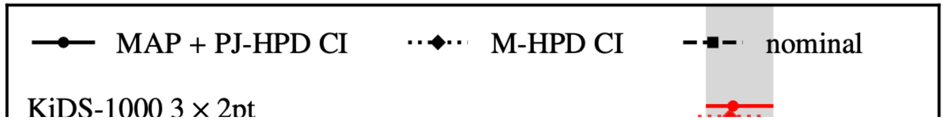
Scaling Solutions in Generalized Proca Theory and its Cosmological Implications

Santiago García-Serna

J. Bayron Orjuela-Quintana
César A. Valenzuela-Toledo

Facultad de Ciencias Naturales y Exactas
Universidad del Valle

Motivation

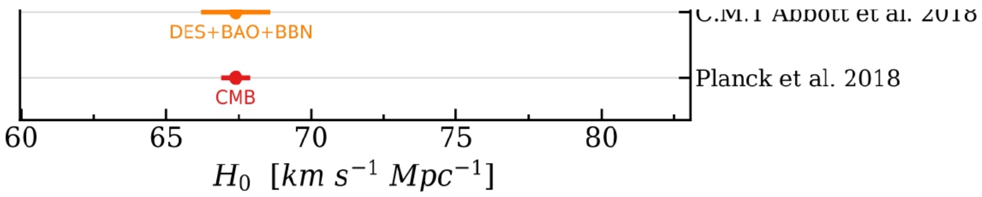


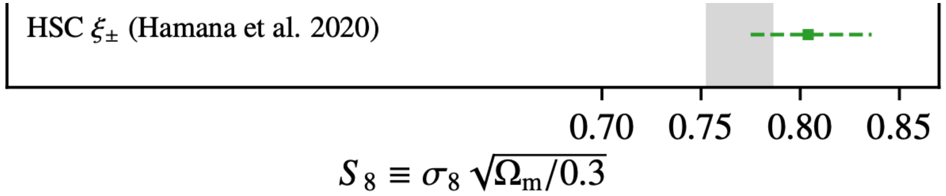
Modified/Extended gravities

- High-order derivatives theories
- Extra fields theories
- Non-local theories
- Higher dimensions theories
- Extensions: torsion/emergent theories

Prescription for dark energy dark energy

- **Standard models:** non-minimally coupled to curvature
- **Clustering:** scenarios with fluctuations on subhorizon scales
- **Modified gravity:** presence of fifth force and anisotropic clustering





arxiv.org/abs/2203.06142

Dynamical Dark Energy
arXiv:1701.08165
arXiv:2201.09306

Modified Gravity models

- **Horndeski theory (arXiv:1901.07183)**

Interacting Dark Energy
arXiv:1910.09853
arXiv:Astro-ph/9908023.

What about the Generalized Proca theory coupled?

Generalized Proca Theory

$$S = \int \mathrm{d}x^4 \sqrt{-g}\left(\mathcal{L}_F + \sum_i \mathcal{L}_i + \mathcal{L}_M\right)$$

$$X = -\frac{1}{2}A_\mu A^\mu, \quad F = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu}, \quad \tilde{F} = -\frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$\mathcal{L}_F = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

$$\mathcal{L}_2 = G_2\left(X, F, \tilde{F}\right)$$

$$\mathcal{L}_3 = G_3(X)\nabla_\mu A^\mu$$

$$\mathcal{L}_4 = G_4(X)R + G_{4X}\Big[(\nabla_\mu A^\mu)^2 + c_2\nabla_\rho A_\sigma \nabla^\rho A^\sigma - (1+c_2)\nabla_\rho A_\sigma \nabla^\sigma A^\rho\Big]$$

$$\begin{aligned}\mathcal{L}_5 = G_5(X)G_{\mu\nu}\nabla^\mu A^\nu - \frac{1}{6}G_{5X}\Big[&(\nabla_\mu A^\mu)^3 \\ &- 3d_2\nabla_\mu A^\mu \nabla_\rho A_\sigma \nabla^\rho A^\sigma \\ &- 3(1-d_2)\nabla_\mu A^\mu \nabla_\rho A_\sigma \nabla^\sigma A^\rho \\ &+ (2-3d_2)\nabla_\rho A_\sigma \nabla^\gamma A^\rho \nabla^\sigma A_\gamma \\ &+ 3d_2\nabla_\rho A_\sigma \nabla^\gamma A^\rho \nabla_\gamma A^\sigma\Big].\end{aligned}$$

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$$
$$A_\mu = (A_0(t), 0, 0, 0)$$
$$X = \frac{1}{2}A_0^2, \quad F = \tilde{F} = 0$$

Background

$$ds^2 = a^2(\eta)\left[-d\eta^2 + \{\delta_{ij} + h_{ij}(x,\eta)\}dx^i dx^j\right]$$
$$c_T^2 = \frac{2G_4 - A_0^2 A_0' G_{5X}}{2G_4 - 2A_0^2 G_{4X} - HA_0^3 G_{5X}} = 1$$
$$G_4 = \frac{m_P^2}{2}, \quad G_5 = \text{const}^{\text{st}}$$

Tensor perturbations

$$\mathcal{L}_F = 0$$
$$\mathcal{L}_2 = G_2$$
$$\mathcal{L}_3 = G_3\dot{A}_0$$
$$\mathcal{L}_4 = \frac{m_P^2}{2}R \equiv \mathcal{L}_{EH}$$
$$\mathcal{L}_5 = \text{Boundary term}$$

$$\mathcal{L}_{GP} = G_2(X) + G_3(X)\nabla_\mu A^\mu$$

Generalized Proca theory coupled to Cold Dark Matter and Scaling Solution

$$\mathcal{L} = \mathcal{L}_{\rm EH} + \mathcal{L}_{\rm GP} + f\mathcal{L}_{\rm CDM} + \mathcal{L}_b + \mathcal{L}_r$$

$$\rho_{\rm DE} = -G_2, \quad w \equiv \frac{p}{\rho}, \quad \Omega \equiv \frac{\rho}{3m_{\rm P}^2 H^2}, \quad w_{\rm eff} = \sum_i \Omega_i w_i$$

$$\frac{\rho_{\rm DE}}{\rho_{\rm CDM}} = \text{const}$$

Scaling condition

$$\frac{\mathrm{d}\ln\mathcal{L}_{\rm GP}}{\mathrm{d}X} X' = -3(1 + w_{\rm eff})$$

$$\mathcal{L}_{GP} = G_2(X) = V_0 e^{-\gamma\int \mathrm{d}X (f_X/f)}$$

$$3m_{\rm P}^2 H^2 = -G_2 + f\rho_{CDM} + \rho_b + \rho_r$$
$$-2m_{\rm P}^2 \dot{H} = f\rho_{\rm CDM} + \rho_b + \frac{4}{3}\rho_r$$

Friedmann equations

$$v^2 = -\frac{G_2}{3m_{\rm P}^2 H^2}, \quad \tilde{\Omega}_{\rm CDM} = \frac{f\rho_{\rm CDM}}{3m_{\rm P}^2 H^2}$$
$$\Omega_b = \frac{\rho_b}{3m_{\rm P}^2 H^2}, \quad \Omega_r = \frac{\rho_r}{3m_{\rm P}^2 H^2}$$

$$\left(G_{2X} - \frac{f_X}{f}\rho_{CDM}\right) A_0 = 0$$

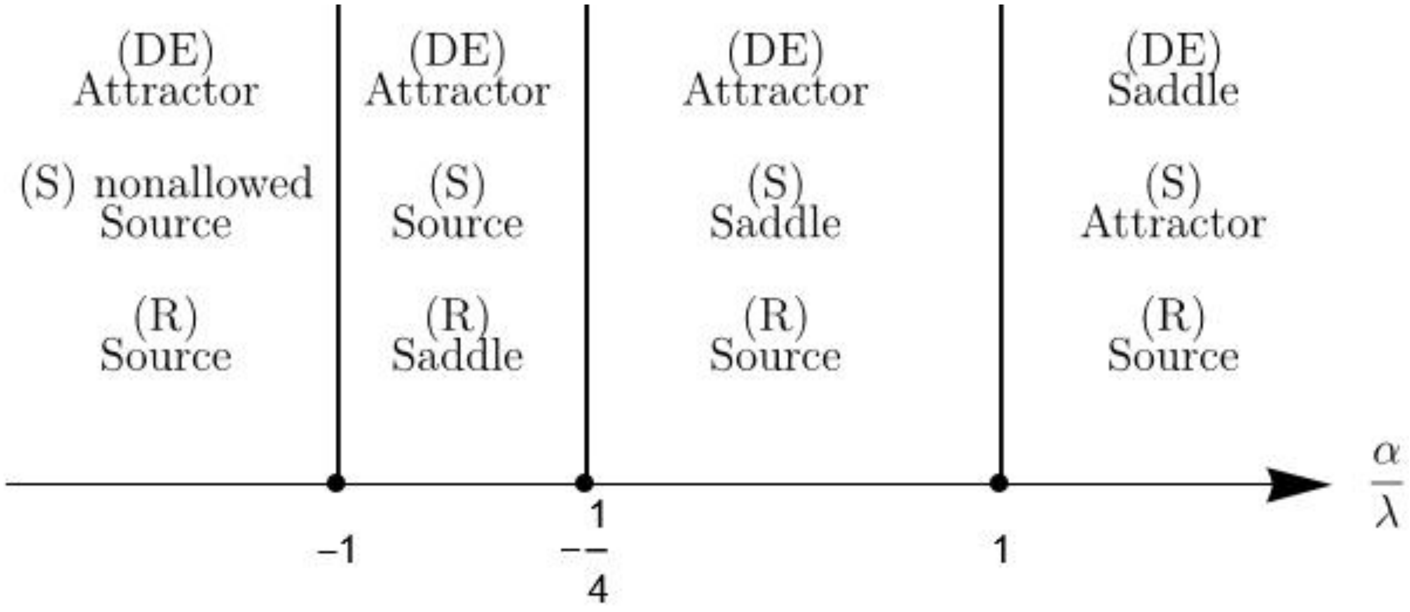
Motion equation of A_0

$$f(X) = f_0 e^{\alpha X/m_{\rm P}^2}, \quad G_2(X) = V_0 e^{-\lambda X/m_{\rm P}^2}$$

Fixed points and stability

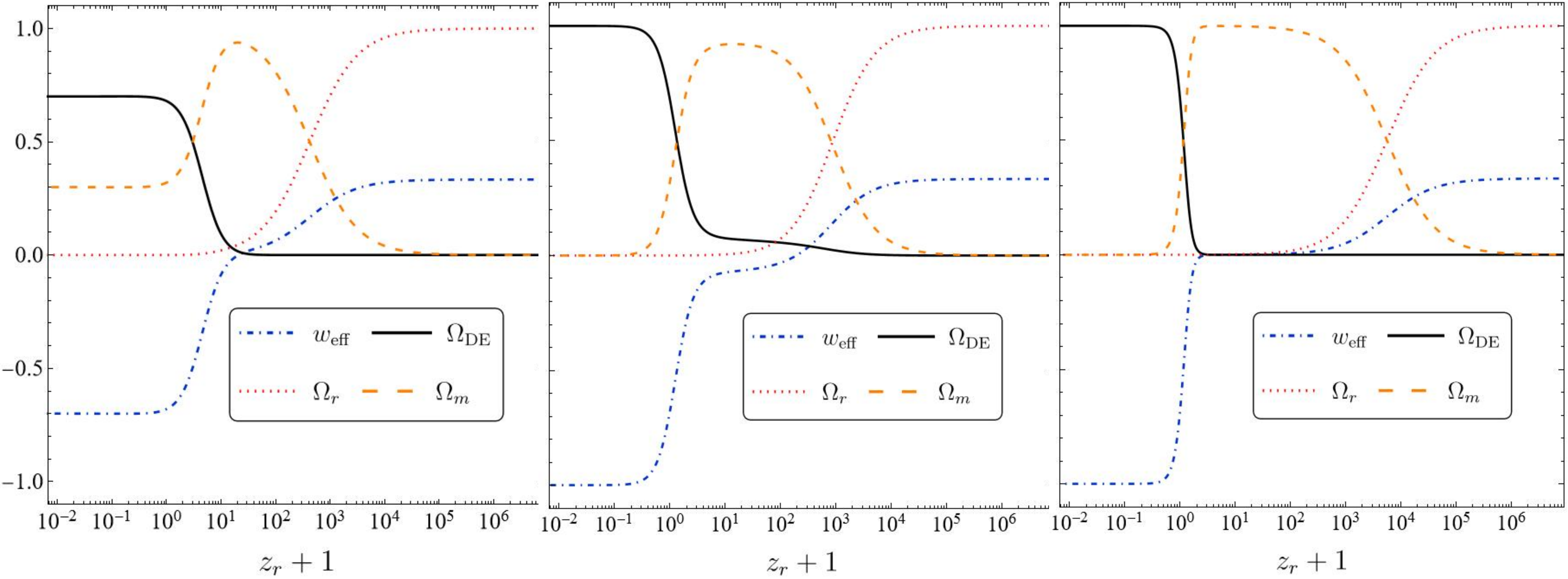
Fixed Point	v	$\tilde{\Omega}_{\rm CDM}$	Ω_b	Ω_b	$w_{\rm eff}$	Stability
(R)	0	0	0	1	$1/3$	source or saddle
(M)	0	$1-\Omega_b$	Ω_b	0	0	saddle
(DE)	1	0	0	0	-1	saddle or attractor
(S)	$\sqrt{\frac{\alpha}{\alpha+\lambda}}$	$1-\frac{\alpha}{\alpha+\lambda}$	0	0	$-\frac{\alpha}{\alpha+\lambda}$	can be anyone

$$\frac{\alpha}{\lambda} > \frac{1}{2} \rightarrow w_{\rm eff} < -\frac{1}{3}$$



Evolution of the background

(S) $\frac{\alpha}{\lambda} = \frac{7}{3}$ **(DE) with (S) as saddle** $\frac{\alpha}{\lambda} = 0.1$ **(DE) De Sitter** $\frac{\alpha}{\lambda} = -0.7$



Linear perturbations

$$\mathrm{d}s^2 = a(\eta)^2\left[-\{1+2\Phi(\boldsymbol{x},\eta)\}\mathrm{d}\eta^2 + \{1-2\Psi(\boldsymbol{x},\eta)\}\mathrm{d}\boldsymbol{x}^2\right]$$

$$\tilde{\delta}_c \equiv \frac{\delta(f\rho_c)}{f\rho_c}$$

$$\tilde{\delta}_c' = -V - 3\Phi'$$

$$V' = -\mathcal{H}V + \frac{f_X(-A_0A_0' + A_0^2\mathcal{H})}{a^2 f}V + k^2\Psi$$

$$\Psi \to -\Phi$$

$$\Phi \to \frac{3a^2}{2k^4m_P^2}f\rho_c + \frac{a^2\tilde{\delta}_c f\rho_c}{2k^2m_{\mathrm{P}}^2} - \frac{A_0^2 f_X\tilde{\delta}_c f\rho_c}{2k^2 f}$$

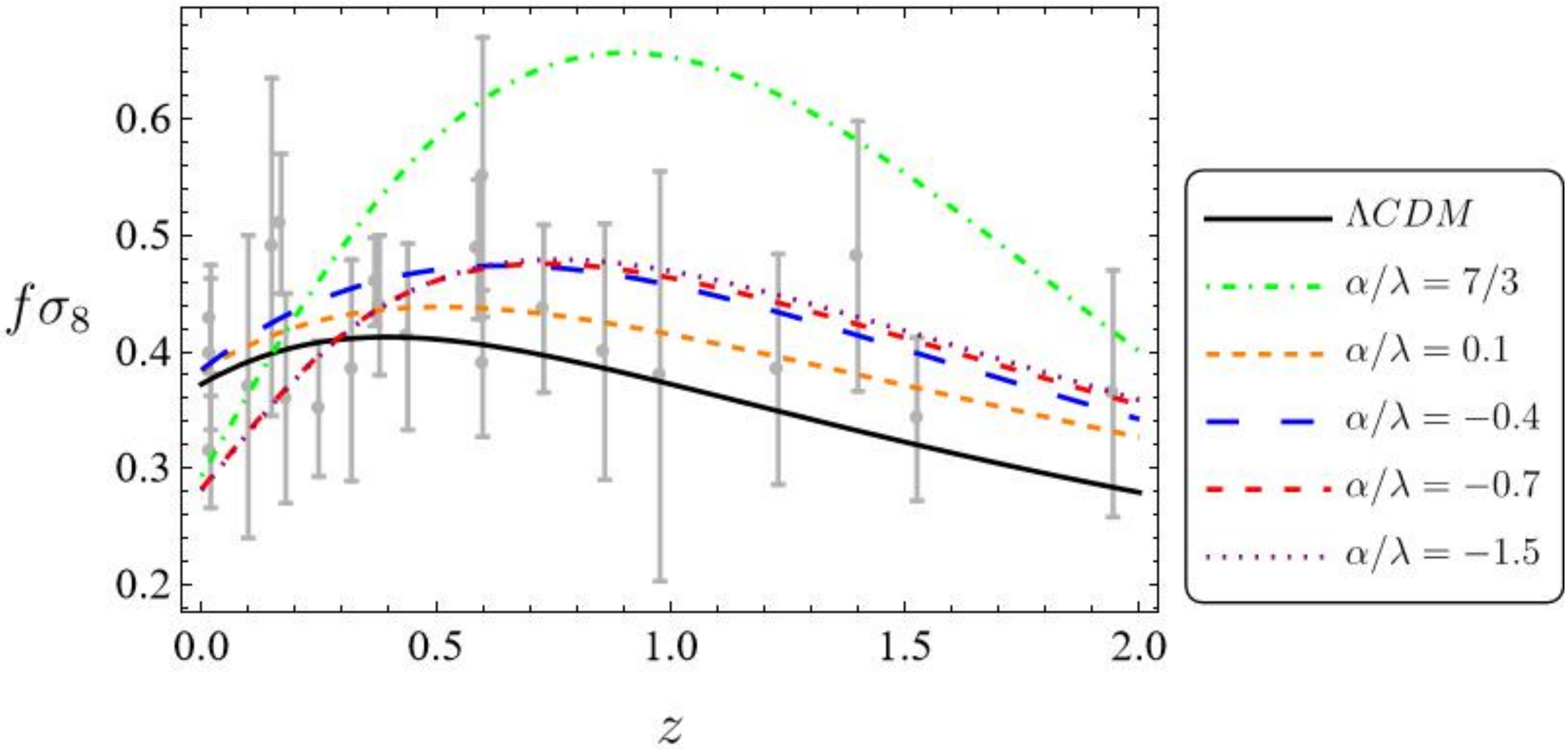
$$\Phi'' \to 0, \quad \Phi' \to 0$$

QSA-SHA approximation

$$\tilde{\delta}_c''(a) + \left(\frac{\mathrm{d}\ln(a^3H)}{\mathrm{d}a} + \alpha X'(a)\right)\tilde{\delta}_c' - \frac{3\Omega_{m0}G_{\mathrm{eff}}/G_N}{2a^5H^2(a)/H_0^2}\tilde{\delta}_c^0 = 0$$

$$G_{\mathrm{eff}} \equiv (1-2X(a)\alpha)G_N$$

Evolution of the $f\sigma_8$ factor



Conclusion

- The interaction between DE and CDM allows for modification of the gravity effects during the matter domination
- Despite the interactions is possible to reproduce the evolution at the background level

Prospects

- MCMC
- Does this model fix H_0 and σ_8 or at least one?
- How is this model compared with the others concerning the predicted observables?


DESCUBRE EL CONOCIMIENTO
HECHO EN
UNIVALLE
QUE TRANSFORMA VIDAS