Non-Gaussianity in rapid-turn multi-field inflation

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Cosmology from Home July 2023



Early Universe cosmology



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In the beginning, there was (probably) inflation



CMB observations constrain the power spectra of primordial scalar and tensor perturbations

$$P_{\mathcal{R}}(k) = \left(\frac{H}{\dot{\phi}}\right)^2 P_{\delta\phi}(k)$$

 $\langle \mathcal{R}_k \mathcal{R}_{k'} \rangle = (2\pi)^3 \delta^{(3)} (k+k') P_{\mathcal{R}}(k)$

$$\Delta_{\mathcal{R}}^2(k) = \frac{k^3}{2\pi^2} P_{\mathcal{R}}(k) = A_s(k_*) \left(\frac{k}{k_*}\right)^{n_s - 1}$$

amplitude of the scalar power spectrum

$$n_s - 1 = \frac{d \ln \Delta_{\mathcal{R}}^2(k)}{d \ln k}$$

scalar spectral index

Current observational bounds from CMB

 $n_s = 0.9603 \pm 0.0073,$ r < 0.044



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r =

Bi-spectrum

 $\left\langle \mathcal{R}_{\vec{k}_1} \mathcal{R}_{\vec{k}_2} \mathcal{R}_{\vec{k}_3} \right\rangle = (2\pi)^3 \delta^{(3)} (\vec{k}_1 + \vec{k}_2 + \vec{k}_3) B_{\mathcal{R}}(k_1, k_2, k_3)$

Result of non-linear evolution of initially Gaussian fluctuations.

Non-Gaussianity

Bi-spectrum

$$\left\langle \mathcal{R}_{\vec{k}_1} \mathcal{R}_{\vec{k}_2} \mathcal{R}_{\vec{k}_3} \right\rangle = (2\pi)^3 \delta^{(3)} (\vec{k}_1 + \vec{k}_2 + \vec{k}_3) B_{\mathcal{R}}(k_1, k_2, k_3)$$

Result of non-linear evolution of initially Gaussian fluctuations.

$$B_{\mathcal{R}}(k_1, k_2, k_3) \propto \sum_{\mathrm{type}} f_{\mathrm{NL}}^{\mathrm{type}} S_{\mathrm{type}}(k_1, k_2, k_3)$$

equilateral

local 🔏

Non-Gaussianity

$$\begin{array}{c} & \text{Bi-spectrum} \\ & & \text{B$$

$$-\frac{6}{5}f_{\rm NL} = \frac{B_{\mathcal{R}}(k_1, k_2, k_3)}{P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_2) + (\vec{k} \text{ cyclic perms})}$$

Non-Gaussianity in single-field inflation



Single-field models of inflation most strongly couple momenta of similar wavelengths and result in bispectra that are highly suppressed in the 'squeezed limit' where one long-wavelength-mode couple to two short-wavelength-modes.

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Non-Gaussianity in single-field inflation



Detection of $f_{\rm NL}^{\rm loc} \simeq \mathcal{O}(1)$ would rule out all attractor models of single-field inflation!

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Multi-field inflation



[A. Achucarro, E. Copeland, O.I. et al]



[O.I., E. Sfakianakis, D.-G.Wang, A. Achucarro]



[A. Achucarro, R. Kallosh, A. Linde et al]

Multi-field inflation and turning trajectory

Multi-field inflation and turning trajectory



Turn-rate: $D_N e^a_{\parallel} = \eta_{\perp} e^a_{\perp}$

Trajectory turns couple the fluctuations and modify their dispersion relations and correlators.

Multi-field inflation and turning trajectory



field-space metric

multi-field potential

Two types of field perturbations:

- Adiabatic (curvature) - along trajectory ${\cal R}$

- Non-Adiabatic (isocurvature) ----- orthogonal to trajectory ${\cal S}$



Turn-rate:

$$D_N e^a_{\parallel} = \eta_{\perp} e^a_{\perp}$$

[D. Wands, K.Malik, D. Lyth, A. Liddle, 2000] [L. Amendola, C. Gordon, D. Wands, M. Sasaki, 2002] [D. Wands, N. Bartolo, S. Matarrese, A. Riotto,2002]



 $egin{array}{rll} \dot{\mathcal{R}} &\simeq lpha H \mathcal{S} \ \dot{\mathcal{S}} &\simeq eta H \mathcal{S} \end{array}$

$$\alpha = 2 \eta_{\perp}$$

$$\beta = -2\epsilon - \frac{\mathcal{M}_{\perp\perp}}{V} + \frac{\mathcal{M}_{\parallel\parallel}}{V} - \frac{4}{3} (\eta_{\perp})^2$$

 $\overline{\mathcal{M}^{a}}_{b} = \overline{G^{ac}} \nabla_{b} \nabla_{c} V - \overline{R^{a}_{dfb}} \dot{\phi}^{d} \dot{\phi}^{f}$

[D. Wands, K.Malik, D. Lyth, A. Liddle, 2000] [L. Amendola, C. Gordon, D. Wands, M. Sasaki, 2002] [D. Wands, N. Bartolo, S. Matarrese, A. Riotto,2002]



 $\begin{cases} \dot{\mathcal{R}} \simeq \alpha H \mathcal{S} \\ \dot{\mathcal{S}} \simeq \beta H \mathcal{S} \\ \alpha = 2 \eta_{\perp} \\ \beta = -2\epsilon - \frac{\mathcal{M}_{\perp \perp}}{V} + \frac{\mathcal{M}_{\parallel \parallel}}{V} - \frac{4}{S} (\eta_{\perp})^2 \end{cases}$

The sourcing of curvature perturbations by isocurvature perturbations is proportional to the turn-rate!

[D. Wands, K.Malik, D. Lyth, A. Liddle, 2000] [L. Amendola, C. Gordon, D. Wands, M. Sasaki, 2002] [D. Wands, N. Bartolo, S. Matarrese, A. Riotto,2002]



 $\begin{aligned} \dot{\mathcal{R}} &\simeq \alpha H \mathcal{S} \\ \dot{\mathcal{S}} &\simeq \beta H \mathcal{S} \end{aligned}$

$$\alpha = 2 \eta_{\perp}$$

$$\beta = -2\epsilon - \underbrace{\underbrace{\mathcal{M}_{\perp\perp}}_{V}}_{V} + \underbrace{\underbrace{\mathcal{M}_{\parallel\parallel}}_{V}}_{V} - \frac{4}{3} (\eta_{\perp})^{2}$$

$$\mathcal{M}^{a}{}_{b} = G^{ac} \nabla_{b} \nabla_{c} V - R^{a}_{dfb} \dot{\phi}^{d} \dot{\phi}^{j}$$

Determined by potential & geometry of field-space.

[D. Wands, K.Malik, D. Lyth, A. Liddle, 2000] [L. Amendola, C. Gordon, D. Wands, M. Sasaki, 2002] [D. Wands, N. Bartolo, S. Matarrese, A. Riotto,2002]



$$\begin{cases} \dot{\mathcal{R}} &\simeq \alpha HS \\ \dot{\mathcal{S}} &\simeq \beta HS \\ \alpha &= 2 \eta_{\perp} \\ \beta &= -2\epsilon - \frac{\mathcal{M}_{\perp\perp}}{V} + \frac{\mathcal{M}_{\parallel\parallel}}{V} - \frac{4}{3} (\eta_{\perp})^2 \end{cases}$$

$$\begin{split} \langle \mathcal{R}_{\vec{k}_1} \mathcal{R}_{\vec{k}_2} \rangle &= \frac{1}{2\epsilon} \langle \delta \phi_{\parallel \vec{k}_1} \delta \phi_{\parallel \vec{k}_2} \rangle \implies P_{\mathcal{R}} & \text{Power curve} \\ \langle \mathcal{R}_{\vec{k}_1} \mathcal{S}_{\vec{k}_2} \rangle &= \frac{1}{2\epsilon} \langle \delta \phi_{\parallel \vec{k}_1} \delta \phi_{\perp \vec{k}_2} \rangle \implies C_{\mathcal{RS}} & \text{crosser curve} \\ \langle \mathcal{S}_{\vec{k}_1} \mathcal{S}_{\vec{k}_2} \rangle &= \frac{1}{2\epsilon} \langle \delta \phi_{\perp \vec{k}_1} \delta \phi_{\perp \vec{k}_2} \rangle \implies P_{\mathcal{S}} & \text{isoccurve} \end{split}$$

Power spectrum of curvature perturbations,

cross-correlation,

isocurvature perturbations.

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Slow-turn vs rapid-turn inflation

[C. Peterson, M. Tegmark, 2011]

Slow-turn: $\eta_{\perp} \ll 1$

$$f_{\rm NL}^{\rm loc} \supset \frac{5}{6} \sqrt{\frac{r}{8}} \left(\frac{T_{\mathcal{RS}}}{\sqrt{1 + T_{\mathcal{RS}}^2}} \right)^3 \partial_{\perp *} \ln T_{\mathcal{RS}}$$

$$\begin{pmatrix} \mathcal{R} \\ \mathcal{S} \end{pmatrix} = \begin{pmatrix} 1 \ T_{\mathcal{RS}} \\ 0 \ T_{\mathcal{SS}} \end{pmatrix} \begin{pmatrix} \mathcal{R}_* \\ \mathcal{S}_* \end{pmatrix}$$

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What is differrent in the rapid-turn regime? $~\eta_{\perp}\gg 1$



[arXiv:2303.14156]





M.C. David Marsh

Gustavo Salinas

Non-Gaussianity in rapid-turn multi-field inflation

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Non-Gaussianity from the δN -formalism

[D.H. Lyth and Y. Rodriguez, (2005)]

$$\left\langle \mathcal{R}_{\vec{k}_1} \mathcal{R}_{\vec{k}_2} \mathcal{R}_{\vec{k}_3} \right\rangle = N_a N_b N_c \left\langle \delta \phi^a_{*\vec{k}_1} \delta \phi^b_{*\vec{k}_2} \delta \phi^c_{*\vec{k}_3} \right\rangle$$
$$+ \frac{1}{2} N_a N_b N_{cd} \left\langle \delta \phi^a_{*\vec{k}_1} \delta \phi^b_{*\vec{k}_2} \left(\delta \phi^c_{*} * \delta \phi^d_{*} \right)_{\vec{k}_3} \right\rangle + (\vec{k} \text{ cyclic perms}) + \cdots$$



Slow-turn: $\eta_{\perp} \ll 1$

$$\langle \delta \phi^a_{*\vec{k}_1} \delta \phi^b_{*\vec{k}_2} \rangle = (2\pi)^3 \delta^{(3)} (\vec{k}_1 + \vec{k}_2) P^*_{\phi}(k_1) \delta^{ab}$$

Slow-turn: $\eta_{\perp} \ll 1$ $\langle \delta \phi^a_{*\vec{k}_1} \delta \phi^b_{*\vec{k}_2} \rangle = (2\pi)^3 \delta^{(3)} (\vec{k}_1 + \vec{k}_2) P^*_{\phi} (k_1) \delta^{ab}$ $P_{\mathcal{R}*} = P_{\mathcal{S}*}, \quad C_{\mathcal{R}\mathcal{S}*} = 0$

[OI, D. Marsh, G. Salinas, 2023]

Slow-turn: $\eta_{\perp} \ll 1$

$$\langle \delta \phi^{a}_{*\vec{k}_{1}} \delta \phi^{b}_{*\vec{k}_{2}} \rangle = (2\pi)^{3} \delta^{(3)} (\vec{k}_{1} + \vec{k}_{2}) P^{*}_{\phi} (k_{1}) \delta^{ab}$$

$$P_{\mathcal{R}*} = P_{\mathcal{S}*}, \quad C_{\mathcal{R}\mathcal{S}*} = 0$$

Rapid-turn: $\eta_{\perp} \gg 1$ $\langle \delta \phi^a_{*\vec{k}_1} \delta \phi^b_{*\vec{k}_2} \rangle = (2\pi)^3 \delta^{(3)} (\vec{k}_1 + \vec{k}_2) P_{\phi}^{*ab}(k_1)$ $P_{\mathcal{R}*} \neq P_{\mathcal{S}*} \neq C_{\mathcal{RS}*} \neq 0$

[OI, D. Marsh, G. Salinas, 2023]

Slow-turn: $\eta_{\perp} \ll 1$

$$-\frac{6}{5}f_{\rm NL}^{(4)} = \frac{N_a N_b N^{ab}}{(N_c N^c)^2}$$

Rapid-turn: $\eta_{\perp} \gg 1$

$$-\frac{6}{5}f_{\rm NL}^{(4)} = \frac{N_a N_b N_{cd} \left[P_{\phi}^{*ac} \left(k_1\right) P_{\phi}^{*bd} \left(k_2\right) + \left(\vec{k} \text{ cyclic perms}\right)\right]}{N_e N_f N_g N_h \left[P_{\phi}^{*ef} \left(k_1\right) P_{\phi}^{*gh} \left(k_2\right) + \left(\vec{k} \text{ cyclic perms}\right)\right]}$$

Rapid-turn: $\eta_{\perp} \gg 1$

[OI, D. Marsh, G. Salinas, 2023]

$$-\frac{6}{5}f_{\mathrm{NL}}^{(4)}(k_1, k_2, k_3) = \sum_{I,J=\mathcal{R},\mathcal{C}} f_{\mathrm{NL}}^{IJ} \frac{\tilde{\mathcal{P}}^I(k_1)\tilde{\mathcal{P}}^J(k_2) + (\vec{k} \text{ cyclic perms})}{P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_2) + (\vec{k} \text{ cyclic perms})}$$
$$\downarrow$$
$$f_{\mathrm{NL}}^{\mathcal{R}\mathcal{R}}, f_{\mathrm{NL}}^{\mathcal{R}\mathcal{C}}, f_{\mathrm{NL}}^{\mathcal{C}\mathcal{R}}, f_{\mathrm{NL}}^{\mathcal{C}\mathcal{C}}$$
$$\tilde{\mathcal{P}}^{\mathcal{R}}(k) = P_{\mathcal{R}}(k) \ , \ \tilde{\mathcal{P}}^{\mathcal{C}}(k) = C_{\mathcal{RS}}(k)$$

Rapid-turn: $\eta_{\perp} \gg 1$

[OI, D. Marsh, G. Salinas, 2023]

Scale dependence and new shape functions!

Rapid-turn: $\eta_{\perp} \gg 1$

[OI, D. Marsh, G. Salinas, 2023]

Assuming the scale-invariant power spectrum, it reduces to:

$$f_{\mathrm{NL}}^{(4)} \supset \eta_{\perp} I_4 + \tilde{M}_{\perp \perp *} I_5 + \tilde{M}_{\perp \parallel *} I_6$$

 $I_i = I_i \left(T_{\mathcal{RS}}, \mathcal{C}_{\mathcal{RS}}, P_{\mathcal{S}}/P_{\mathcal{R}} \right)$

New model-independent potentially large contributions to the non-Gaussianity parameter!

[P. Christodoulidis, D. Roest, E. I. Sfakianakis, 2019]

$$V(\phi,\chi) = \frac{\tilde{\alpha}}{2} \left(m_{\phi}^2 \phi^2 + m_{\chi}^2 \chi^2 \right)$$

$$G_{ab} = \frac{6\tilde{\alpha}}{\left(1 - \phi^2 - \chi^2\right)^2} \delta_{ab}$$

[OI, D. Marsh, G. Salinas, 2023]

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[OI, D. Marsh, G. Salinas, 2023]



Power spectrum of curvature perturbations,

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[OI, D. Marsh, G. Salinas, 2023]



$$f_{\rm NL}^{\rm loc} = -\frac{5}{6} \left(0.006 \, I_{1*} + 1.89 \, I_{2*} + 0.004 \, I_{3*} - 2.35 \, I_{4*} - 0.015 \, I_{5*} + 2.3 \, I_{6*} \right)$$

[OI, D. Marsh, G. Salinas, 2023]



$$f_{\rm NL}^{\rm loc} = -\frac{5}{6} \left(0.006 \, I_{1*} + 1.89 \, I_{2*} + 0.004 \, I_{3*} - 2.35 \, I_{4*} - 0.015 \, I_{5*} + 2.3 \, I_{6*} \right)$$

 $f_{\rm NL}^{\rm loc} = 0.705 \simeq \mathcal{O}\left(1\right)$

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Conclusions

1. Extended the δN -formalism to rapid-turn inflation.

2. Identified new model-independent potentially large contributions to the non-Gaussianity parameter.

3. The resulting bispectrum in general is not of the local shape.

- 4. Detection of Non-Gaussianity $f_{\mathrm{NL}}^{\mathrm{loc}} \simeq \mathcal{O}\left(1\right)$ signals:
- OR non-inflationary perturbations?





Conclusions

1. Extended the δN -formalism to rapid-turn inflation.

2. Identified new model-independent potentially large contributions to the non-Gaussianity parameter.

3. The resulting bispectrum in general is not of the local shape.

- 4. Detection of Non-Gaussianity $f_{\mathrm{NL}}^{\mathrm{loc}} \simeq \mathcal{O}\left(1\right)$ signals:
- New particles inflation with more than one field, curved field-space, steep potentials, UV competions...
- **OR** non-inflationary perturbations?

