Can EFT reveal if there was an electroweak phase transition?

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Cosmology from Home, 07/2023

P. Schicho, T. V. I. Tenkanen, and G. White, *Combining thermal resummation and gauge invariance for electroweak phase transition,* JHEP **11** (2022) 047 [2203.04284], A. Ekstedt, P. Schicho, and T. V. I. Tenkanen, DRalgo*: A package for effective field theory approach for thermal phase transitions,* Comput. Phys. Commun. **288** (2023) 108725 [2205.08815]

Motivation

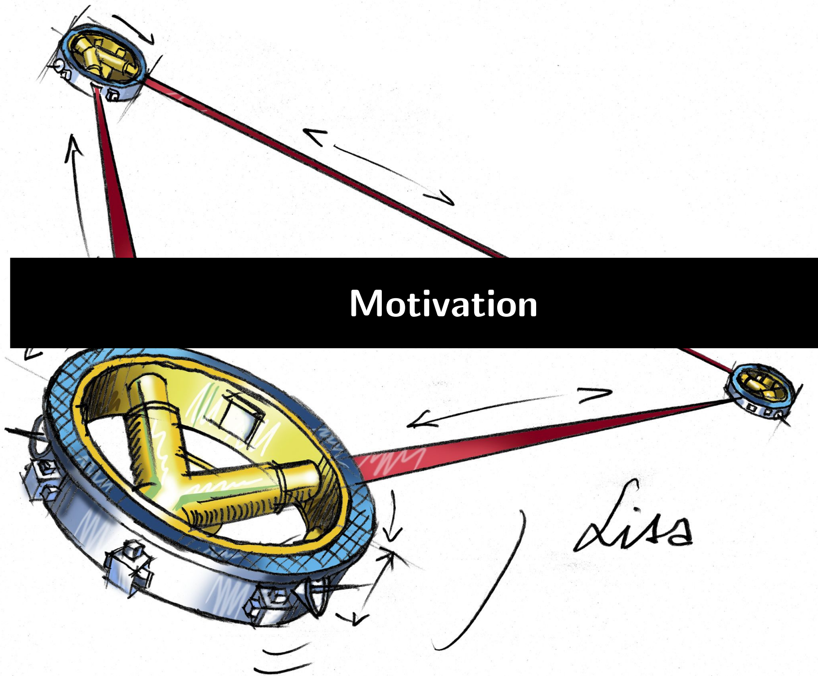

figure by ESA-C. Vijoux

The thermal history of electroweak symmetry breaking

Strong first-order cosmic phase transition at EW scale $T_{\mathrm{c}} \sim 100$ GeV:

- ▷ Baryogenesis — Baryon asymmetry of the universe
- ▷ Colliding bubbles — Gravitational wave (GW) production

In Standard Model EWSB occurs via a smooth crossover but possible that it is first-order in Beyond the Standard Model (BSM) extensions.

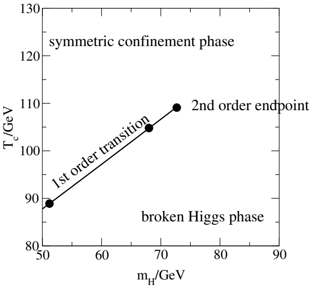

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figure by M. Laine, *Electroweak phase transition beyond the standard model*, in *4th International Conference on Strong and Electroweak Matter*, pp. 58–69, 6, 2000 [hep-ph/0010275]

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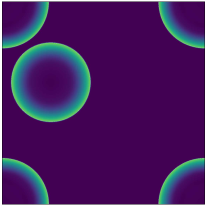
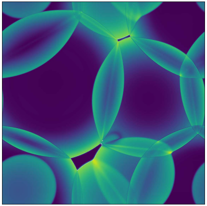
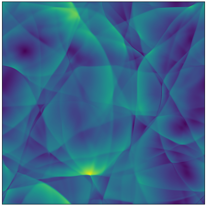
In Standard Model EWSB occurs via a smooth crossover but possible that it is first-order in Beyond the Standard Model (BSM) extensions. Study **BSM physics** near EW scale in context of phase transitions:

- Light fields strongly coupled to Higgs
- Collider targets. BSM testing pipeline: Collider phenomenology

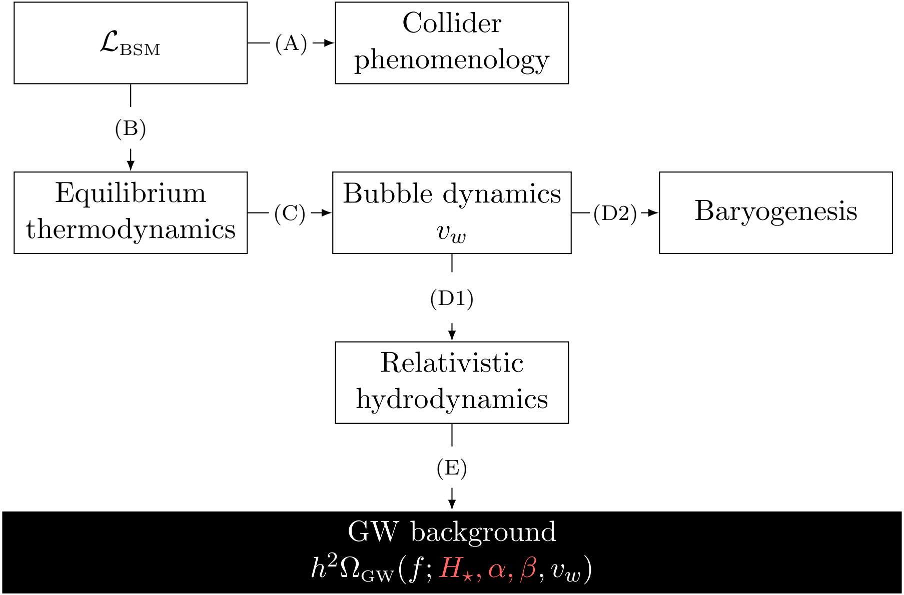






figures by D. Cutting, M. Hindmarsh, and D. J. Weir, *Vorticity, kinetic energy, and suppressed gravitational wave production in strong first order phase transitions*, Phys. Rev. Lett. **125** (2020) 021302 [1906.00480]

Uncertainties of the gravitational wave pipeline



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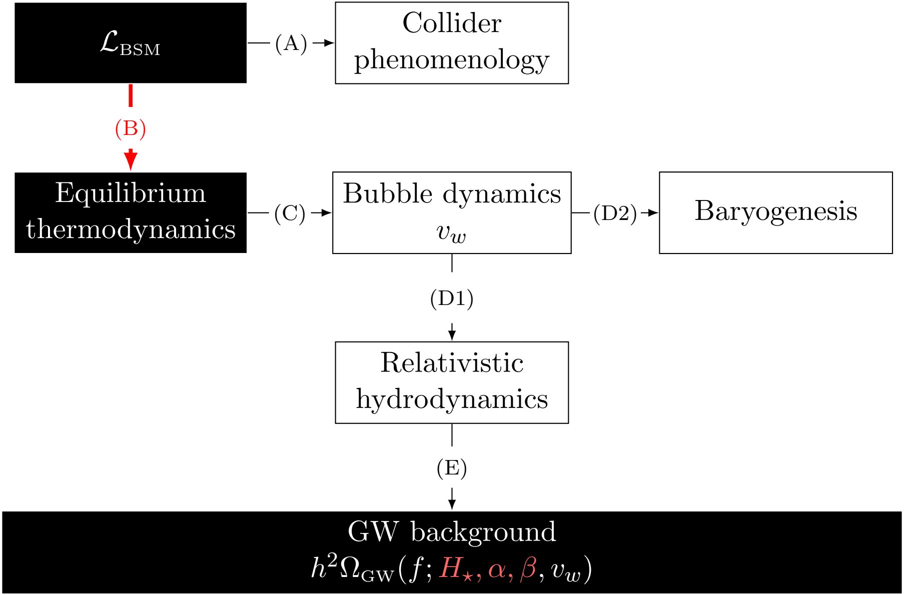
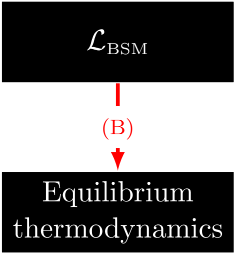
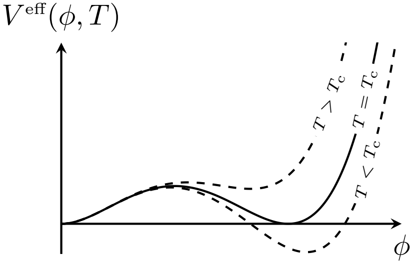


The effective potential in perturbation theory[1]





(B): Equilibrium thermodynamics as a function of BSM theory parameters. Encoded in **effective potential**, V^{eff}. Origin of uncertainty.

Important for v_w in the context of local thermal equilibrium (LTE).

[1] R. Jackiw, *Functional evaluation of the effective potential*, Phys. Rev. D **9** (1974) 1686

Theoretical predictions are not robust

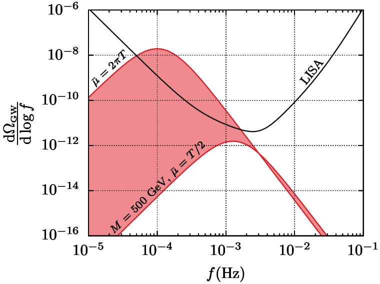
$\mathcal{O}(10^4)$ uncertainty even for purely perturbative regimes[2] as $\Omega_{\rm GW}$ depends strongly on the transition temperature, T_*, in simulation fits:

$$\Omega_{\rm GW} \propto \frac{(\Delta V_*)^2}{T_*^8}$$

- Ensure (improve) quantitative precision at finite T?

Minimal SM extensions e.g.:

- SMEFT: SM + $\frac{1}{M^2}(\phi^\dagger\phi)^3$



[2] D. Croon, O. Gould, P. Schicho, T. V. I. Tenkanen, and G. White, *Theoretical uncertainties for cosmological first-order phase transitions*, JHEP **04** (2021) 055 [2009.10080], O. Gould and T. V. I. Tenkanen, *On the perturbative expansion at high temperature and implications for cosmological phase transitions*, JHEP **06** (2021) 069 [2104.04399]

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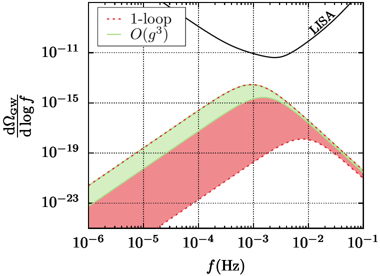
- ▷ Ensure (improve) quantitative precision at finite T?

Minimal SM extensions e.g.:

- ▷ SMEFT: SM + $\frac{1}{M^2}(\phi^\dagger\phi)^3$
- ▷ xSM: SM + singlet



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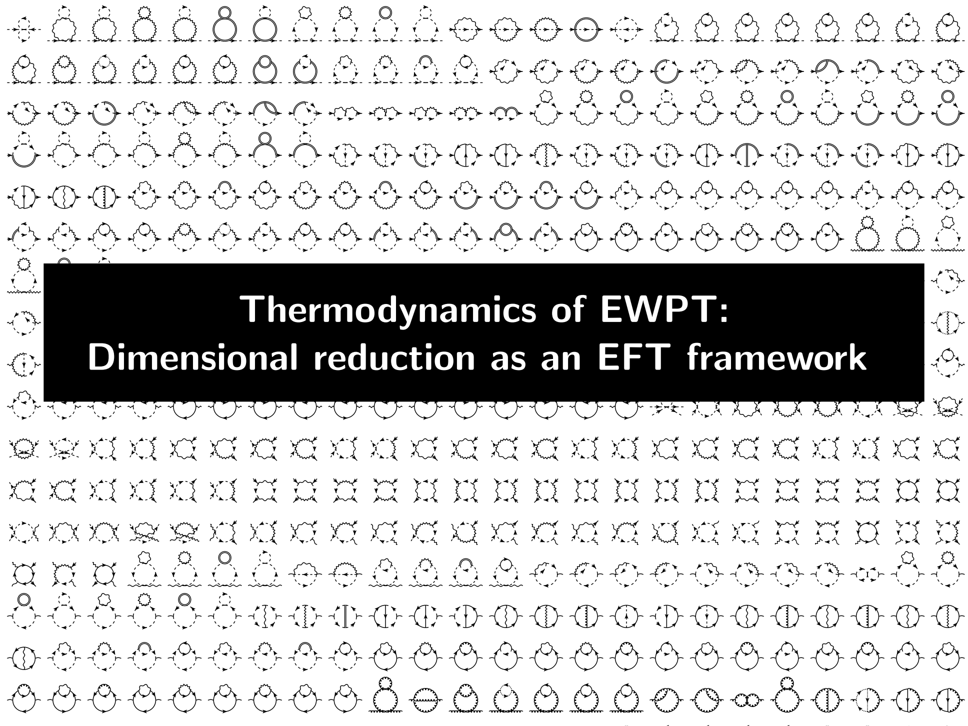
Thermodynamics of EWPT: Dimensional reduction as an EFT framework



Perturbative phase transitions need scale hierarchies

for quantum effects ΔV_{fluct} to influence the tree-level potential

$$V^{\text{eff}} = V_{\text{tree}} + \Delta V_{\text{fluct}}\ .$$

As relevant operators ($\sigma > 0$) in the IR get large UV contributions

$$\frac{\Delta V_{\text{fluct}}}{V_{\text{tree}}} \sim g^2 N \left[\frac{\Lambda_{\text{fluct}}}{\Lambda_{\text{tree}}}\right]^{\sigma} \overset{!}{\sim} 1\ .$$

Realised for

1. strong coupling $g^2 N \gtrsim 1$
2. scale hierarchies $\frac{\Lambda_{\text{fluct}}}{\Lambda_{\text{tree}}} \gg \frac{1}{(g^2 N)^{\frac{1}{\sigma}}}$

Equilibrium Thermodynamics: Imaginary Time Formalism

$\rho(\beta) = e^{-\beta\mathcal{H}} \to \mathcal{U}(t) = e^{-i\mathcal{H}t}$. Relating density operator to time evolution corresponds to path integral over imaginary-time $t \to -i\tau$,

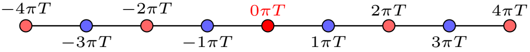
$$\mathcal{Z} = C \int_{\text{b.c.}} \mathcal{D}\phi \, \exp\left[-\int_0^{\beta=1/T} \mathrm{d}\tau \int_{\mathbf{x}} \mathcal{L}_{\text{E}} \right] , \quad \phi(0,\mathbf{x}) = \pm\phi(\beta,\mathbf{x}) \, .$$

(Anti-)periodic bosonic(fermionic) fields at boundaries $\to$ compact time direction: $\mathbb{R}^3 \times S^1_\beta$.

Finite-τ and (b.c.) induce a discrete Fourier sum for time component $P = (\omega_n, \mathbf{p})$ with **Matsubara frequencies**

$$\omega_n = \begin{cases} 2n\pi T & \text{bosonic} \\ (2n+1)\pi T & \text{fermionic} \end{cases}$$

Absent zero mode $\omega_{n=0}$ for fermions:



Multi-scale hierarchy in hot gauge theories

Evaluated **Matsubara sums** yield Bose(Fermi) distribution. At asymptotically high-T and weak $g \ll 1$ the effective expansion parameter

$$g^2 n_{\mathrm{B}}(|p|) = \frac{g^2}{e^{|p|/T} - 1} \approx \frac{g^2 T}{|p|}$$

differs from the weak coupling g^2. Fermions are IR-safe $g^2 n_{\mathrm{F}} |p| \sim g^2/2$.

Theory separates scales rigorously:

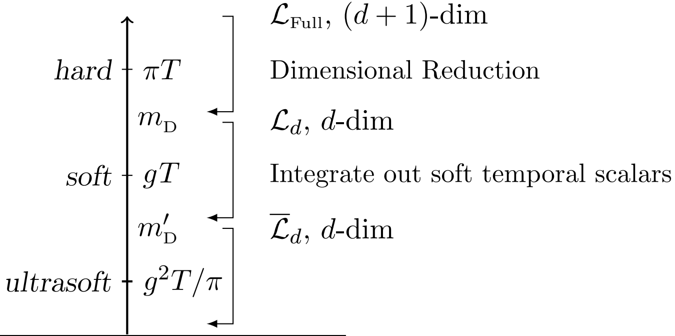
$$|p| \sim \begin{cases} \pi T & \textit{hard}\ \text{scale} \\ gT & \textit{soft}\ \text{scale} \\ g^{3/2} T & \boxed{\textit{supersoft}\ \text{scale} \quad \text{symmetry breaking}} \\ g^2 T/\pi & \textit{ultrasoft}\ \text{scale} \end{cases}$$

Limit: Confinement-like behavior in ultrasoft sector $g^2 n_{\mathrm{B}}(g^2 T) \sim \mathcal{O}(1)$. Ultrasoft bosons are non-perturbative at finite T: Linde IR problem.[3]

[3] A. Linde, *Infrared problem in the thermodynamics of the Yang-Mills gas*, Phys. Lett. B **96** (1980) 289

Effective Field Theory (EFT): Dimensional Reduction (DR)

Integrate out hard modes perturbatively → EFT for static modes.
Incorporates an all order thermal resummation to by-pass IR problem.
Precision thermodynamics of non-Abelian gauge theories as QCD and (EW) phase transition[4] using e.g. `DRalgo`[5]

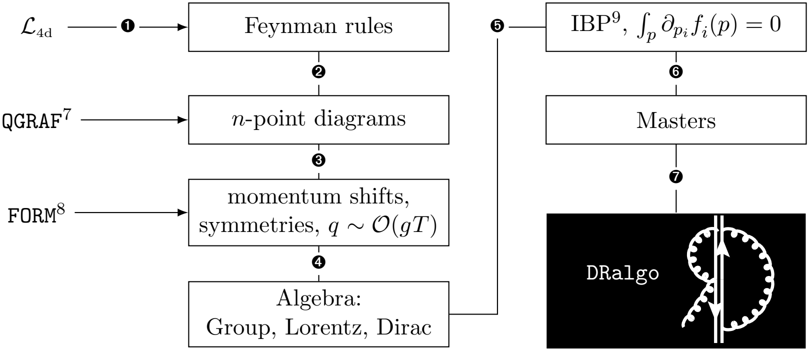


[4] K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *Generic rules for high temperature dimensional reduction and their application to the standard model,* Nucl. Phys. B **458** (1996) 90 [hep-ph/9508379], K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *The Electroweak phase transition: A Nonperturbative analysis,* Nucl. Phys. B **466** (1996) 189 [hep-lat/9510020]

[5] A. Ekstedt, P. Schicho, and T. V. I. Tenkanen, `DRalgo`*: A package for effective field theory approach for thermal phase transitions,* Comput. Phys. Commun. **288** (2023) 108725 [2205.08815]

A Dimensional Reduction algorithm (DRalgo)

State-of-the-art `Mathematica` package `DRalgo`.[6]



[6] `github.com/DR-algo/DRalgo`, A. Ekstedt, P. Schicho, and T. V. I. Tenkanen, `DRalgo`*: A package for effective field theory approach for thermal phase transitions,* Comput. Phys. Commun. **288** (2023) 108725 [`2205.08815`]

[7] P. Nogueira, *Automatic Feynman Graph Generation,* J. Comput. Phys. **105** (1993) 279

[8] B. Ruijl, T. Ueda, and J. Vermaseren, *FORM version 4.2* arXiv (2017) [`1707.06453`]

[9] S. Laporta, *High precision calculation of multiloop Feynman integrals by difference equations,* Int. J. Mod. Phys. A **15** (2000) 5087 [`hep-ph/0102033`]

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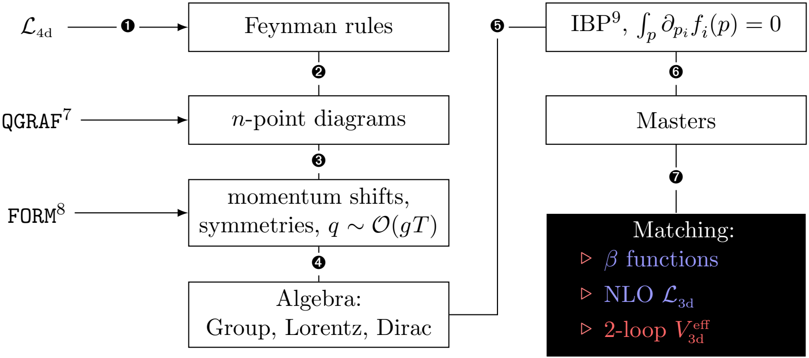

[6] github.com/DR-algo/DRalgo, A. Ekstedt, P. Schicho, and T. V. I. Tenkanen, DRalgo*: A package for effective field theory approach for thermal phase transitions*, Comput. Phys. Commun. **288** (2023) 108725 [2205.08815]

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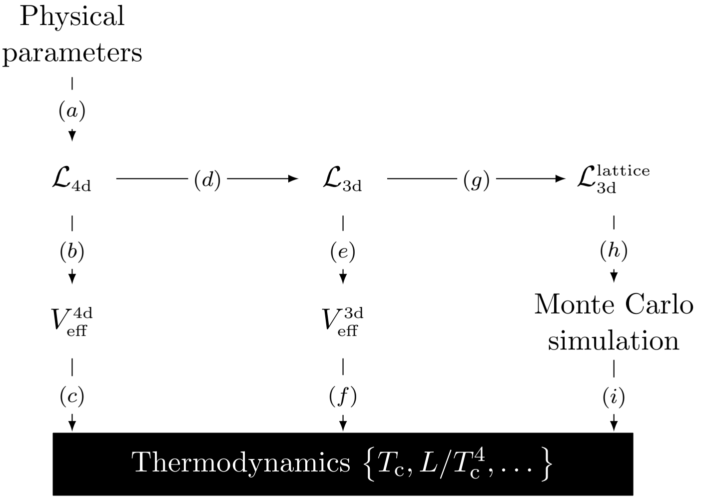
Thermodynamics of electroweak phase transition



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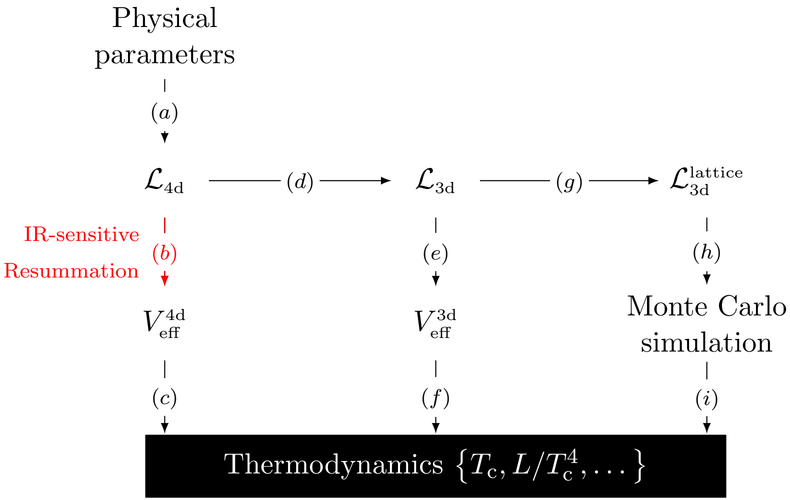
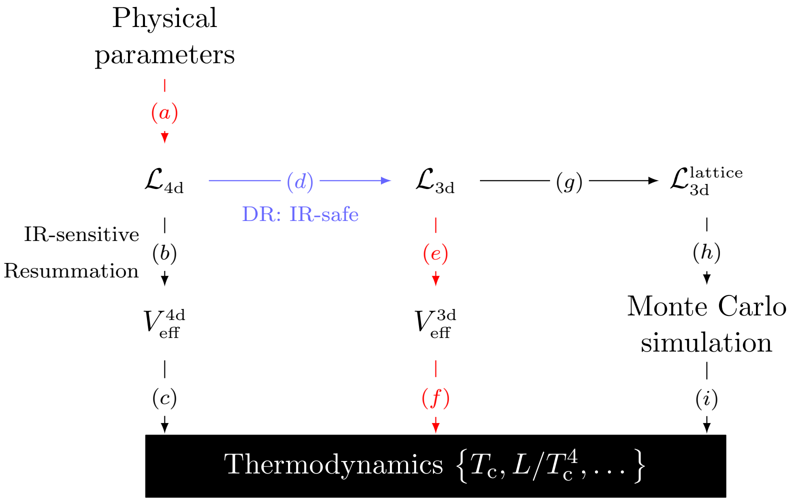


▷ $4d$ approach: $(a) \to (b) \to (c)$

Thermodynamics of electroweak phase transition



- $4d$ approach: $(a) \to (b) \to (c)$
- Perturbative $3d$ approach: $(a) \to (d) \to (e) \to (f)$

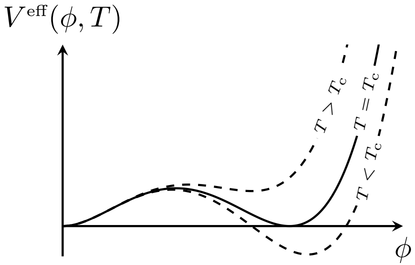
Improving accuracy of EWPT: Effective potential

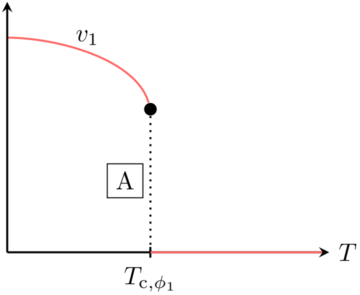
The effective potential in perturbation theory

receives thermal corrections $\Pi_T \sim \gamma T^2$ with $\gamma \sim g^n$. Dynamical generation of scales close to critical temperature T_c:

$$V^{\text{eff}} \simeq \frac{1}{2}(-\mu^2 + \Pi_T)\phi^2 + \frac{1}{2}\lambda\phi^4 - \frac{g^3 T}{16\pi}\phi^3 + \dots$$

$$(-\mu^2 + g^n T^2) \sim \underbrace{0 \times (gT)^2}_{\text{soft}} + \underbrace{0 \times (g^{3/2}T)^2}_{\text{supersoft}} + \underbrace{\#(g^2T)^2}_{\text{ultrasoft}}.$$





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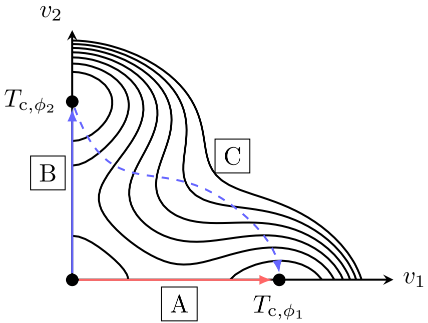
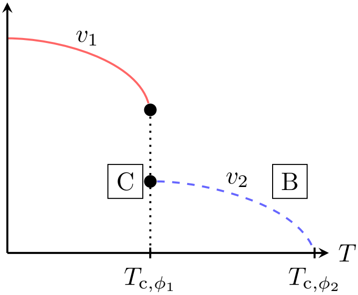


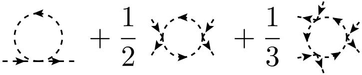


The thermal effective potential at LO

$$V^{\rm eff} = V^{\rm eff}_{\rm tree} + V^{\rm eff}_{1\ell}\,.$$

At 1-loop sum over n-point functions at $Q_i = 0$ external momenta

$$V^{\rm eff}_{1\ell} =$$



$$+ \dots \Big|_{Q_i=0}$$

$$= \frac{1}{2}\sum\!\!\!\!\!\!\!\int_P \ln\left(P^2 + m^2\right)$$

$$V^{\rm eff}_{1\ell} = \underbrace{\frac{1}{2}\int_P \ln(P^2+m^2)}_{\equiv V_{\rm CW}(m)} \underbrace{-T\int_p \ln\Big(1 \mp n_{\rm B/F}(E_p,T)\Big)}_{\equiv V_{T,b/f}\left(\frac{m^2}{T^2}\right)}$$

$$= \underbrace{\frac{T}{2}\int_p \ln(p^2+m^2)}_{\equiv T V_{\rm soft}(m)} + \underbrace{\frac{1}{2}\sum\!\!\!\!\!\!\!\int_{P/\{P\}}{}' \ln(P^2+m^2)}_{\equiv V_{\rm hard}(m)}\,.$$

Renormalization scale (in)dependence at finite T

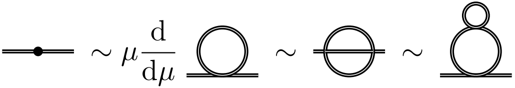
Zero temperature

$$V^{\text{eff}}(\phi,\bar{\mu}) = \underbrace{V^{\text{eff}}_{\text{tree}}}_{\mathcal{O}(g^2)} + \underbrace{V_{\text{CW},1\ell}}_{\mathcal{O}(g^4)}\,, \quad \mu\frac{\mathrm{d}}{\mathrm{d}\mu}\Big(V^{\text{eff}}_{\text{tree}} + V_{\text{CW},1\ell}\Big) = 0\,.$$

At finite temperature[10]

$$V^{\text{eff}}_{\text{res.}}(\phi,T,\bar{\mu}) = \underbrace{V^{\text{eff}}_{\text{tree}}}_{\mathcal{O}(g^2)} + \underbrace{V_{\text{res.,soft}}}_{\mathcal{O}(g^3)} + \underbrace{V_{\text{hard}}}_{\mathcal{O}(g^2T^2)+\mathcal{O}(g^4)}\,,$$

running of 1-loop thermal masses is of the same order as 2-loop thermal-mass logarithms.
Automatically included in dimensionally reduced $3d$-approach:

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}$$

$$\sim \mathcal{O}(g^4T^2)$$

[10] O. Gould and T. V. I. Tenkanen, *On the perturbative expansion at high temperature and implications for cosmological phase transitions*, JHEP **06** (2021) 069 [2104.04399]

The effective potential at NLO and beyond

$$V^{\text{eff}} = V^{\text{eff}}_{\text{tree}} + V^{\text{eff}}_{1\ell} + V^{\text{eff}}_{2\ell} + V^{\text{eff}}_{3\ell} \; .$$

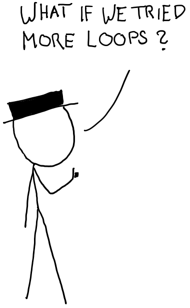


figure adapted from `xkcd.com`

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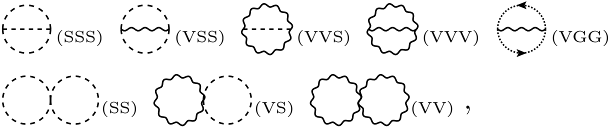
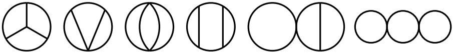
Computing 2-loop[11] and 3-loop[12] V^{eff} via vacuum integrals in $3d$ EFT:

$V^{\text{eff}}_{2\ell} \supset$



,

$V^{\text{eff}}_{3\ell} \supset$



.

Todo: extend 3-loop V^{eff} to general models.

[11] K. Farakos, K. Kajantie, K. Rummukainen, and M. E. Shaposhnikov, *3-D physics and the electroweak phase transition: Perturbation theory,* Nucl. Phys. B **425** (1994) 67 [hep-ph/9404201], M. Laine, *The Two loop effective potential of the 3-d SU(2) Higgs model in a general covariant gauge,* Phys. Lett. B **335** (1994) 173 [hep-ph/9406268], L. Niemi, M. Ramsey-Musolf, T. V. I. Tenkanen, and D. J. Weir, *Thermodynamics of a two-step electroweak phase transition,* [2005.11332], L. Niemi, P. Schicho, and T. V. I. Tenkanen, *Singlet-assisted electroweak phase transition at two loops,* Phys. Rev. D **103** (2021) 115035 [2103.07467]

[12] A. K. Rajantie, *Feynman diagrams to three loops in three-dimensional field theory,* Nucl. Phys. B **480** (1996) 729 [hep-ph/9606216]

figure adapted from xkcd.com

A minimal scheme for gauge invariance and resummation

1. Determine 3d EFT at NLO (gauge-invariant)
2. Compute $V_{3\mathrm{d}}^{\mathrm{eff}}$ within 3d EFT at 1-loop level
3. Determine T_c, condensates e.g. $\langle \phi^\dagger \phi \rangle$, and latent heat

Minimum of V^{eff} is gauge parameter independent (Nielsen identities[13]); use $\hbar$-expansion. Improve previous schemes.[14]

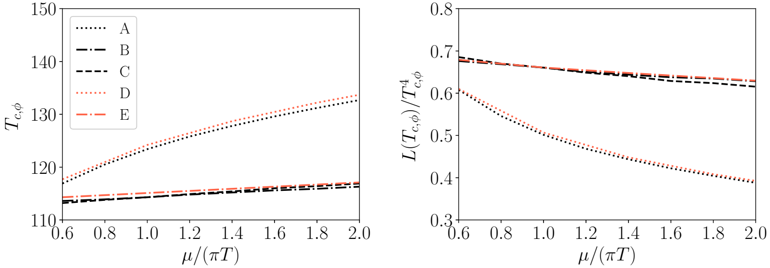
[13] N. Nielsen, *On the gauge dependence of spontaneous symmetry breaking in gauge theories*, Nucl. Phys. B **101** (1975) 173

[14] PRM scheme: H. H. Patel and M. J. Ramsey-Musolf, *Baryon Washout, Electroweak Phase Transition, and Perturbation Theory*, JHEP **2011** (2011) 29 [1101.4665]

Increasing accuracy to $\mathcal{O}(g^4)$: cxSM (complex singlet)

Augment SM with **complex singlet scalar**[15], $\mathbb{S} \to v_{\mathbb{S}} + \mathbb{S} + iA$ at

Benchmark	$M_{\mathbb{S}}$	M_A	λ_p	$\lambda_{\mathbb{S}}$
BM1	62.5 GeV	62.5 GeV	0.55	0.5



- A, D: 1-loop level dimensional reduction
- B, E: 2-loop level dimensional reduction
- C: as B, with varying $\mu_3 = \mu/(\pi T) g_3^2$

[15] P. Schicho, T. V. I. Tenkanen, and G. White, *Combining thermal resummation and gauge invariance for electroweak phase transition*, JHEP **11** (2022) 047 [2203.04284], C.-W. Chiang, M. J. Ramsey-Musolf, and E. Senaha, *Standard Model with a Complex Scalar Singlet: Cosmological Implications and Theoretical Considerations*, Phys. Rev. D **97** (2017) 1 [1707.09960]

Conclusions

Precision thermodynamics of BSM theories:

- reliably describe cosmological FOPT and GW production,
- practical approach: **Effective Theories**.

Precision cosmology with dimensionally reduced 3d EFT:

- multi-loop sport – automatic all-order high-T resummation,
- analytic fermions, numerical on the lattice at $T_c \sim 100$ GeV,
- systematic higher-loop/operator improvement,
- **universality**,
- apply to **supercooled phase transitions**(?),
- accurate description of the phase transition.[a]

[a] D. Croon, O. Gould, P. Schicho, T. V. I. Tenkanen, and G. White, *Theoretical uncertainties for cosmological first-order phase transitions*, JHEP **04** (2021) 055 [2009.10080]