შოთა რუსთაველის საქართველოს ეროვნული სამეცნიერო ფონდი
SHOTA RUSTAVELI NATIONAL SCIENCE FOUNDATION OF GEORGIA

Cosmological Scalar Field ϕCDM Models

Olga Avsajanishvili

E.Kharadze Georgian National Astrophysical Observatory

The presentation is supported in part by the Rustaveli National Science Foundation of Georgia under Grant YS-22-998

Online Conference *Cosmology from Home 2023*, United Kingdom
3-14 July 2023

Hubble - Lemaître law

$$V = H_0 D$$

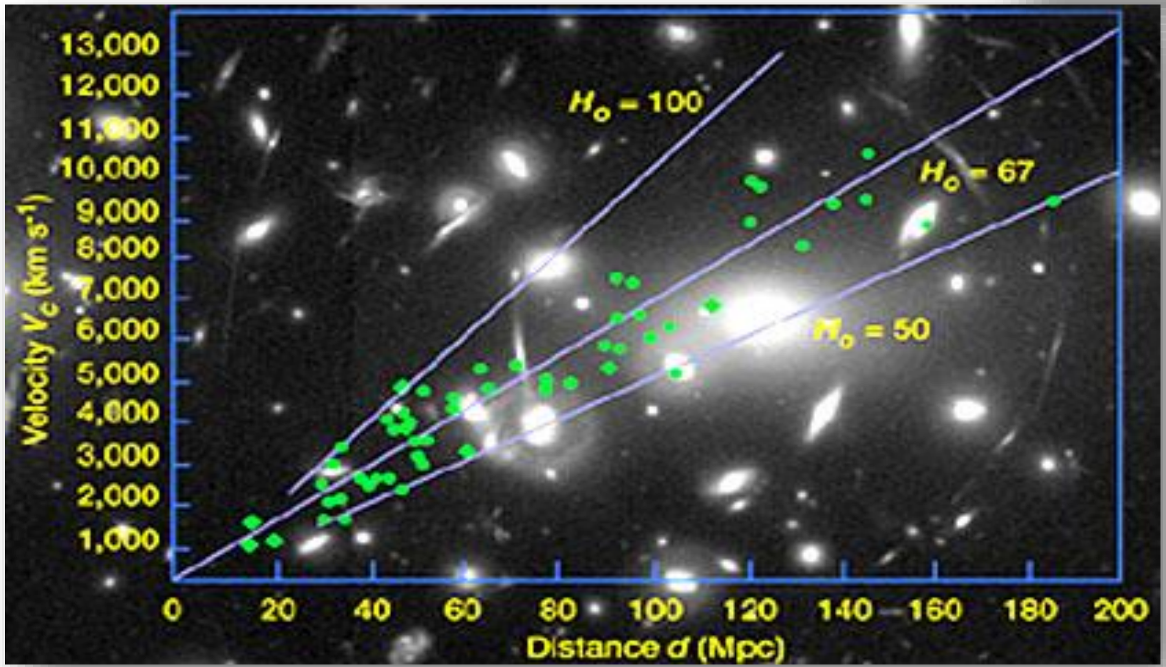
H_0 is a Hubble constant

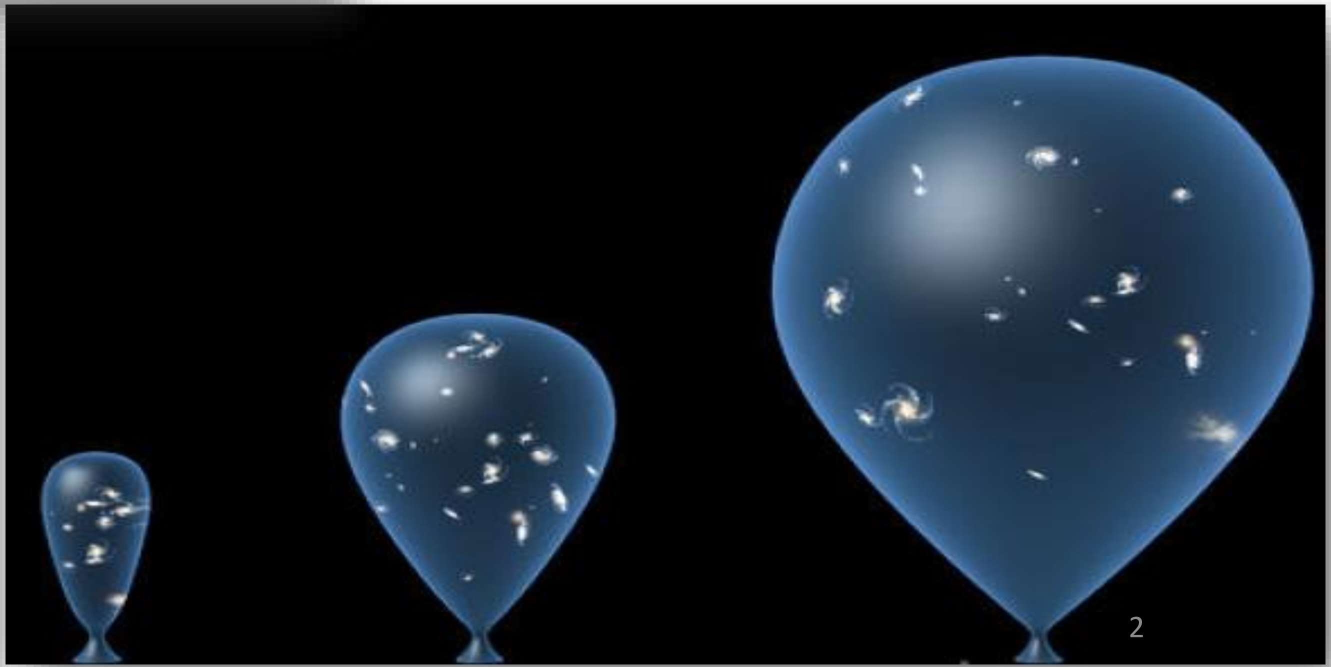
H_0= 67.4 km s^{-1}Mpc^{-1} (Planck 2018 results)

E.Hubble

G.Lemaître

V.Slipher





Accelerated expansion of the Universe

Nobel Prize in Physics 2011

Photo: Lawrence Berkeley National Lab — Photo: Belinda Pratten, Australian National University — Photo: Scanpix/AFP

Saul Perlmutter — **Brian P. Schmidt** — **Adam G. Riess**

The Nobel Prize in Physics 2011 was awarded *"for the discovery of the accelerating expansion of the Universe through observations of distant supernovae"* with one half to Saul Perlmutter and the other half jointly to Brian P. Schmidt and Adam G. Riess.

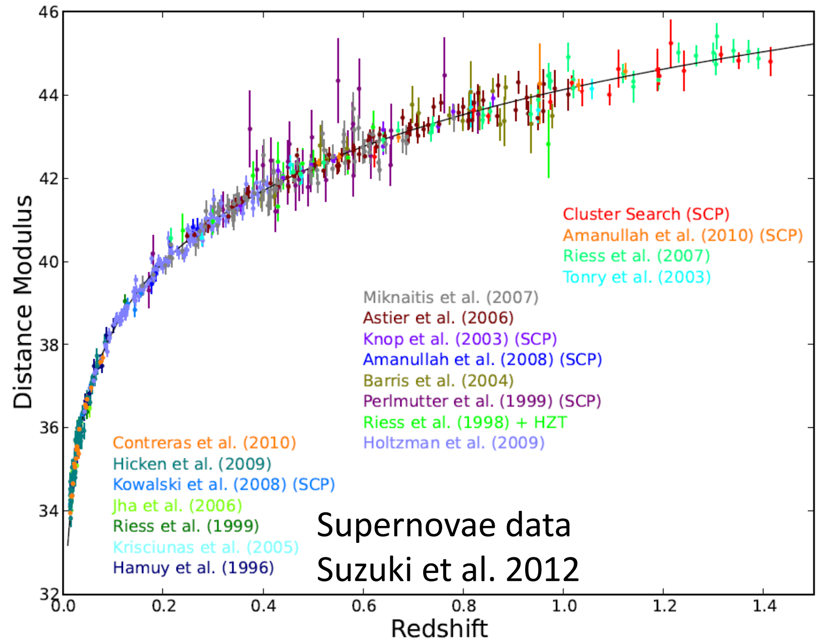
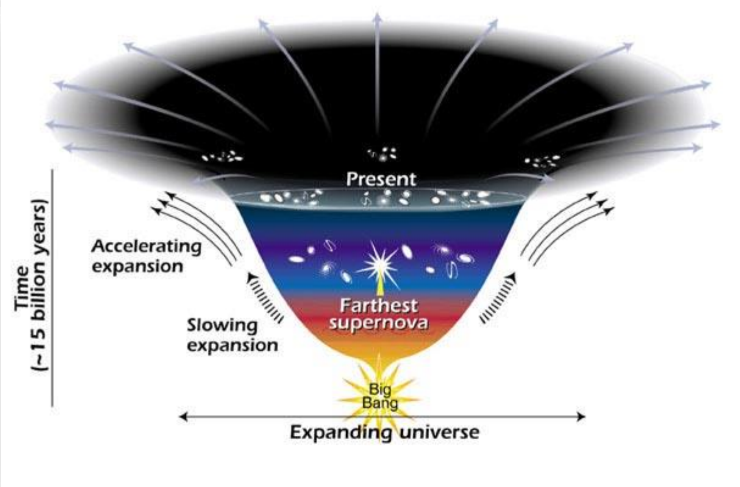


Figure 4. Hubble diagram for the Union2.1 compilation. The solid line represents the best-fit cosmology for a flat ΛCDM Universe for supernovae alone. SN SCP06U4 falls outside the allowed x_1 range and is excluded from the current analysis. When fit with a newer version of SALT2, this supernova passes the cut and would be included, so we plot it on the Hubble diagram, but with a red triangle symbol.

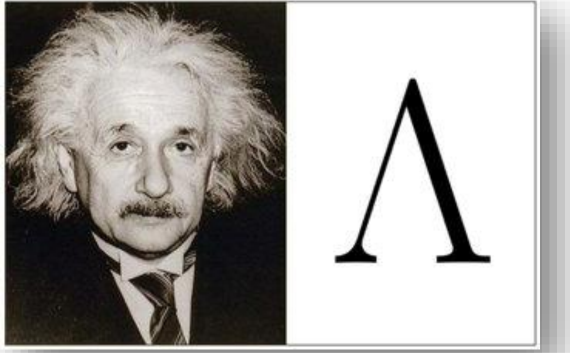


Modified Gravity? Dark Energy?

Dark energy models

- Cosmological standard ΛCDM model (the concept of vacuum energy)
- Scalar field ϕCDM models
  - Quintessence (freezing/thawing) models
  - Phantom models
- Tachyon field models
- Dilaton field models
- K-essence models
- Holographic dark energy models
- Barotropic fluid or Chaplygin gas models
- Coupled dark matter (neutrino) - dark energy models

and many more…



Cosmological Constant Λ

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

"Einstein's Greatest Blunder"

- **The simplest description for dark energy is the concept of vacuum energy or the time-independent cosmological constant Λ, first introduced by Albert Einstein;**
- **The cosmological model based on such a description of dark energy in the spatially flat Universe is called the standard, concordance or fiducial Lambda Cold Dark Matter (ΛCDM) model;**
- **The ΛCDM model is based on GR for description of gravity in the Universe on large scales;**
- **The energy density associated with the cosmological constant is about 70% of the total energy density of the Universe at present;**
- **The action:** $$S = -\frac{1}{16\pi G} \int d^4x \sqrt{-g}(R + 2\Lambda) + S_M$$
- **Friedmann's equations:** $$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho_\Lambda - \frac{k}{a^2} + \frac{\Lambda}{3}, \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho_\Lambda + 3p_\Lambda) + \frac{\Lambda}{3},$$

where g is the determinant of the metric tensor $g_{\mu\nu}$; R is the Ricci scalar, a is a scale factor, S_m denotes the matter action, p_Λ and $\rho_\Lambda = \frac{\Lambda}{8\pi G}$ are the pressure and dark energy density of the cosmological constant Λ, respectively; k is a curvature parameter; $\Lambda = 4.33 \cdot 10^{-66}$ eV.

Problems of the ΛCDM model as a crisis in modern cosmology

Being still a fiducial cosmological model at present, the ΛCDM model has several still unsolved problems, the number of which increases as more accurate observational data are obtained.

The main of which are:

- **Fine tuning or cosmological constant problem** (the observed value of the cosmological constant Λ is 120 values less than its theoretically predicted value); Type equation here.

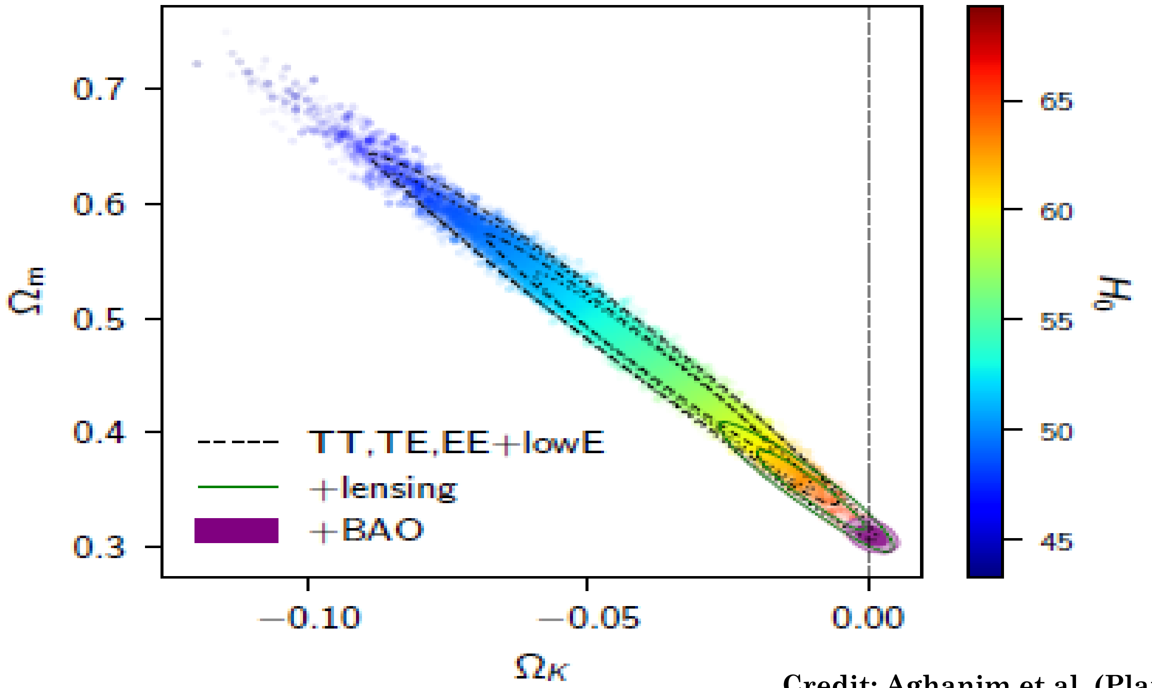
- **Coincidence problem** (according to the data of the Planck satellite, at present epoch the cosmological constant energy density (68.5%) is comparable (in order of magnitude) with the energy density of matter (31.5%), despite the fact that these quantities have evolved differently);

- **Hubble parameter tension problem** (a discrepancy between the value of Hubble parameter H_0 at present epoch obtained by the local measurements and CMB temperature anisotropy data);

- **The parameter $S_8 = \sigma_8\sqrt{\Omega_m/0.3}$ tension problem** (a discrepancy between the primary CMB temperature anisotropy measurements by the Planck satellite in the strength of matter clustering compared to lower redshift measurements such as the weak gravitational lensing and galaxy clustering.) Here σ_8 is the clustering amplitude of the matter power spectrum on scales of $8h^{-1}$Mpc, Ω_m is a fractional matter density parameter.

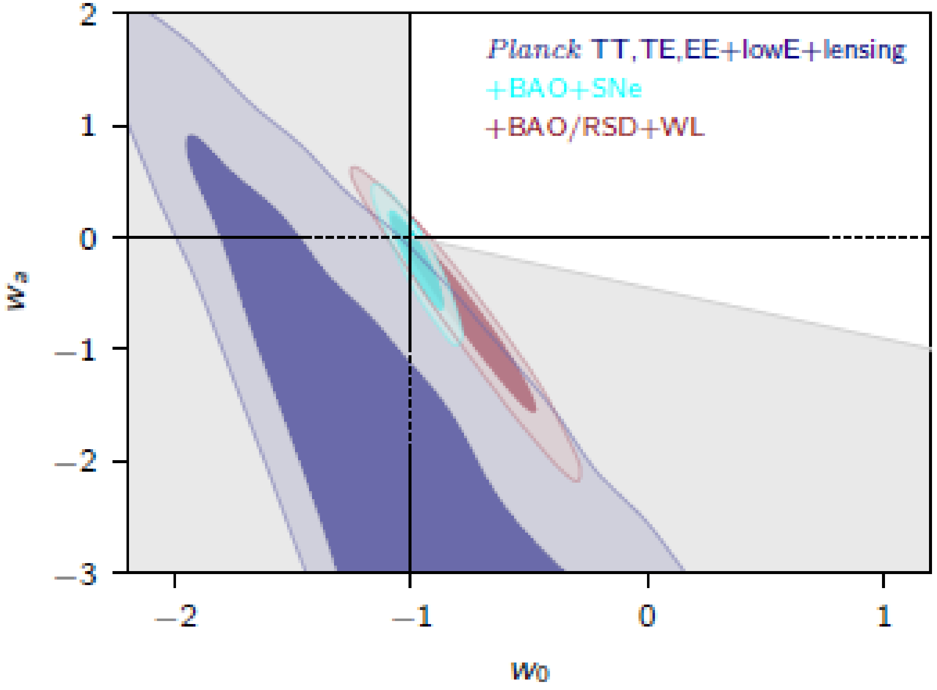
Problems of the ΛCDM model as a crisis in modern cosmology

- **The problem of the shape of the Universe (the preference for observational data of closed hyperspaces)**



Credit: Aghanim et al. (Planck 2018 results)

- **The preference for observational data of dynamical dark energy (in particular, phantom dark energy).**



Credit: Aghanim et al. (Planck 2018 results)

The presence of all the above discrepancies of the ΛCDM model is interpreted as a crisis of modern cosmology. Although some of them may be due to systematic errors, their persistence strongly points to the need for new physics and new cosmological models that go beyond the standard ΛCDM scenario, on the one hand, and on tensions and anomalies in the current CMB data, the "queen" of cosmological data, on the other.

Dynamical scalar field ϕCDM models

- **In these models, dark energy is represented in the form of a slowly varying uniform cosmological scalar field at present;**
- **This family of models avoids the coincidence problem of the ΛCDM model;**
- **In these models, the energy density $\rho(t)$ and pressure $p(t)$ are time dependent functions under the assumption that the scalar field is described by the ideal barotropic fluid model;**

Type equation here.

- **Dark energy models are characterized by the equation of state parameter, $\omega = p/\rho$:**

- **for the ΛCDM model:** $p_\Lambda = -\rho_\Lambda = -\frac{\Lambda}{8\pi G} = const \rightarrow \omega_\Lambda = \frac{p_\Lambda}{\rho_\Lambda} = -1$
- **for ϕCDM models:** $\omega_\phi(t) \neq -1$

  $\omega_\phi(t) \approx -1$ **(nowadays)**

- **Dynamical dark energy can mimic the cosmological constant Λ at present, while becoming almost indistinguishable from it.**

Quintessence and Phantom scalar field ϕCDM models

$$S = \int d^4x\sqrt{-g}\Big[-\frac{M_{\rm pl}^2}{16\pi}R + \mathcal{L}_\phi\Big] + S_{\rm M}$$

Quintessence models	Phantom models
$-1 < w_0 < -1/3$	$w_0 < -1$
$\mathcal{L}_\phi = \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)$	$\mathcal{L}_\phi = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)$
$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial\phi} = 0$	$\ddot{\phi} + 3H\dot{\phi} - \frac{\partial V(\phi)}{\partial\phi} = 0$
By the dynamics of scalar fields: the quintessence field rolls gradually to the minimum of its potential.	The phantom field rolls to the maximum of its potential.
By the temporal evolution of dark energy: the dark energy density for the quintessence field remains almost unchanging with time.	The dark energy density for the phantom field increases with time.
By forecasting the future of the Universe: depending on the spatial curvature of the Universe, the quintessence models predict either an eternal expansion of the Universe, or a repeated collapse.	Phantom models predict the destruction of any gravitationally-related structures in the Universe.

The quintessence Ratra-Peebles scalar field ϕCDM model

- **Inverse power-law Ratra-Peebles potential:**

$$V(\phi) = \frac{\kappa}{2} M_{\rm pl}^2 \phi^{-\alpha}$$

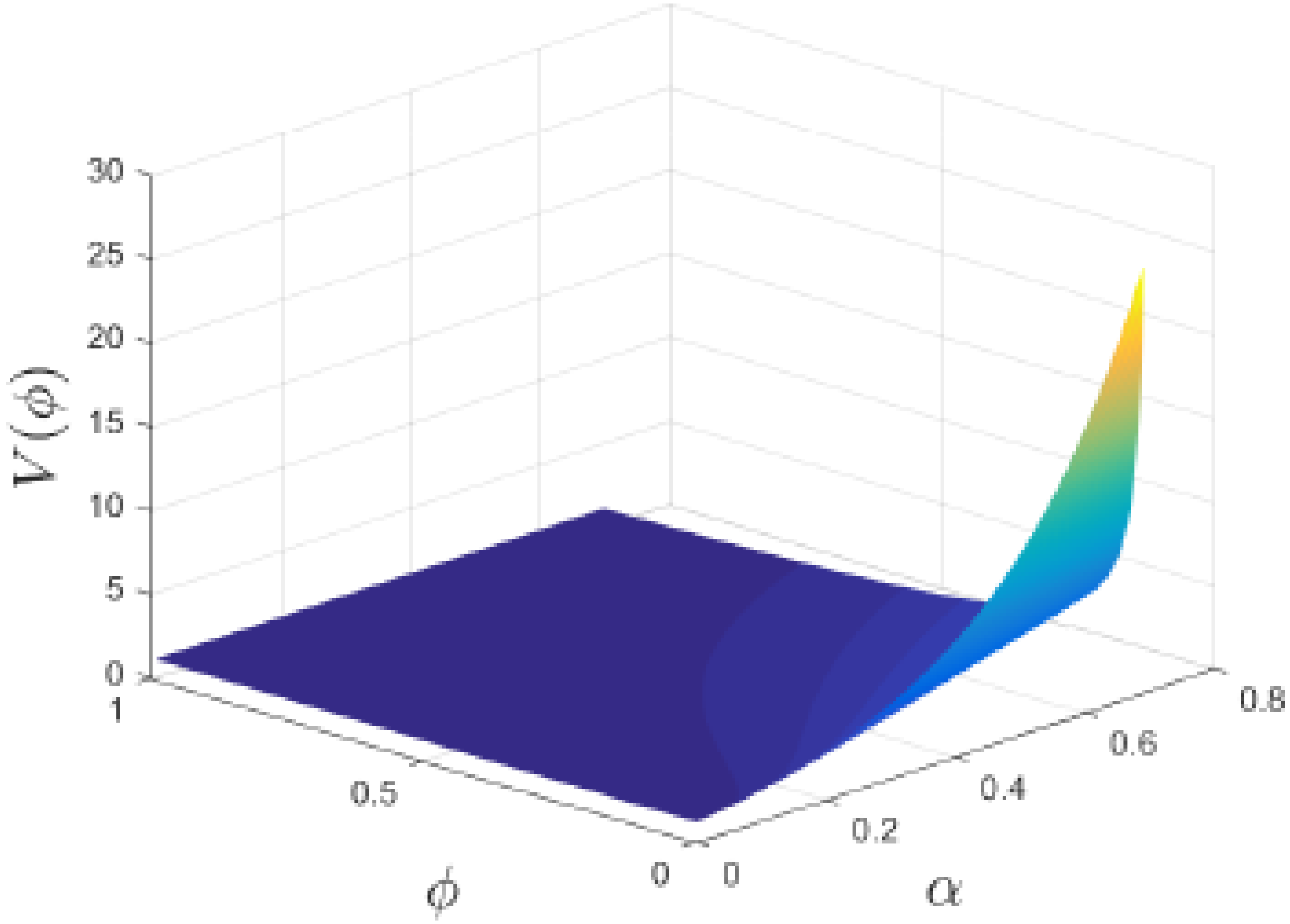
B. Ratra and P. J. E. Peebles, Phys. Rev. **D37**, 3406 (1988)

- **Model parameters:** $\alpha > 0 \qquad k > 0$

$$0 < \alpha \leq 0.7$$

L. Samushia, arXiv:0908.4597

$$\alpha \longrightarrow 0 \qquad \phi\text{CDM} \longrightarrow \Lambda\text{CDM}$$



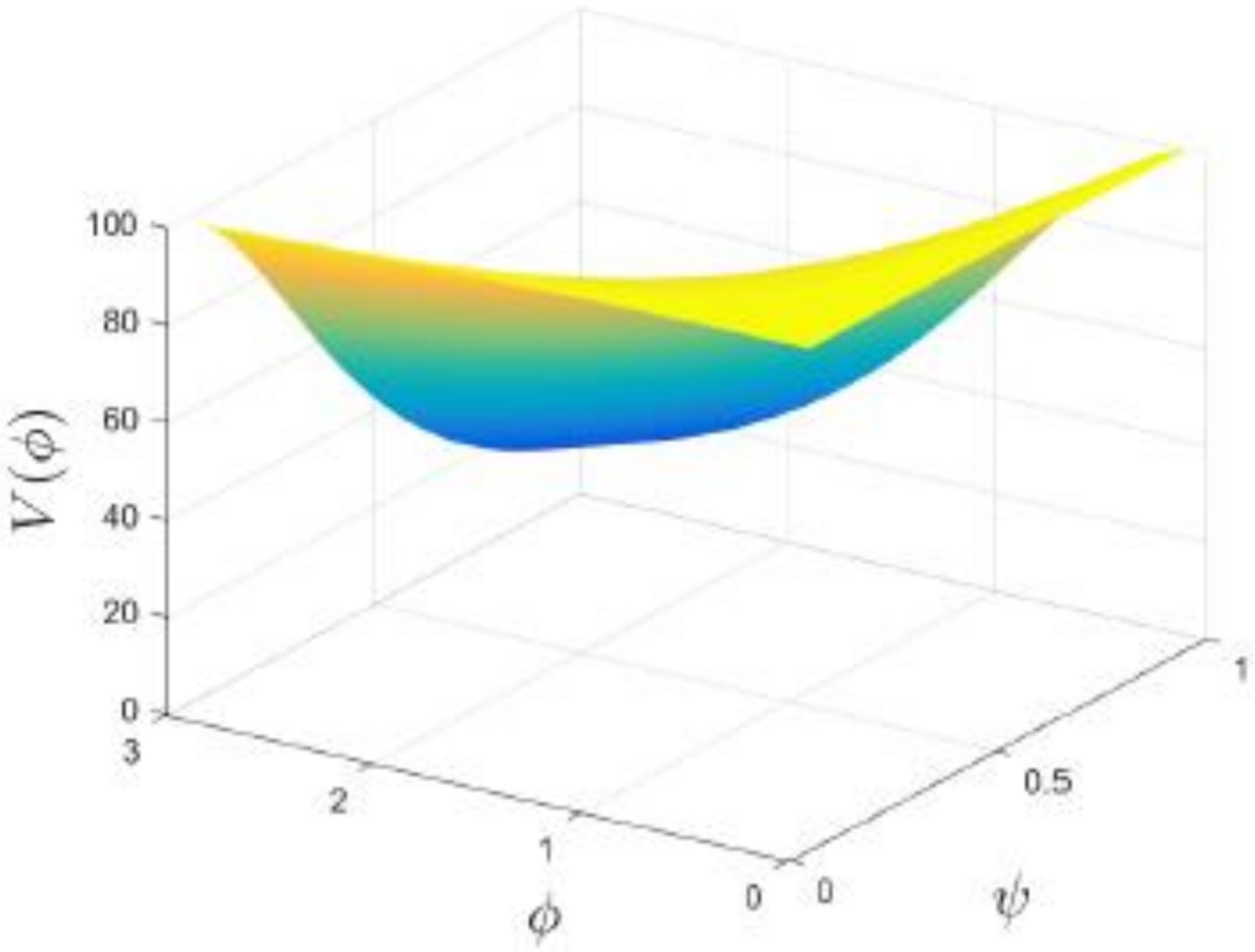
(In units: M_{pl}=1.)

The phantom inverse hyperbolic cosine scalar field ϕCDM model

- **Phantom inverse hyperbolic cosine potential:**

$$V(\phi) = V_0 \cosh^{-1}(\psi\phi)$$

R. Rakhi and K. Indulekha, arXiv:0910.5406 (2009)



Model parameters:

$V_0 > 0 \qquad \psi > 0$

The quintessence Ratra-Peebles scalar field ϕCDM model

- **The expansion rate** $$E(a) = H(a)/H_0 = \left(\Omega_{\rm r0}a^{-4} + \Omega_{\rm m0}a^{-3} + \frac{1}{12H_0^2}\left(\dot{\phi}^2 + \kappa M_{\rm pl}^2\phi^{-\alpha}\right)\right)^{1/2}$$

- **The scalar field equation** $$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} - \frac{1}{2}\kappa\alpha M_{\rm pl}^2\phi^{-(\alpha+1)} = 0$$

- **Fractional matter density** $$\Omega_{\rm m} = \frac{\Omega_{\rm m0}a^{-3}}{\Omega_{\rm r0}a^{-4} + \Omega_{\rm m0}a^{-3} + \frac{1}{12H_0^2}\left(\dot{\phi} + \kappa M_{\rm pl}^2\phi^{-\alpha}\right)}$$

- **The scalar field density parameter** $$\Omega_\phi = \frac{1}{12H_0^2}\left(\dot{\phi}^2 + \kappa M_{\rm pl}^2\phi^{-\alpha}\right)$$

- **The energy density of the scalar field** $$\rho_\phi = \frac{M_{\rm pl}^2}{32\pi}\left(\dot{\phi}^2 + \kappa M_{\rm pl}^2\phi^{-\alpha}\right)$$ 0

- **The pressure of the scalar field** $$p_\phi = \frac{M_{\rm pl}^2}{32\pi}\left(\dot{\phi}^2 - \kappa M_{\rm pl}^2\phi^{-\alpha}\right)$$

- **The equation of state parameter** $$w_\phi = \frac{\dot{\phi}^2 - \kappa M_{\rm pl}^2\phi^{-\alpha}}{\dot{\phi}^2 + \kappa M_{\rm pl}^2\phi^{-\alpha}}$$

- **We consider a flat and isotropic universe, which is described by the space time Friedmann-Lemaître -Robertson-Walker (FLRW) metric: $ds^2 = -dt^2 + a(t)^2dx^2$, with normalization of the scale factor to be equal to one at present time, $a_{today} = a_0 = 1$, $H(a)$ is a Hubble parameter, Ω_{ro} is a density parameter for radiation at present epoch, Ω_{mo} is a density parameter for matter at present epoch.**

The phantom inverse hyperbolic cosine scalar field ϕCDM model

- **The expansion rate**

$$E(a) = H(a)/H_0 = \left(\Omega_{\rm r0}a^{-4} + \Omega_{\rm m0}a^{-3} + \frac{1}{6H_0^2}\left(-\frac{\dot{\phi}^2}{2} + V_0\cosh^{-1}(\psi\phi)\right)\right)^{1/2}$$

- **The scalar field equation**

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + V_0\psi\tanh(\psi\phi)\cosh^{-1}(\psi\phi) = 0$$

- **Fractional matter density**

$$\Omega_{\rm m} = \frac{\Omega_{\rm m0}a^{-3}}{\Omega_{\rm r0}a^{-4} + \Omega_{\rm m0}a^{-3} + \frac{1}{6H_0^2}\left(-\frac{\dot{\phi}^2}{2} + V_0\cosh^{-1}(\psi\phi)\right)}$$

- **The scalar field density parameter**

$$\Omega_\phi = \frac{1}{6H_0^2}\left(-\dot{\phi}^2/2 + V_0\cosh^{-1}(\psi\phi)\right)$$

- **The energy density of the scalar field**

$$\rho_\phi = \frac{M_{\rm pl}^2}{16\pi}\left(-\dot{\phi}^2/2 + V_0\cosh^{-1}(\psi\phi)\right)$$

$$p_\phi = \frac{M_{\rm pl}^2}{16\pi}\left(-\dot{\phi}^2/2 - V_0\cosh^{-1}(\psi\phi)\right)$$

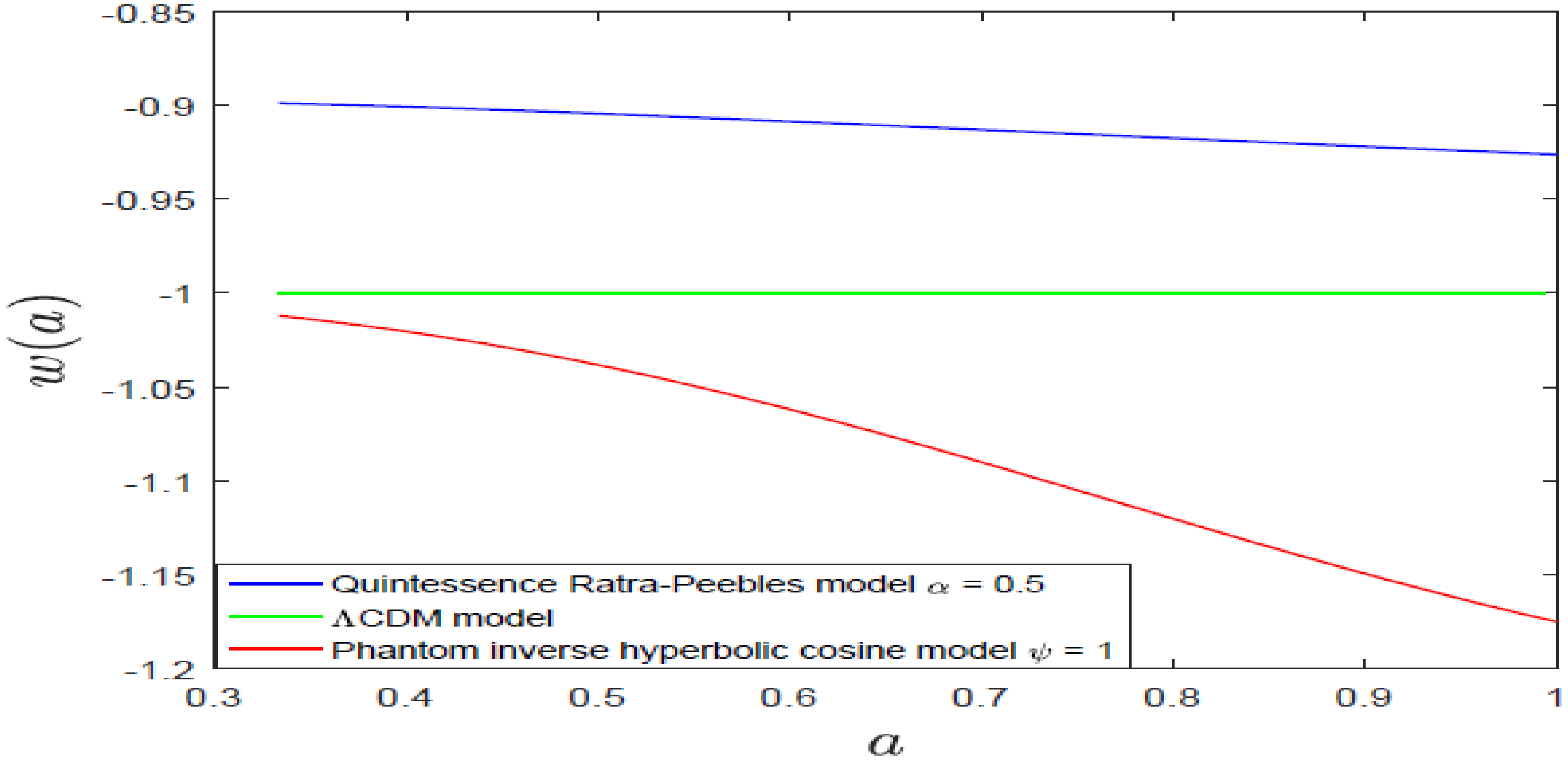
- **The pressure of the scalar field**

$$w_\phi = \frac{-\dot{\phi}^2/2 - V_0\cosh^{-1}(\psi\phi)}{-\dot{\phi}^2/2 + V_0\cosh^{-1}(\psi\phi)}$$

- **The equation of state parameter**

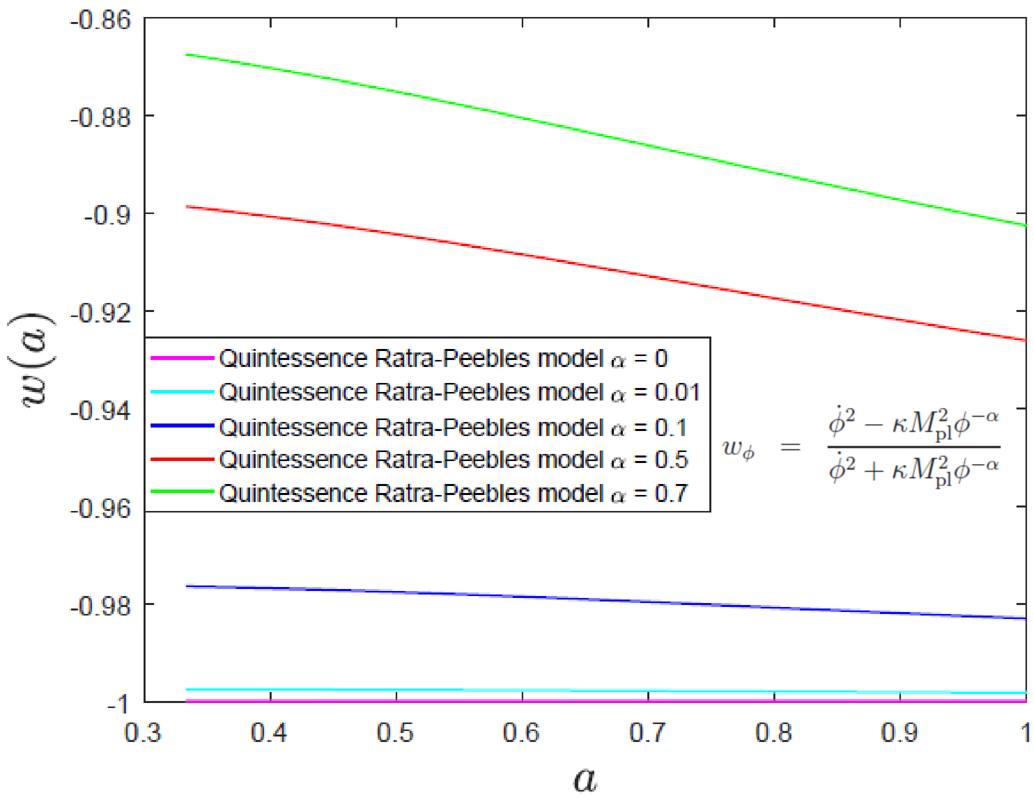
- **We consider a flat and isotropic universe, which is described by the space time Friedmann-Lemaître - Robertson-Walker (FLRW) metric: $ds^2 = -dt^2 + a(t)^2 dx^2$, with normalization of the scale factor to be equal to one at present time, $a_{today} = a_0 = 1$.**

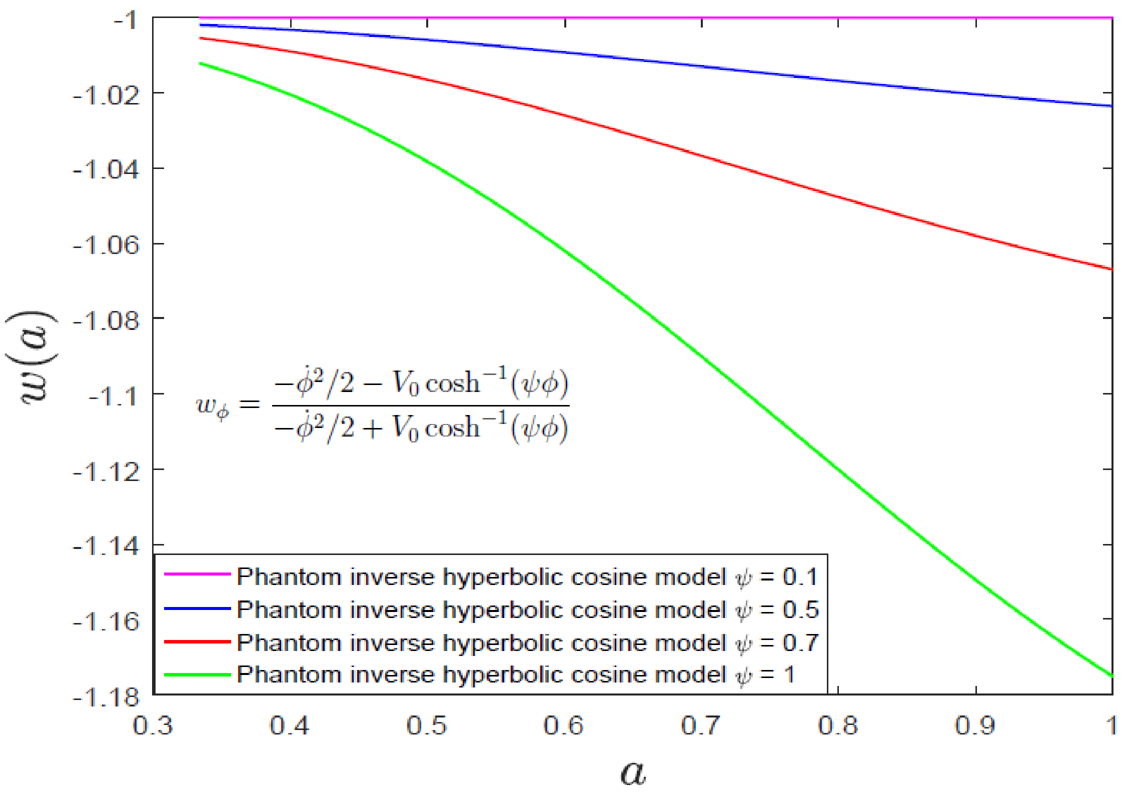
Evolution of the equation of state parameters in ϕCDM models



- The evolution of the equation of state parameters for the quintessence and phantom scalar field models for fixed model parameters in them is presented here.

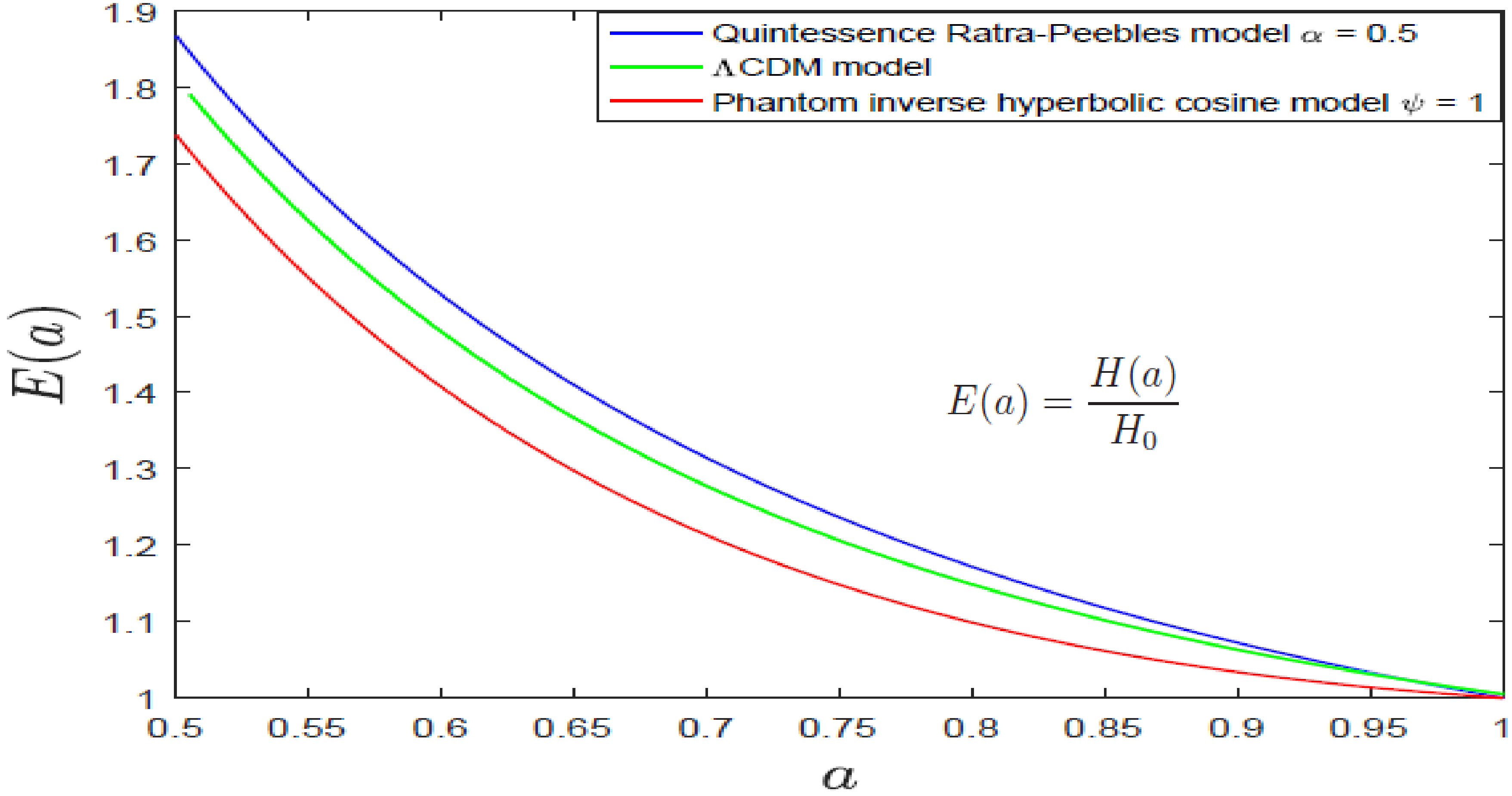
Evolution of the equation of state parameters in ϕCDM models





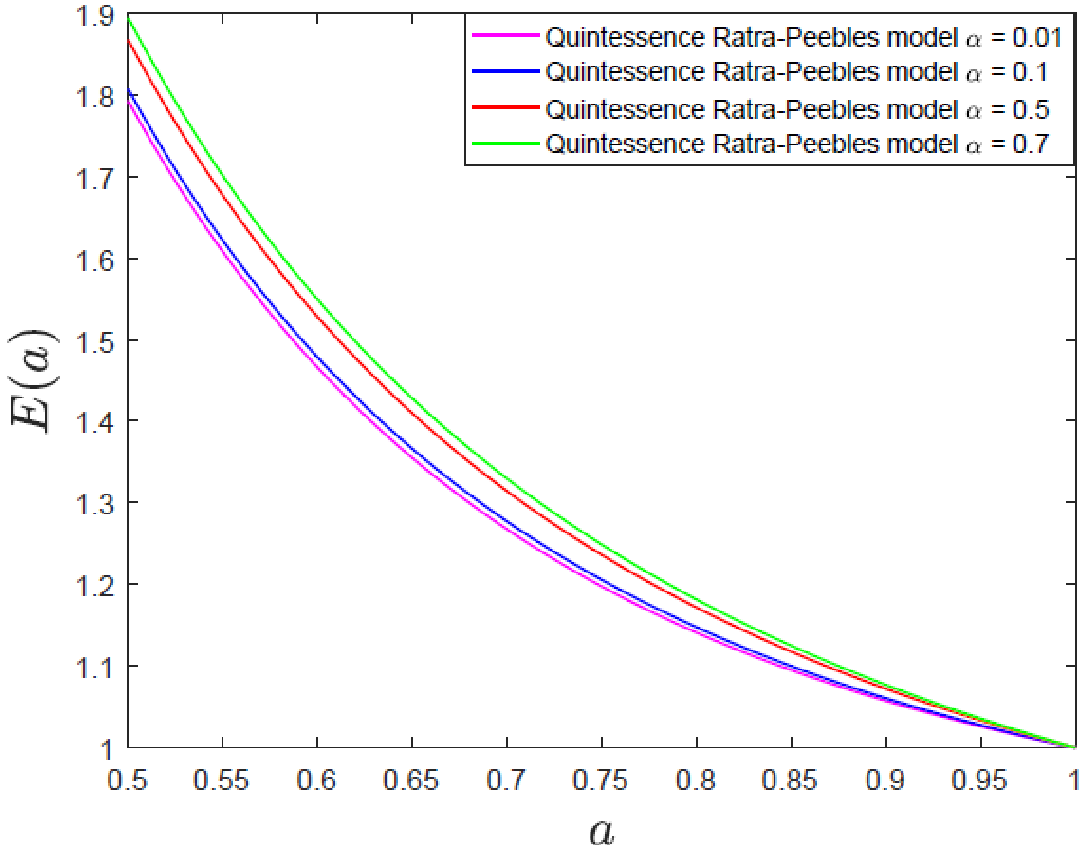
- A larger value of the parameter α in the quintessence Ratra-Peebles model and the parameter ψ in the phantom inverse hyperbolic cosine model causes an increase in dark energy and, thus, a stronger time dependence of the equation of state parameters in these models and vice versa.

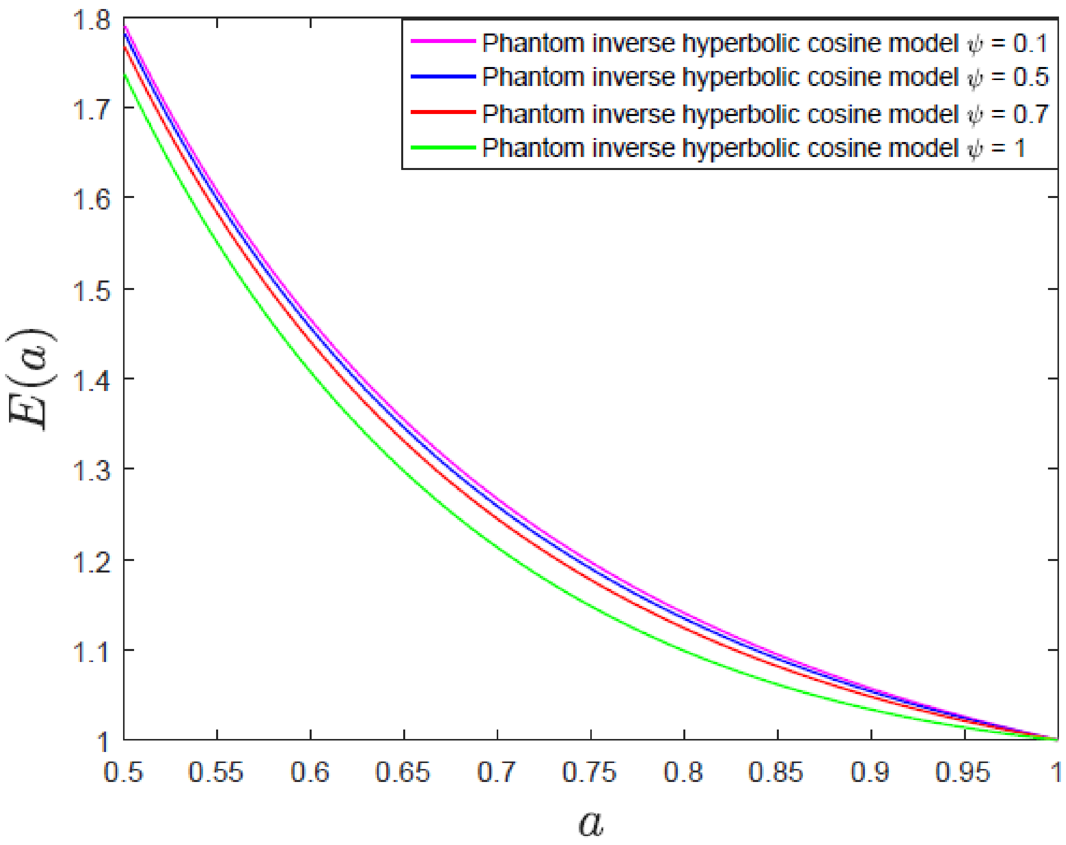
Hubble expansion of the Universe in scalar field ϕCDM models



- The expansion rate of the Universe is faster in quintessence scalar field ϕCDM models and slower in phantom scalar field ϕCDM models, in comparison with the ΛCDM model.

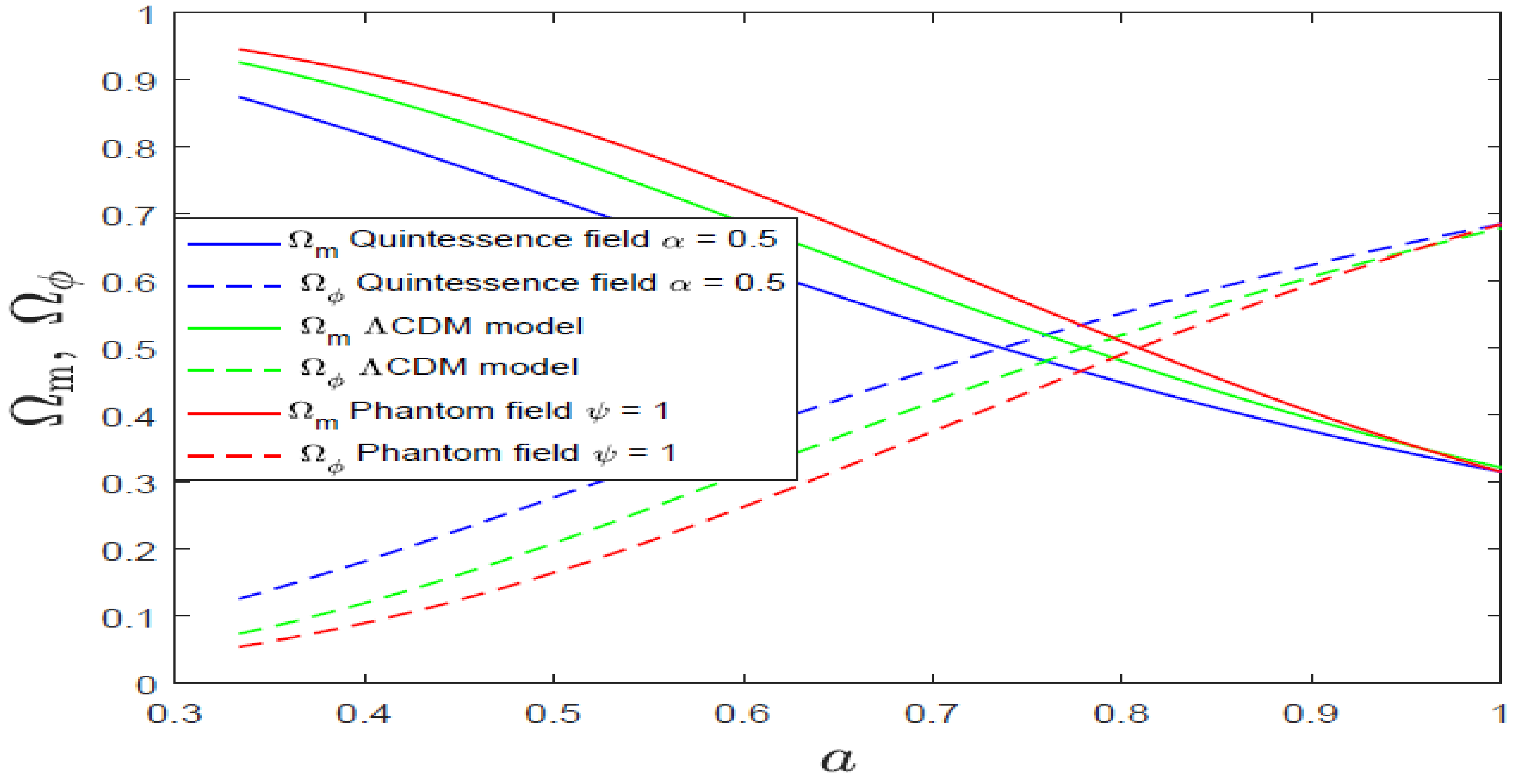
Hubble expansion of the Universe in the quintessence Ratra-Peebles ϕCDM model





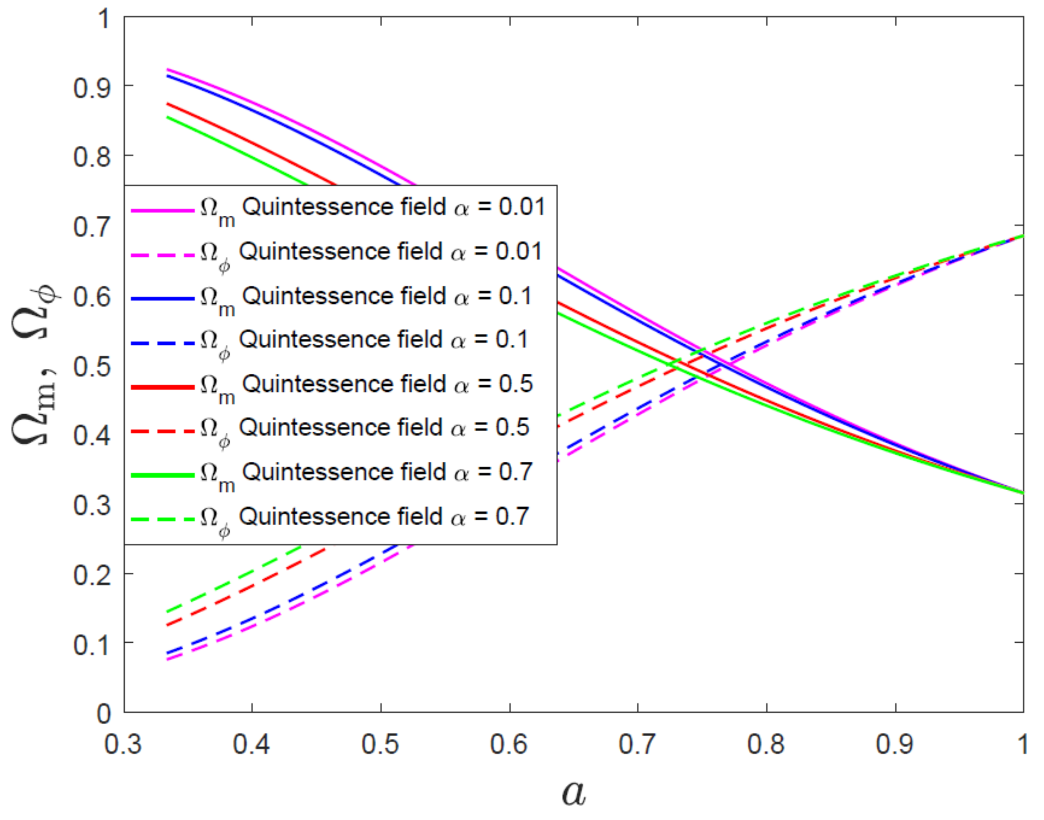
- In quintessence scalar field models, the expansion of the Universe occurs faster with an increase in the value of the parameter α, and, conversely, in phantom scalar field models, with an increase in the value of the parameter ψ, the expansion of the Universe occurs more slowly.

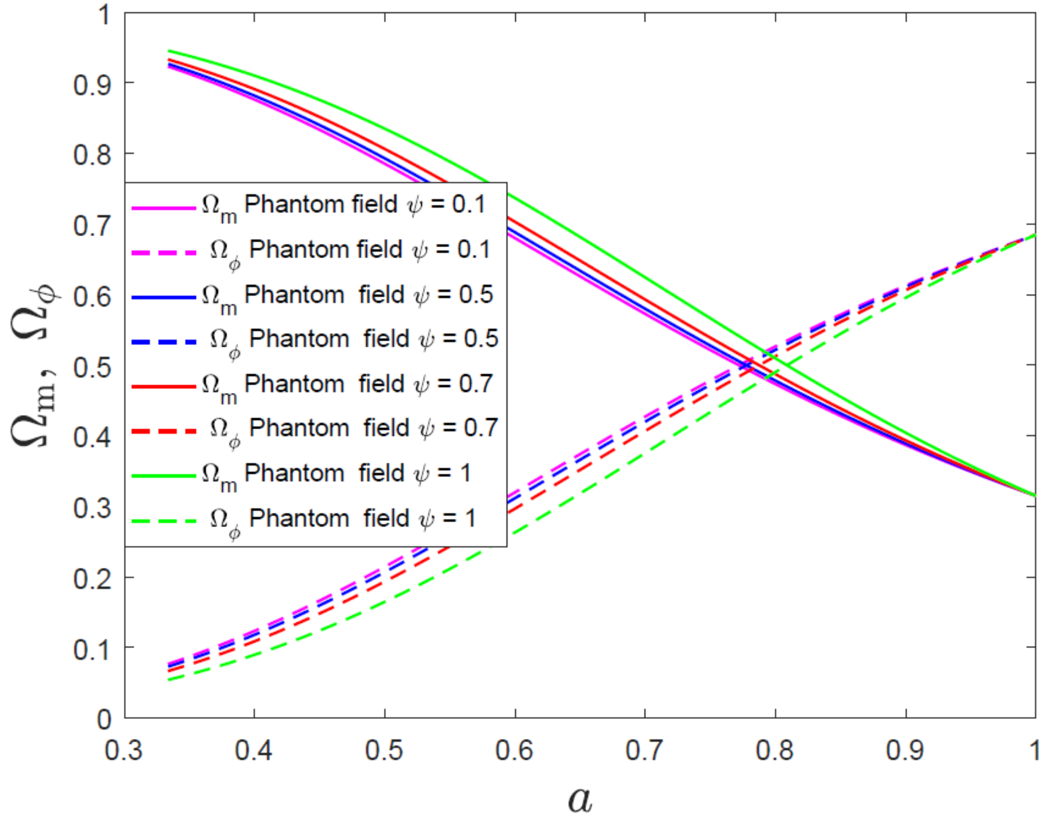
Energy components of the Universe in ϕCDM models



- The epoch of dominance of dark energy is established earlier in quintessence scalar field ϕCDM models and later in phantom scalar field ϕCDM models, in comparison with the ΛCDM model.

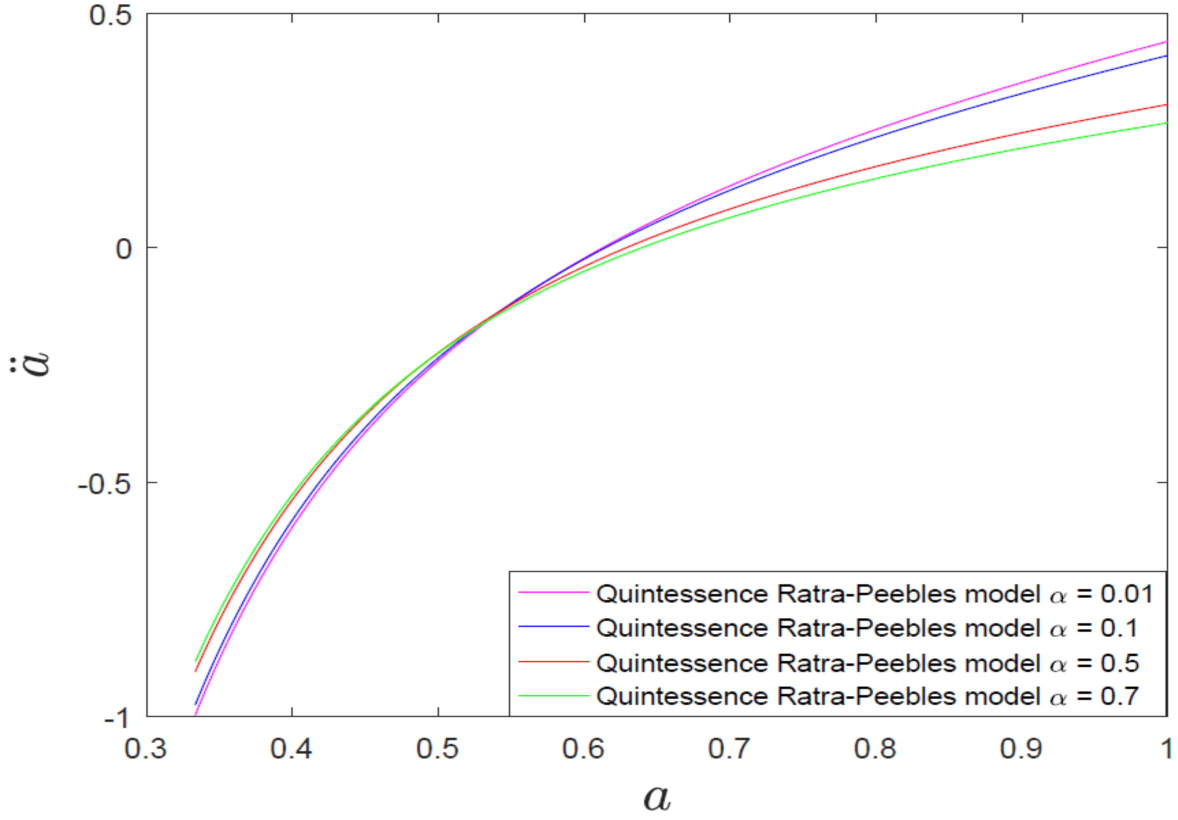
Energy components of the Universe in ϕCDM models

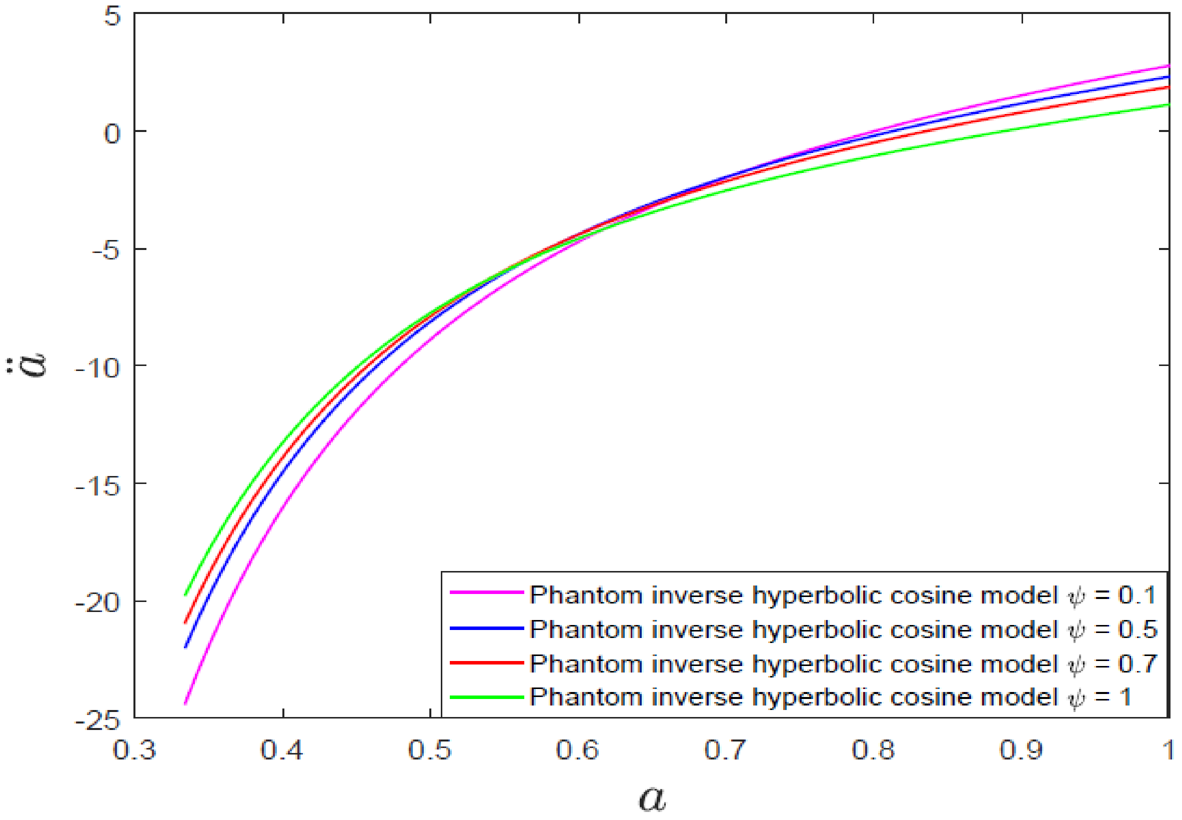




- In quintessence scalar field models, the energetic domination of dark energy began earlier with an increase in the value of the parameter α, and, conversely, in phantom scalar field models, with an increase in the value of the parameter ψ, the energetic domination of dark energy began later.

Dynamics of the Universe in ϕCDM models

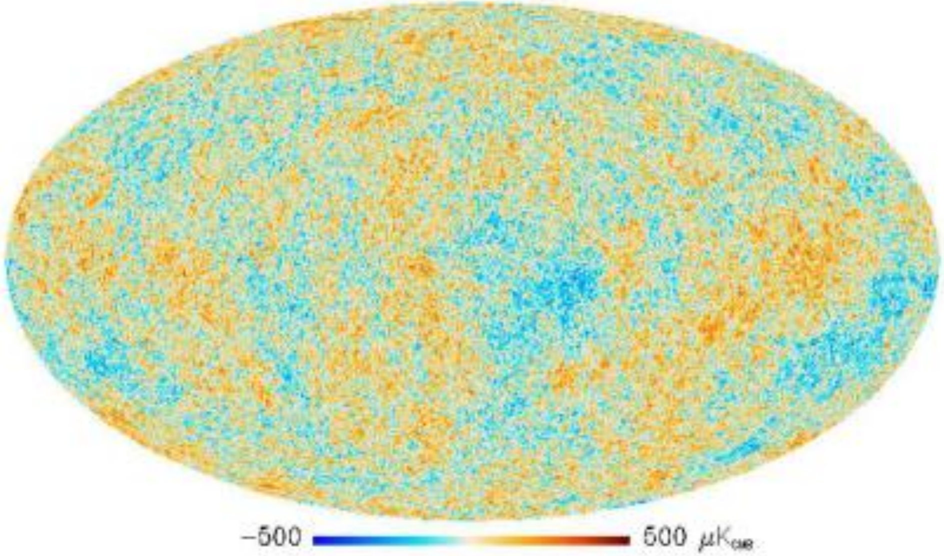




- The dynamic dominance of dark energy began earlier in the quintessence field than in the phantom field.
- Both in the quintessence and in phantom scalar fields models, the dynamic dominance of dark energy began earlier than the energy dominance at a fixed value of the model parameter in these models.

What is the influence of ϕCDM models on the evolution of large-scale structures?

CMB map from Planck space experiment

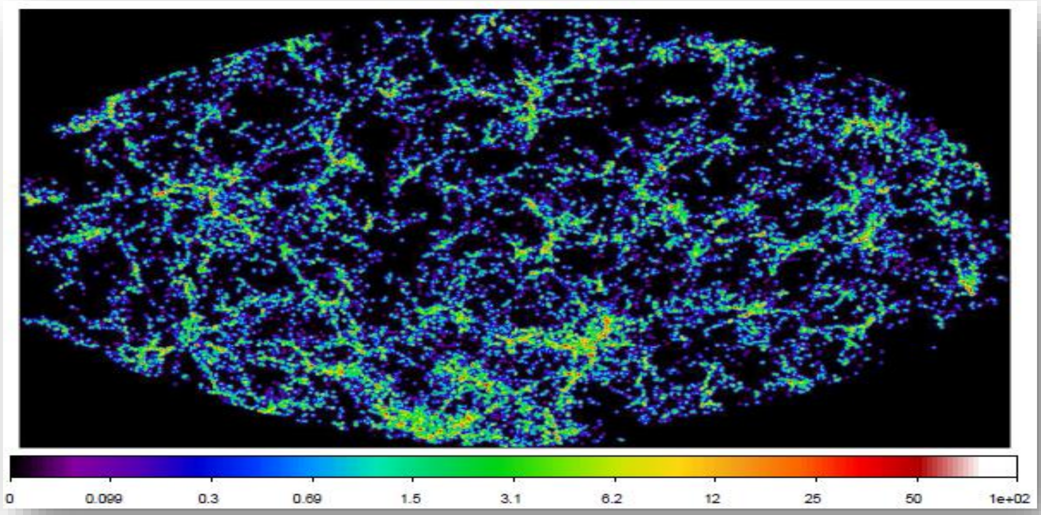


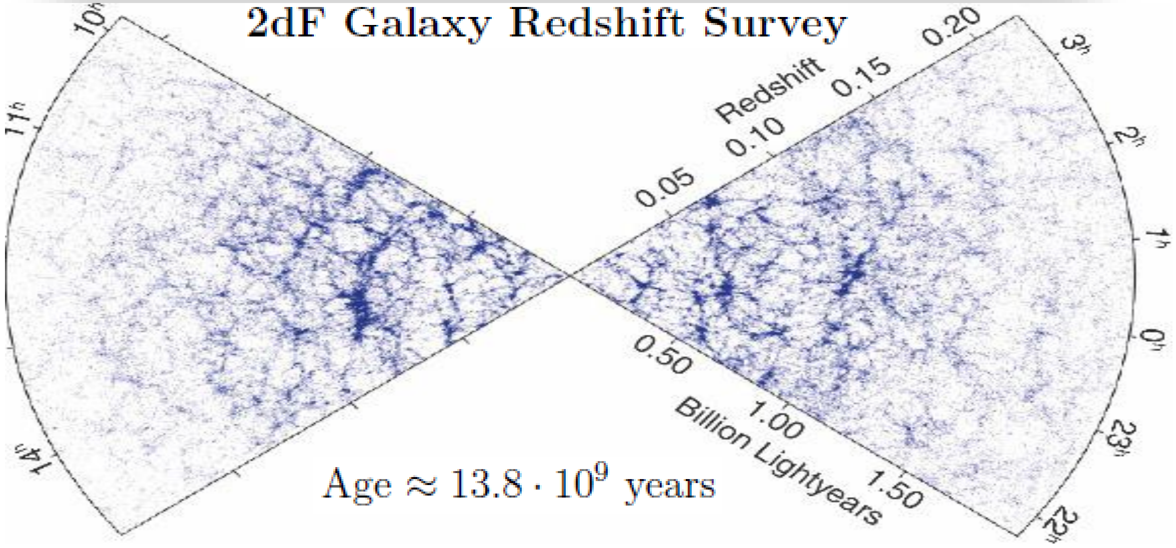
Credit: Ade et al. (Planck 2013 results)

$$\frac{\Delta T}{T} \sim \frac{\Delta \rho}{\rho} \approx 5 \cdot 10^{-5}$$

$$\text{Age} \approx 3.8 \cdot 10^{5} \text{ years}$$

Cosmic Web (SDSS)





Credit: Colless et al. (2003)

Influence of ϕCDM models on the formation of the large-scale structure in the universe

- **The linear perturbation equation**

$$\delta'' + \left(\frac{3}{a} + \frac{E'}{E}\right)\delta' - \frac{3\Omega_{m,0}}{2a^5E^2}\delta = 0$$

F. Pace, J.-C. Waizmann and M. Bartelmann, Mon. Not. Roy. Astron. Soc. **406**, 1865 (2010)

- **The matter density fluctuations**

$$\delta = \frac{\rho(\vec{r},t) - \langle\rho\rangle}{\langle\rho\rangle} \qquad \delta(a_{\rm in}) = \delta'(a_{\rm in}) = 5\cdot 10^{-5}$$

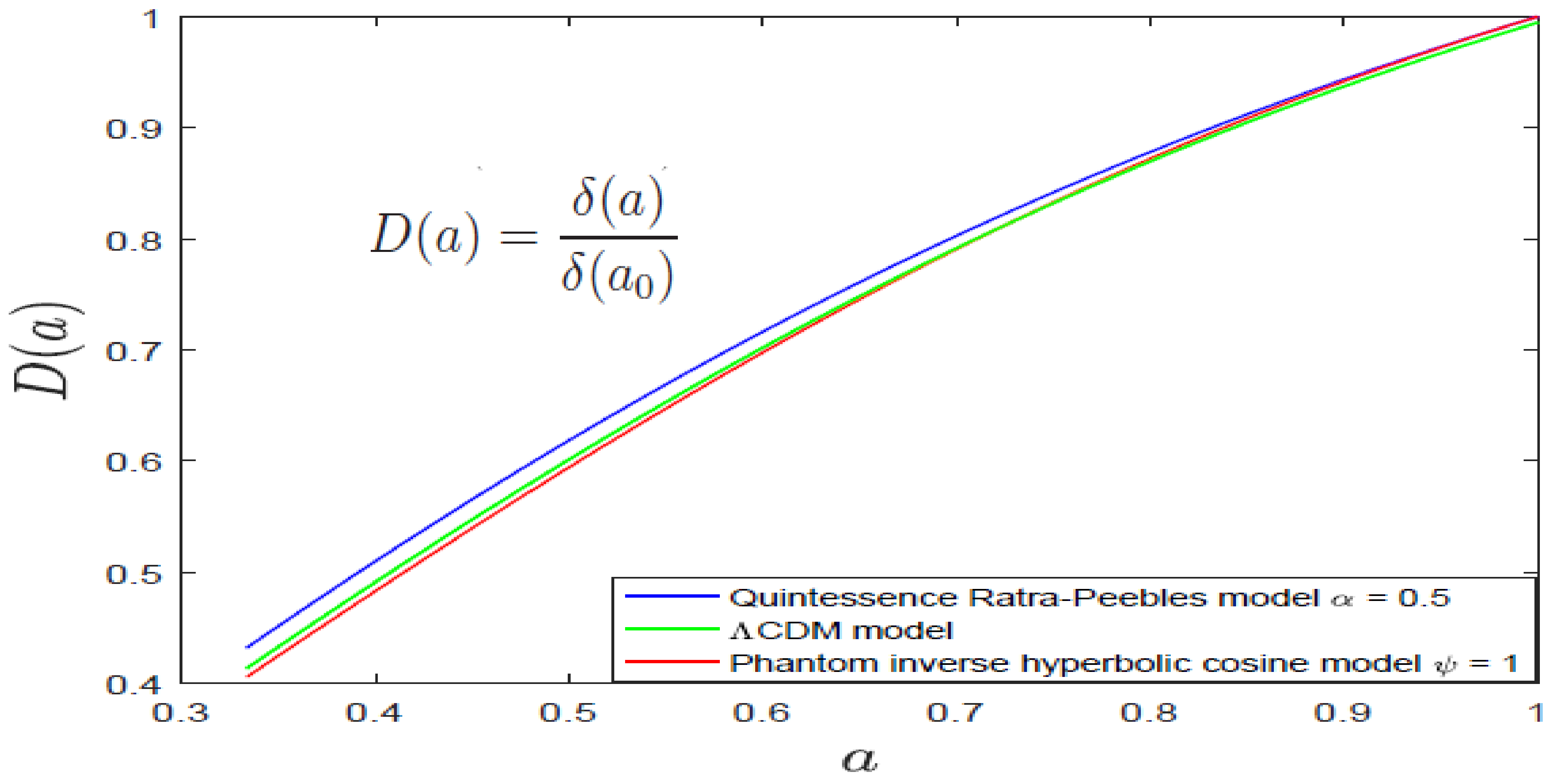
- **The linear growth factor**

$$D(a) = \frac{\delta(a)}{\delta(a_0)}$$

- **The large-scale structure growth rate**

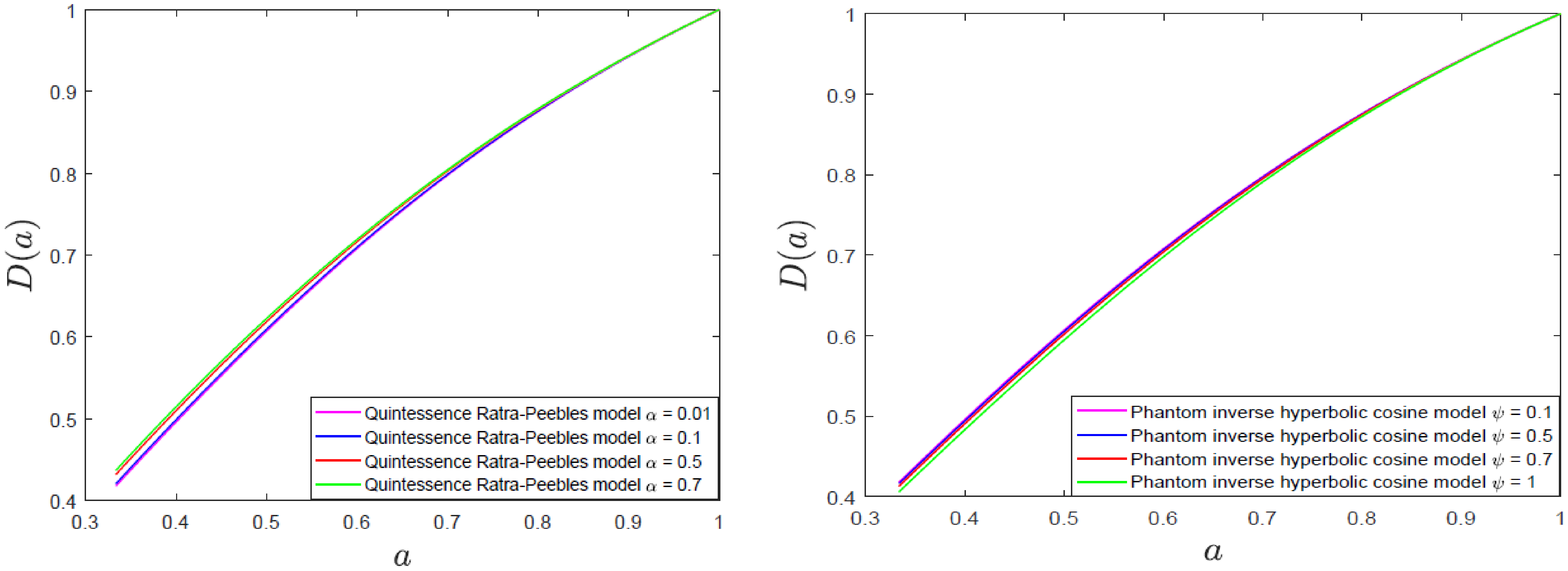
$$f(a) = \frac{d\ln D(a)}{d\ln a}$$

The evolution of the linear growth factor in ϕCDM models



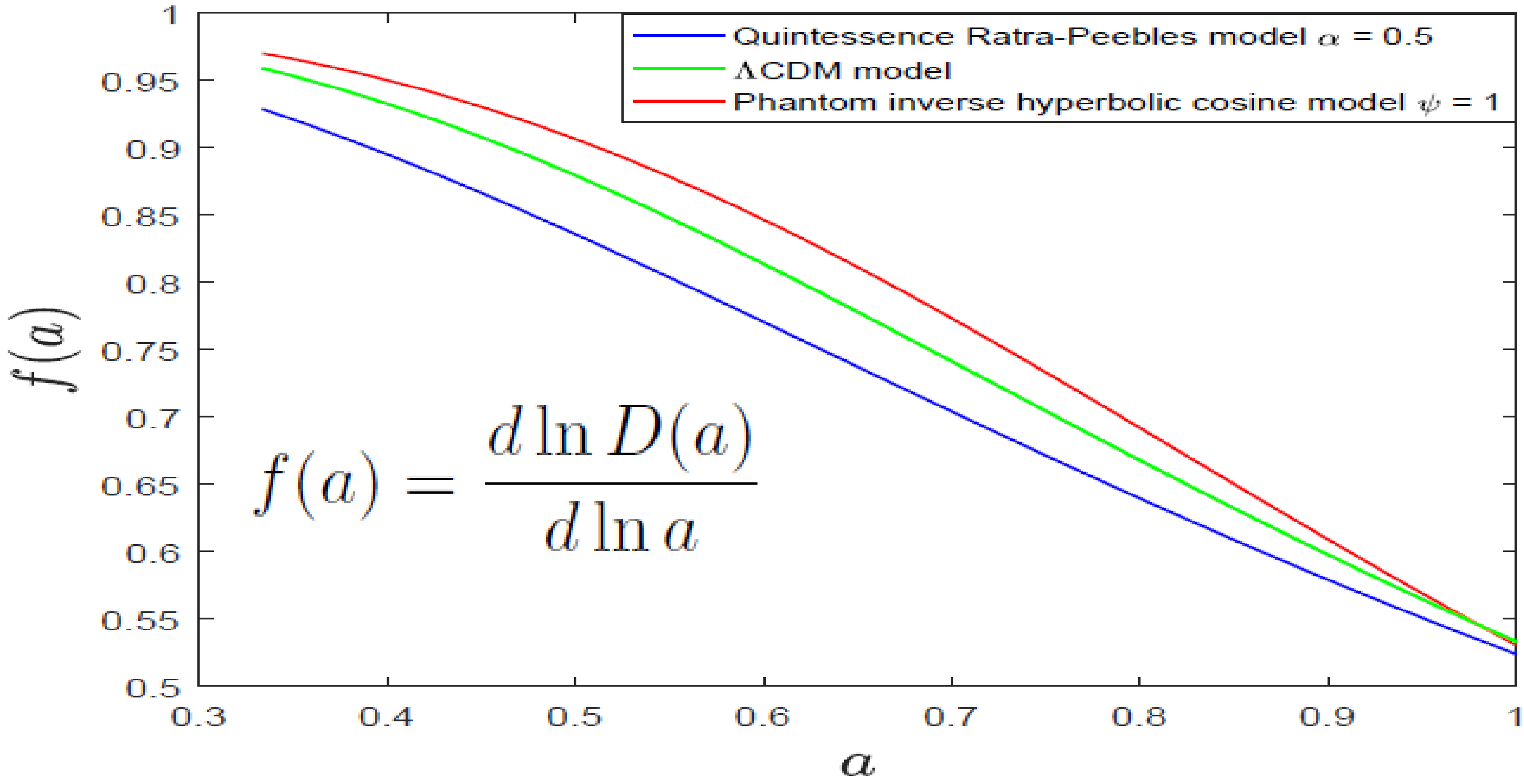
- Larger values of the matter density fluctuations are generated in quintessence scalar field ϕCDM models and smaller values in phantom scalar field ϕCDM models, compared to the ΛCDM model.

The linear growth factor of the matter density fluctuations in the quintessence Ratra-Peebles model



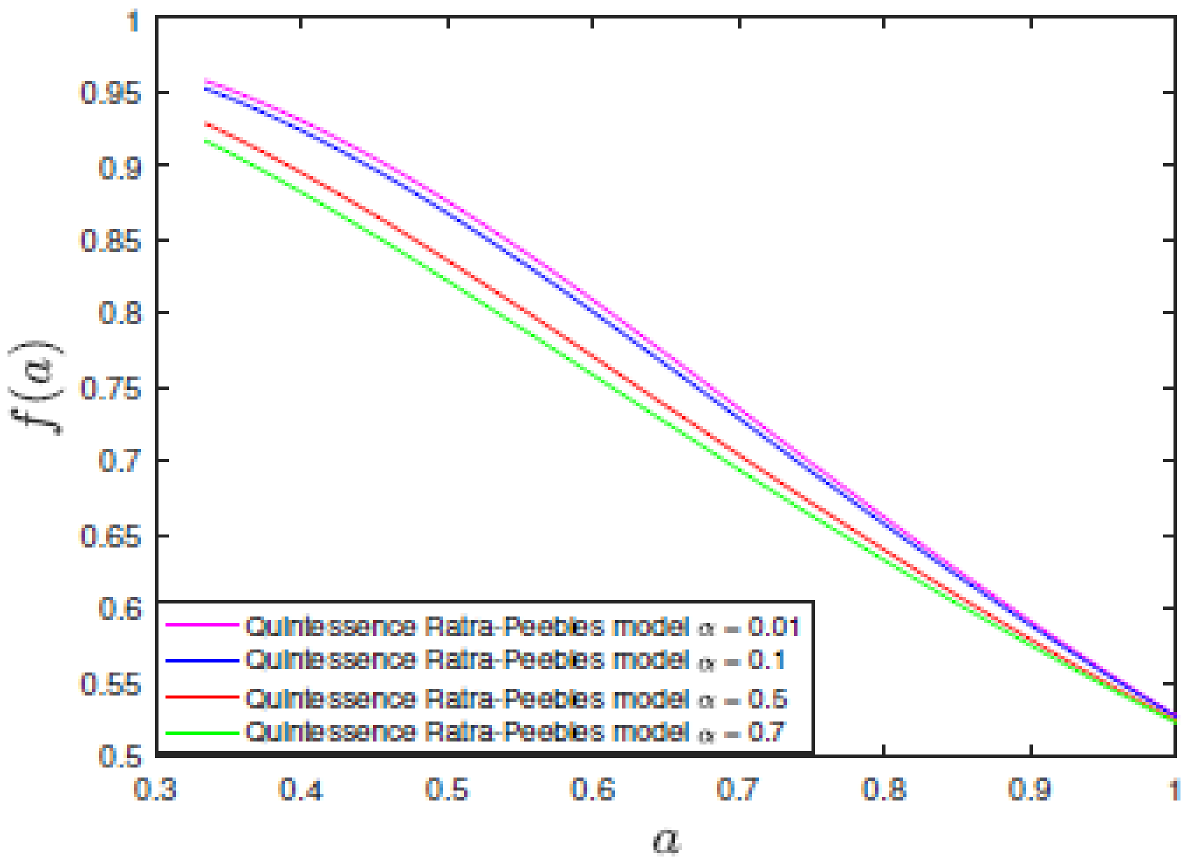
- In quintessence scalar field models, the larger values of the matter density fluctuations are generated with an increase in the value of the parameter α, and, conversely, in phantom scalar field models, the smaller values of the matter density fluctuations are generated with an increase in the value of the parameter ψ.

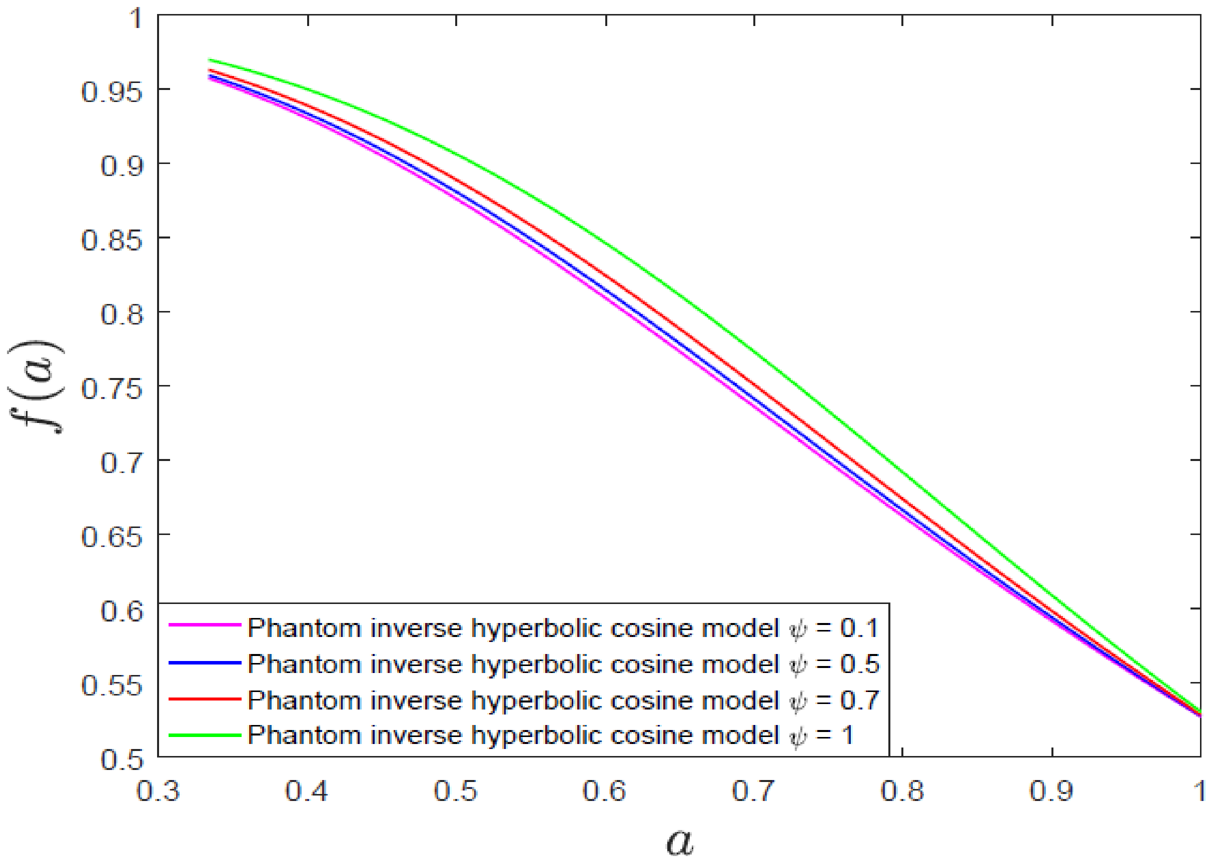
The evolution of the large-scale structure growth rate in ϕCDM models



-The large-scale structure growth rate is slower in quintessence scalar fields, but faster in phantom scalar fields, because in quintessence scalar fields, the Hubble expansion is faster than in phantom scalar fields, which leads to suppression of the large-scale structure growth rate in the Universe.

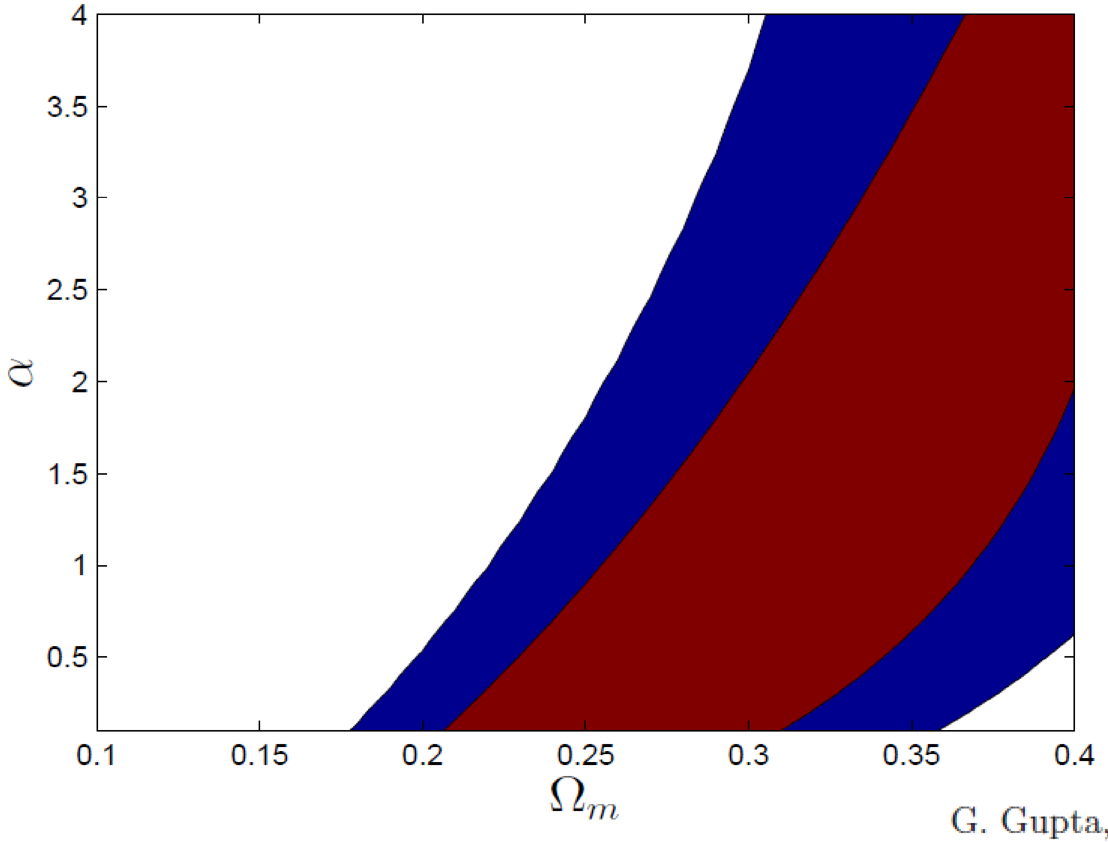
The evolution of the large-scale structure growth rate in ϕCDM models





- In quintessence scalar field models, the large-scale structure growth rate slows down with an increase in the value of the parameter α, and, conversely, in phantom scalar field models, with an increase in the value of the parameter ψ, the large-scale structure growth rate rapids.

Constraints from the growth rate data



$$\chi^2(\alpha, \Omega_{\rm obs}) = \frac{[f_{\rm obs} - f_{\rm th}(\alpha, \Omega_{\rm m})]^2}{\sigma^2}$$

$$\mathcal{L}^{\rm f}(\alpha, \Omega_{\rm m}) \propto \exp[-\chi^2(\alpha, \Omega_{\rm m})/2]$$

G. Gupta, S. Sen and A. A. Sen, JCAP **1204**, 028 (2012)

- Using the growth rate data alone, we have got the highly degenerated likelihood contours in the $\alpha - \Omega_m$ plane.

- If we fix $\alpha = 0$, we get the best fit value of $\Omega_{\rm m} = 0.278 \pm 0.03$, which is within 1σ confedence level of the Planck 2013 data.

Constraints from the BAO data

$$\mathcal{L}^{\rm B}(\alpha, \Omega_{\rm m}, H_0) \propto \exp(-\chi^2_{\rm B}/2)$$

$$\chi^2_{\rm B} = \boldsymbol{X}^{\rm T} \boldsymbol{C}^{-1} \boldsymbol{X} \quad \boldsymbol{X} = \eta_{\rm th} - \eta_{\rm obs}$$

$$\eta(z) \equiv d_{\rm A}(z_{\rm dec})/D_{\rm V}(z_{\rm BAO})$$

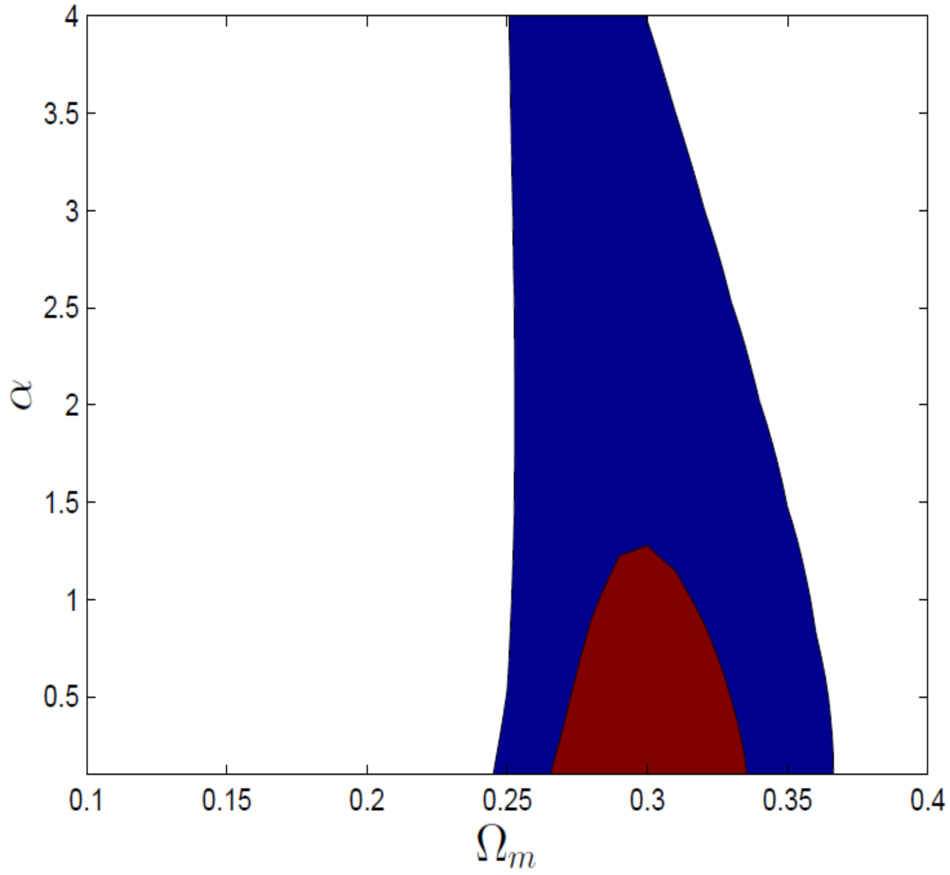
Angular diameter distance:

$$d_{\rm A}(z, \alpha, \Omega_{\rm m}, H_0) = \int_0^z \frac{dz'}{H(z', \alpha, \Omega_{\rm m}, H_0)}$$

Distance scale: $D_{\rm V}(z, \alpha, \Omega_{\rm m}, H_0) =$

$$[d^2_{\rm A}(z, \alpha, \Omega_{\rm m}, H_0) z / H(z, \alpha, \Omega_{\rm m}, H_0)]^{1/3}$$

R. Giostri, M. V. dos Santos, I. Waga, R. R. R. Reis, M. O. Calvo and B. L. Lago, JCAP **1203**, 027 (2012)

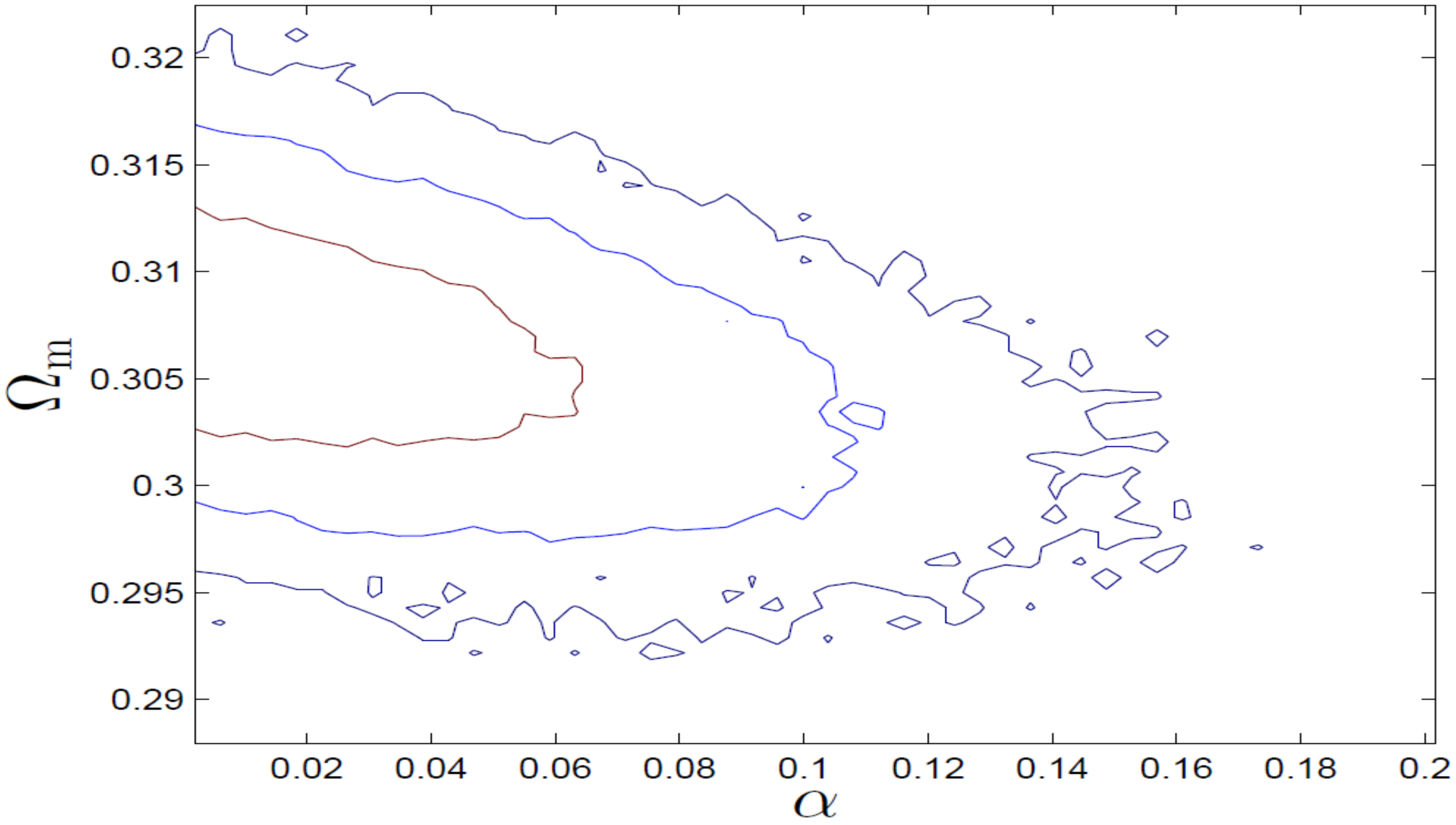


The addition of the BAO data and distance prior from CMB broke the degeneracy:

$0 \leq \alpha \leq 1.3$ at 1σ confidence level

$0.26 \leq \Omega_{\rm m} \leq 1.34$ at 1σ confidence level

Monte Carlo Markov Chains (MCMC) analysis with upcoming DESI data



Carrying out the MCMC analysis with upcoming DESI data, we obtained ranges of α and Ω_m parameters, $0 < \alpha \leq 0.16$ and $0.296 < \Omega_m < 0.32$ at 3σ confidence level, where the Ratra-Peebles ϕCDM model is compliance with the ΛCDM model.

Conclusion

Scalar field ϕCDM models differ from the ΛCDM model in a number of characteristics, which are generic for these models.

Compared to the ΛCDM model,

- **the Hubble expansion rate of the Universe is faster in quintessence scalar field models and slower in phantom scalar field models;**
- **dynamic and energetic domination of dark energy began earlier in quintessence scalar field models and later in phantom scalar field models;**
- **larger values of matter density fluctuations are generated in quintessence scalar field models and smaller values in phantom scalar field models;**
- **the large-scale structures growth rate of the Universe is faster in phantom scalar field models and slower in quintessence scalar field models.**


Thank you for your
kind attention!