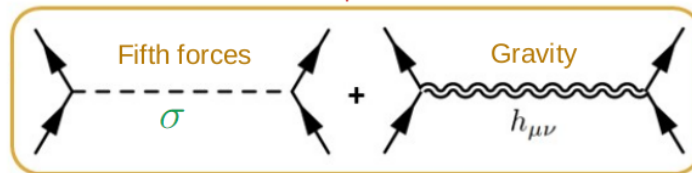
Modified theory of  
gravity  
+  
Standard Model

General Relativity  
+  
Beyond the  
Standard Model

## Fifth forces



Diagonalization ↓





# Beyond Standard Model

Once we have the Beyond Standard Model description, we can calculate from quantum corrections to scattering amplitudes!

However, these calculations are very tedious and model-dependent

## Simplest Jordan frame calculation

Expansion of gravity  
Canonical normalization  
Expansion around non-trivial vevs  
Kinetic mixings to graviton  
Mass mixings

(even for the Einstein frame)

# Beyond Standard Model

Once we have the Beyond Standard Model description, we can calculate from quantum corrections to scattering amplitudes!

However, these calculations are very tedious and model-dependent:

[Simplest Jordan frame calculation](#)

(Graphic representation  
of your collapse)



Stable Diffusion

Almost impossible to study scalar-tensor theories consistently!!!

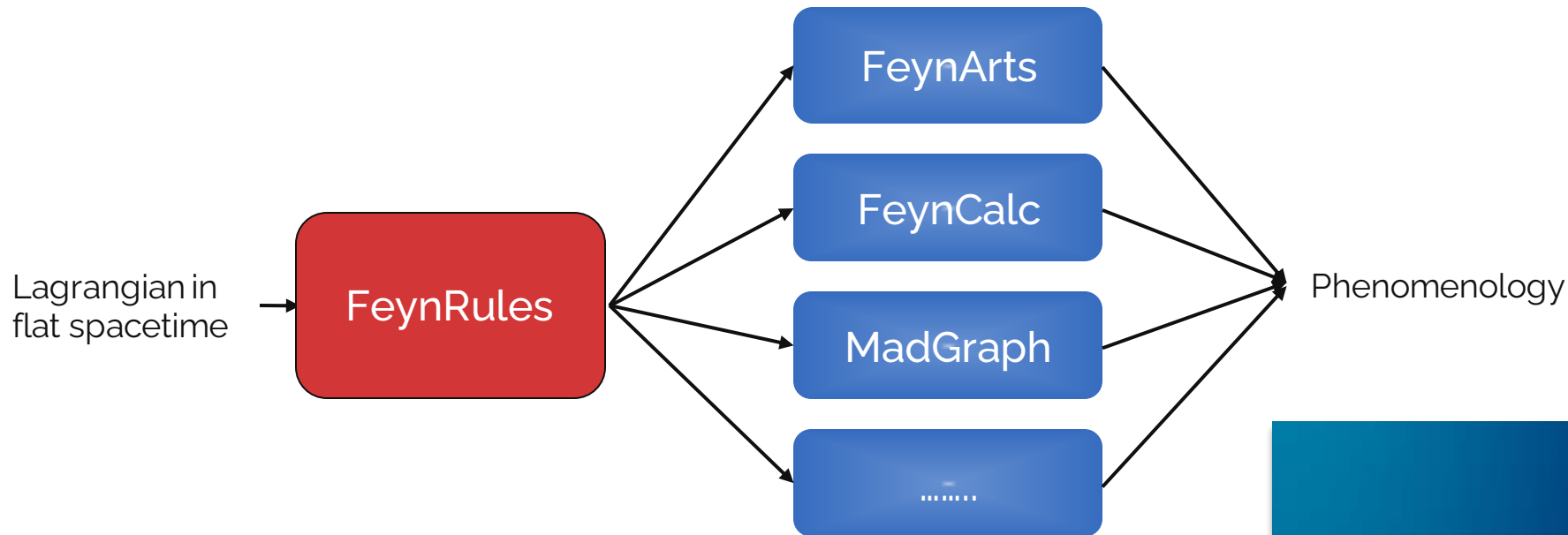
# Can we get a computer to do it?

## Learn from particle phenomenologists:

They use **FeynRules**: a Mathematica Package that from a Lagrangian defined in flat spacetime gives the list of Feynman Rules

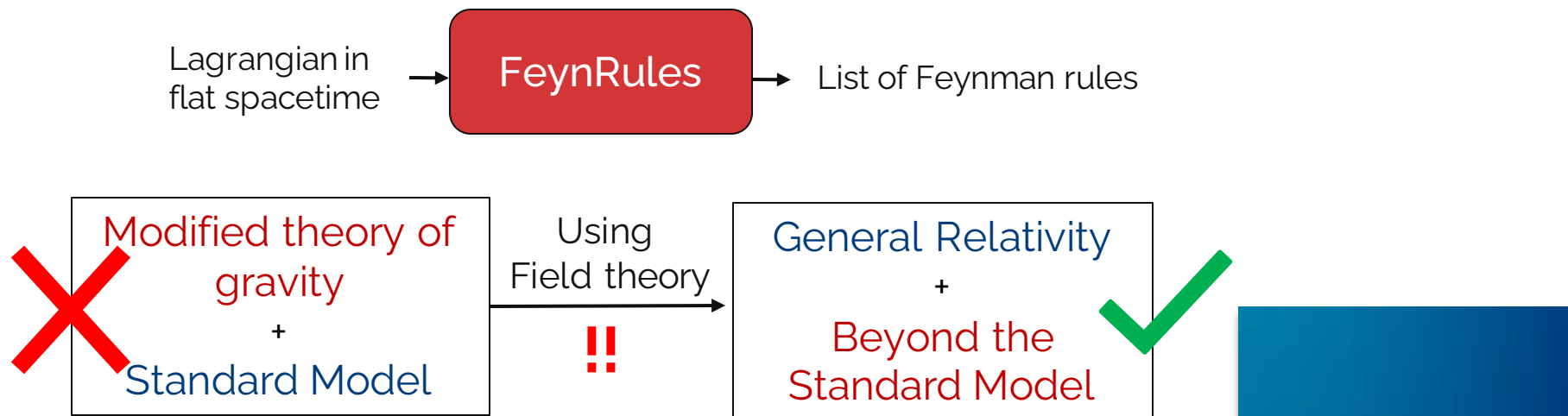


# Can we get a computer to do it?



# Can we get a computer to do it?

However, the difficult thing is getting to the BSM formalism



# Can we get a computer to do it?

We developed [FeynMG](#) to help through the process



Subpackage of FeynRules

# What can FeynMG do?

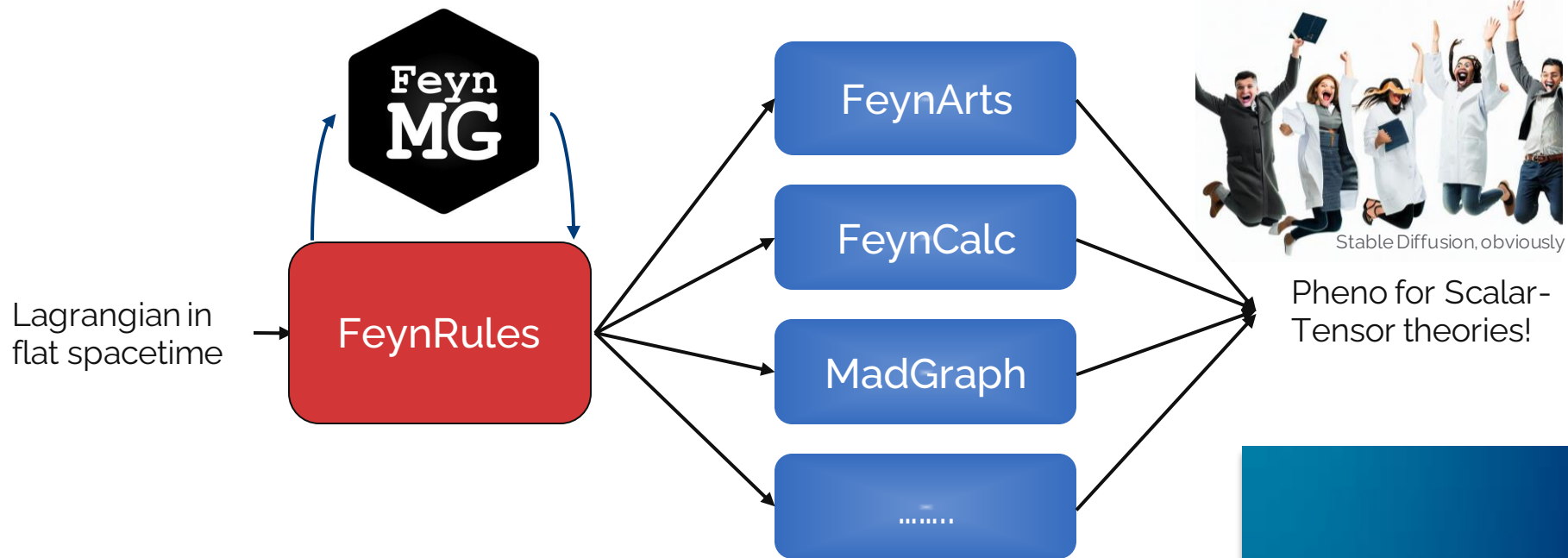
- Allows the user to insert new scalar degrees of freedom and any gravitational theory
- Perform all the necessary operations to calculate the Beyond the Standard Model description

## Simplest Jordan frame calculation

~~CanonScalar~~      Expansion of gravity      LinearizeGravity  
                          Canonical normalization  
~~VevExpand~~      Expansion around non-trivial vevs  
                          Kinetic mixings to graviton      GravKinMixing  
                          Mass mixings      MassDiagMG

Test Scalar-Tensor theories in colliders!

# FeynMG!

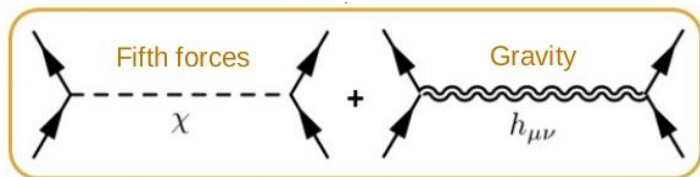




# What can FeynMG do?

A quick example to express my excitement:

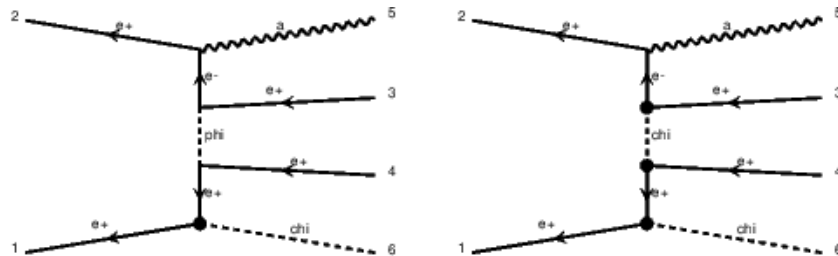
Calculating by hand  
fifth forces for an electron



3-4 months of learning and mistakes  
in the process

VS

Using MadGraph:



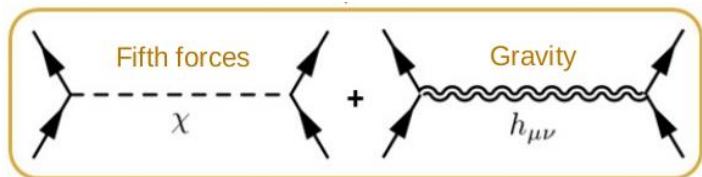
It took **X min.** to generate the  
possible **344** diagrams

It can work with any scalar-tensor theory

# What can FeynMG do?

A quick example to express my excitement:

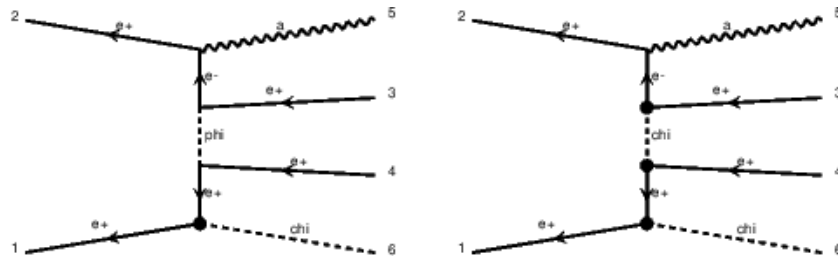
Calculating by hand  
fifth forces for an electron



3-4 months of learning and mistakes  
in the process

VS

Using MadGraph:



It took **1.2 s** to generate the  
possible **344** diagrams

It can work with any scalar-tensor theory

# Conclusion

- Scalar-tensor theories can be studied as a particle theory through a Beyond Standard Model description
- FeynMG help us through the calculation, making it possible to work with the whole Standard Model
- Being inside FeynRules, we can use all the compatible packages to do pheno studies!

If you want to test Scalar-tensor theories of gravity at colliders,

FeynMG is the way

# Thank you for listening



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<https://gitlab.com/feynmg/FeynMG>

