Testing Spacetime Symmetries in the Early Universe

Nils A. Nilsson (SYRTE, Paris Observatory)

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Cosmology from Home 2023

Symmetries?

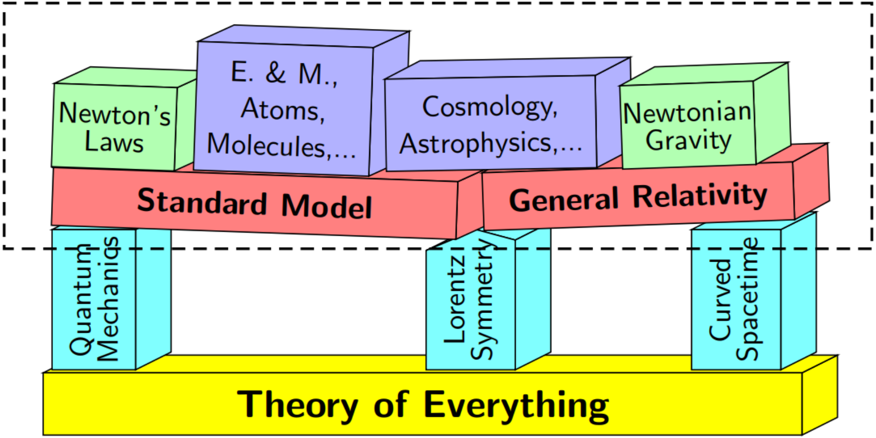


Image credit: Matt Mewes, Cal Poly

Spacetime symmetries

- General relativity defined in geometrical terms: $\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu$, no torsion

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = 8\pi G\, T^{\mu\nu}$$

- Geometry
- Matter

The related conservation laws lead to the Bianchi identities ($+\,T_{[\mu\nu]} = 0$):

$$\nabla_\mu G^{\mu\nu} = 0$$

- Related symmetries:
  - Local Lorentz symmetry (**6 rotations**): $B'^\mu = \Lambda^\mu_\nu B^\nu$
  - Diffeomorphism symmetry (**4 translations**): $B^\mu \to B^\mu + (\partial_\nu \xi^\mu)\, B^\nu - \xi^\nu \partial_\nu B^\mu$

Why test spacetime symmetries?

Why are we interested in this?

- **Theoretical motivation**
  - Spontaneous Lorentz violation from string field theory (Kostelecký and Samuel, PRD 1989)
  - Trickle-down UV Lorentz violation in LQG (Gambini et al, CQG 2011)
  - High-order spatial derivatives for renormalisability (Hořava, PRD 2009)
- **What can be measured with precision?**
  - Symmetries lend themselves easily to high precision tests, easy to describe theoretically
- **Quantum gravity likely alters the structure of spacetime**
  - Are there extra spacetime dimensions? (Kaluza-Klein etc.)
  - Do spacetime coordinates commute? ($[x^\mu, x^\nu] = i\theta^{\mu\nu}$)
  - Is spacetime discrete? (LQG etc.)
  - Spacetime foam?

[Spacetime symmetries are the basis of ALL of modern physics, we should be testing them!]

The Standard-Model Extension

Model A **Model B** **Model C** **???**

- Lorentz violation type 1
- Lorentz violation type 2
- Lorentz violation type 3
- Lorentz violation type 4

Can we find a general framework instead? (Answer: yes, we can!)

Standard-Model Extension

- The Standard-Model Extension (SME): general framework for studying Lorentz/CPT violation, in all sectors

Contains:

- Standard Model of particle physics
- General relativity
- ALL operator breaking Lorentz and/or CPT symmetry

$$\mathcal{L}_{\mathrm{SME}} = \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_{\mathrm{EH}} + \mathcal{L}_{\mathrm{LV}}$$

- operators proportional to *coefficients of Lorentz violation*, e.g. $\left(k^{(4)}\right)_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta}$
- 20 d.o.f

Originally developed by Kostelecky, Colladay + collaborators in the 1990's

Standard-Model Extension

The SME is a **framework**, not a model! Can always map model parameters onto SME coefficients

Model	Link to SME	LV fields	General Test Framework
PPN	Yes	None $(\alpha_1, \alpha_2, \ldots)$	Yes
SME gravity sector	Yes	Tensors	Yes, EFT
Bumblebee	Yes	Vectors	No
Einstein-Aether	Partial	Vector	No
Hořava gravity	Yes	Vector	No
ATT model	Yes	Antisymm. 2-tensor	No
Cardinal model	Yes	Symm. 2-tensor	No
Massive gravity	Yes	2-tensor	?
CS gravity	?	Scalar	No
GW MDR	Yes	None	Yes, DR
NC gravity	Yes	$\theta^{\mu\nu}$	No

The pure-gravity sector

Pure gravity: generic Lagrangian up to dimension-6 operators

$$\mathcal{L} = \frac{\sqrt{-g}}{2\kappa}\Big[R + \underbrace{\left(k^{(4)}\right)_{\alpha\beta\gamma\delta}}_{20} R^{\alpha\beta\gamma\delta} + \underbrace{\left(k^{(5)}\right)_{\alpha\beta\gamma\delta\kappa}}_{60} \nabla^{\kappa} R^{\alpha\beta\gamma\delta}$$

$$+\tfrac{1}{2}\underbrace{\left(k^{(6)}\right)_{\alpha\beta\gamma\delta\kappa\lambda}}_{126} \left\{\nabla^{\kappa}, \nabla^{\lambda}\right\} R^{\alpha\beta\gamma\delta} + \underbrace{\left(k^{(6)}\right)_{\alpha\beta\gamma\delta\kappa\lambda\mu\nu}}_{210} R^{\alpha\beta\gamma\delta} R^{\kappa\lambda\mu\nu}\Big] + \mathcal{L}'$$

- $\nabla_\mu k \neq 0$, thanks to property of Riemannian geometry
- Dynamical coefficients iff spontaneous breaking, contained in $\mathcal{L}'$
- Any spontaneous breaking will involve a choice

Altschul+Bailey PRD 2010 consider generic potentials for antisymmetric tensors. For the pure-gravity sector, see e.g. Kostelecky PRD 2004, Bailey+Kostelecky PRD 2006, O'Neal-Ault+Bailey+NAN PRD 2021

Strong-field regime

- Bumblebee: cosmological anisotropies, superradiance in Kerr, noncommutative spacetime, monopoles (Jiang et al 2021, Maluf et al 2021, Kumar Jha et al JCAP 2020,...)[a]
- Cosmology: Inflation, deceleration, modified matter evolution in FLRW, Hubble tension, early Universe (Bonder PRD 2017, 2020, NAN 2018, Ault-O'Neal, Bailey, NAN PRD 2021, Reyes et al PRD 2021, 2022)[a]

[a]and many more...

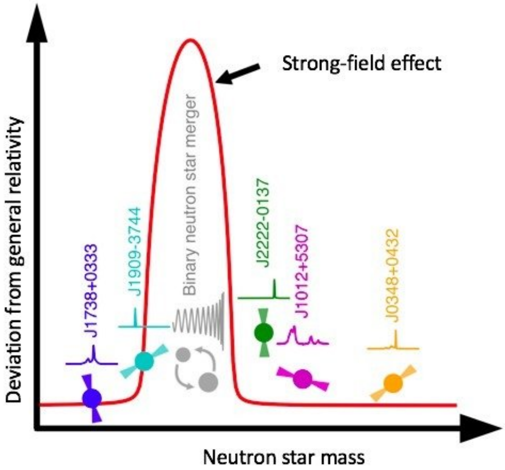


Figure 1: Image credit: L. Shao et al

Cosmological solutions - Global coefficients only

- First cosmological solution within the SME
- Curvature term
- Energy density

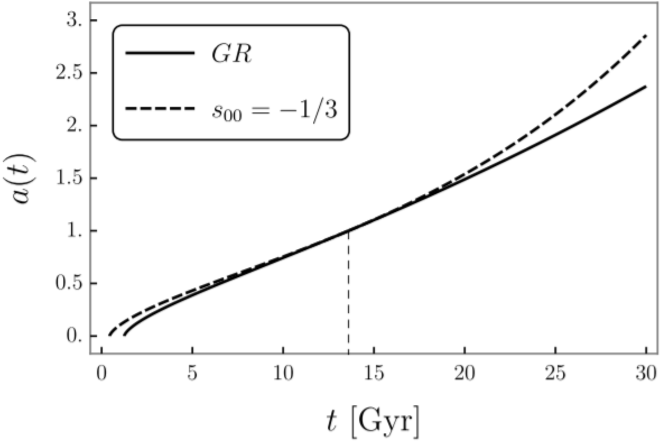
$$\left(\frac{\dot a}{a}\right)^2 (1 - s_{00}) = \frac{\kappa\,\rho}{3} - \frac{k}{a^2} - s_{00}\frac{\ddot a}{a} + \frac{\dot a}{a}\frac{\dot s_{00}}{2},$$

$$\left[\frac{\ddot a}{a} + \frac{1}{2}\left(\frac{\dot a}{a}\right)^2\right](1 - s_{00}) = -\frac{\kappa p}{2} - \frac{k}{2a^2} + \frac{\dot a}{a}\dot s_{00} + \tfrac{1}{4}\ddot s_{00},$$

- Bianchi identities: $\nabla_\mu (T_{\rm s})^{\mu\nu} = -\kappa \nabla_\mu (T_M)^{\mu\nu}$.

$\nabla_\mu (T_s)^{\mu\nu} = \nabla_\mu (T_M)^{\mu\nu} = 0$ OR $\nabla_\mu \left[(T_s)^{\mu\nu} + \kappa (T_M)^{\mu\nu}\right] = 0$

$$\dot{\rho} + 3\frac{\dot{a}}{a} f(w, s_{00})\rho = 0, \quad f(w, s_{00}) = \frac{2(1 + w - s_{00})}{2 + s_{00}(3w - 2)}$$



O'Neal-Ault, Bailey, NAN PRD 2021; NAN, O'Neal-Ault, Bailey CPT'19, 198-200

Existing limits

What are the existing limits on $s^{\mu\nu}$?

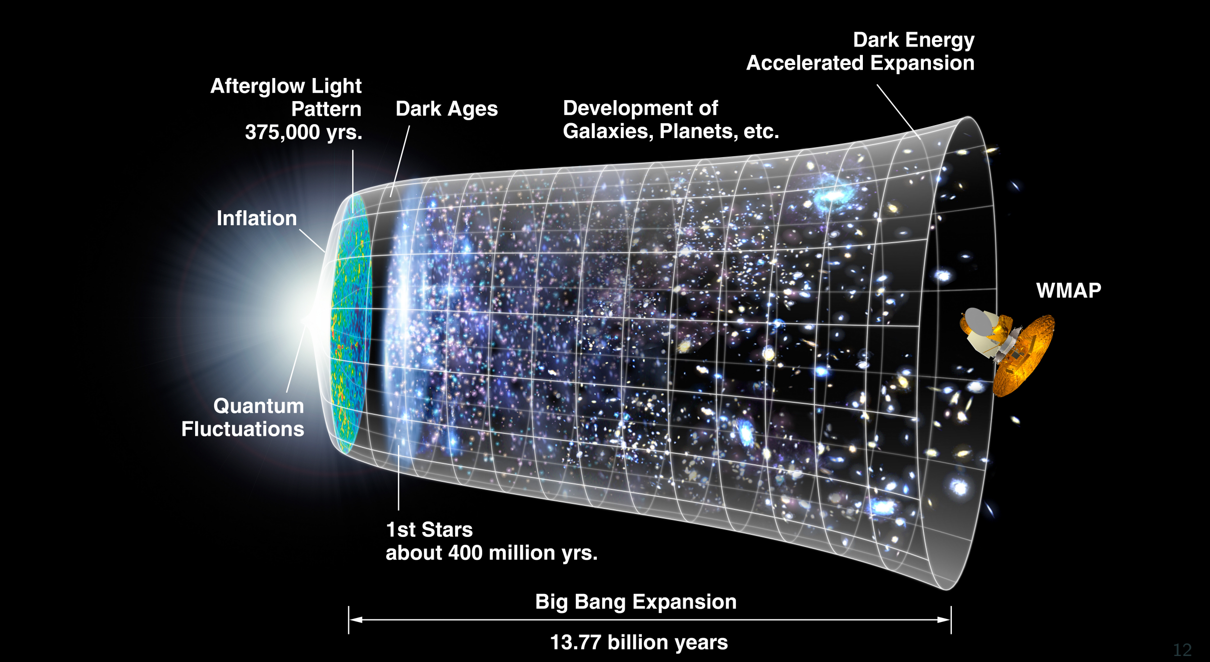
- Choose limits on s^{00} only, as relevant to this work.
- No limits in the exact sector exist! Only linearised gravity, i.e. $\bar{s}^{00}$

Gravitational waves:

- Speed of GW's: $\mathbf{\bar{s}^{00} = -0.2 - 0.07}$ (Liu, He PRD 2020)
- GW170817 and GRB 170817A: $\mathbf{\bar{s}^{00} = (-20 - 2.5) \cdot 10^{-15}}$ (LIGO ApJL 2017)
- Gravitational Cherenkov radiation (cosmic rays): $\mathbf{\bar{s}^{00} > -3 \cdot 10^{-15}}$ (Kostelecky & Tasson PLB 2015)

Can we do better?

Answer: Yes, no, and maybe...


Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
375,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
WMAP
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.77 billion years

Spacetime-symmetry breaking

- The primordial Universe is an excellent probe of fundamental physics
- What happens to primordial gravitational waves if symmetries are broken?
- Primordial gravitational waves produced by quantum fluctuations during inflation; One possible signal, CMB B modes (**not yet observed**)

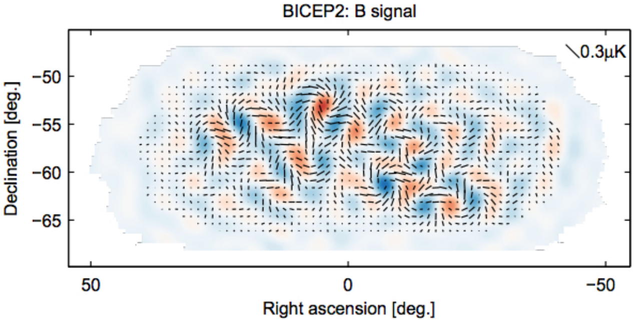


Image: BICEP collaboration

Generating PGW's with spacetime-symmetry breaking

- Introduce tensor perturbations around a flat FLRW background

$$ds^2 = -dt^2 + a(t)(\delta_{ij} + h_{ij}(t, \mathbf{x}))dx^i dx^j$$

- Transverse-traceless perturbation, these are the gravitational waves

$$\delta^2 S = \frac{1}{4}\int dt d^3x a \left[a^2(1 - s_{00})\dot{h}_{ij}\dot{h}^{ij} - \partial_k h_{ij}\partial^k h^{ij}\right]$$

- Equations of motion for the perturbations

$$\ddot{h}_{ij} + \left(3H - \frac{\dot{s}_{00}}{1 - s_{00}}\right)\dot{h}_{ij} - \frac{1}{a^2(1 - s_{00})}\nabla^2 h_{ij} = 0$$

See NAN PRD 2022 for all details

See also Bonder PRD 2017 for a discussion about inflation and spontaneous breaking with a Weyl-symmetric 4-tensor.

Generating PGW's with spacetime-symmetry breaking

- Move to Fourier space using

$$h_{ij}(t,\mathbf{x}) = \sum_{\lambda}\int \frac{d^3k}{(2\pi)^{3/2}}\epsilon_{ij}^{\lambda}(\mathbf{k})k_{\mathbf{k}}(t)e^{i\mathbf{k}\cdot\mathbf{x}} \quad \Longrightarrow \quad \ddot{h}_{\mathbf{k}} + \left(3H - \frac{\dot{s}_{00}}{1-s_{00}}\right)\dot{h}_{\mathbf{k}} + \frac{k^2}{a^2(1-s_{00})}h_{\mathbf{k}} = 0$$

- From here we can read off that

$$c_T^2 = \frac{1}{1-s_{00}}, \qquad m_g = 0, \qquad \frac{d\ln M_{\mathrm{Pl}}^2}{dt} = -\frac{\dot{s}_{00}}{1-s_{00}}.$$

- Propagation of tensor modes ("speed of gravity") constrained with GW170817+GRB170817A to $-3\cdot 10^{-15} < c_T - 1 < +7\cdot 10^{-16}$, leading to

$$\boxed{-6\cdot 10^{-15} < s_{00} < +1.4\cdot 10^{-15}}$$

Background evolution

- We need to plug in the background evolution, which is also modified

$$\frac{H^2}{H_0^2} = \Omega_m^0 a^{-3} + \Omega_r^0 a^{-4x_r} + \Omega_\Lambda^0 a^{-x_\Lambda} + \Omega_k^0 a^{-2}$$

- Evolution of radiation and the cosmological constant (!) different from ΛCDM
- As a result, there is **no pure de Sitter phase**. In conformal time, it reads

$$\mathcal{H}_{\rm dS} = -\frac{2}{(2-x_\Lambda)\eta}, \qquad \mathcal{H}_{\rm RD} = \frac{1}{(2x_r-1)\eta}, \qquad \mathcal{H}_{\rm MD} = \frac{1}{2\eta}$$

Quantising the perturbations

- de Sitter inflation: simple model with exponential expansion and **scale-invariant** power spectrum
- To find the power spectrum, we need quantisation.

$$\hat{\tilde{h}}_{\mathbf{k}}(\eta) = v_{\mathbf{k}}(\eta)\hat{a}_{\mathbf{k}} + v^*_{\mathbf{k}}(\eta)\hat{a}^\dagger_{\mathbf{k}}, \qquad \tilde{h}_{\mathbf{k}} = a h_{\mathbf{k}}$$

- Mode coefficients $v_{\mathbf{k}}(\eta)$ satisfy the **Mukhanov-Sasaki equation**

$$v''_{\mathbf{k}} + \left(\frac{k}{1-s_{00}} - \frac{2}{\eta^2}\gamma\right) v_{\mathbf{k}} = 0, \qquad (s_{00} \to 0 \Rightarrow \gamma \to 1 \Rightarrow \mathrm{GR})$$

- Not possible to rescale away s_{00}'s at both background and perturbation level

Details in NAN PRD 2022 (2205.00496)

The power spectrum

- Now we need the two-point correlation function:

$$\langle 0|\,\hat{h}^{\dagger}_{\mathbf{k}}\hat{h}_{\mathbf{k}}\,|0\rangle \equiv \frac{16\pi G}{a^2}\int_0^{\infty}\frac{dk}{k}\frac{k^3}{2\pi^2}|v_{\mathbf{k}}|^2,$$

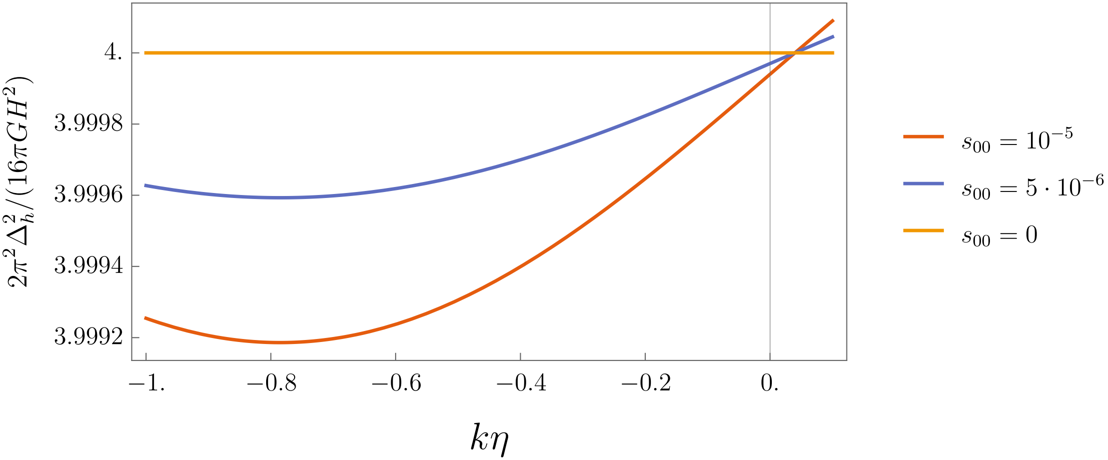
- and the **power spectrum** is the logarithmic derivative of the correlator

$$\Delta_h^2(\eta,k) \equiv \frac{d\,\langle 0|\,\hat{h}^{\dagger}_{\mathbf{k}}\hat{h}_{\mathbf{k}}\,|0\rangle}{d\ln k} = \frac{16\pi G}{a^2}\frac{k^3}{2\pi^2}|v_{\mathbf{k}}|^2 = \frac{16\pi G}{a^2}\frac{k^3}{2\pi^2}H^2(2-x_\Lambda)^2\gamma^2|A_{\mathbf{k}}(\eta)|^2,$$

$$A_{\mathbf{k}}(\eta) = e^{\frac{2ik\eta}{1-s_{00}}}\left(1+e^{i\pi\sqrt{1+8\gamma}}\right) - ie^{\frac{i\pi}{2}\sqrt{1+8\gamma}}$$

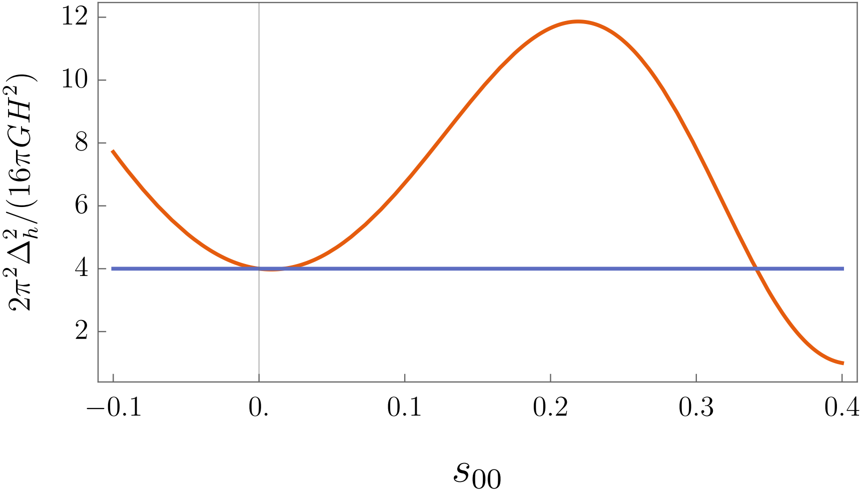
Wronskian condition holds in the GR limit.

Solutions

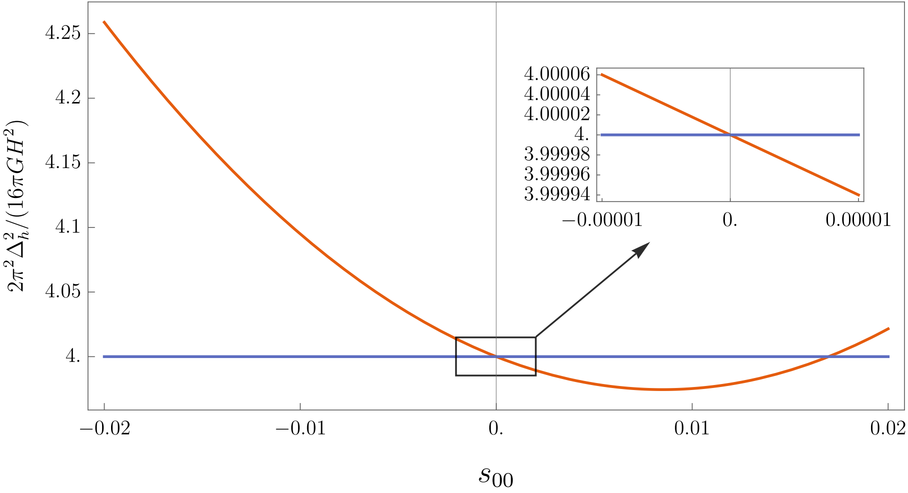


$2\pi^2\Delta_h^2/(16\pi G H^2)$
4.
3.9998
3.9996
3.9994
3.9992
−1.
−0.8
−0.6
−0.4
−0.2
0.
$k\eta$
$s_{00} = 10^{-5}$
$s_{00} = 5 \cdot 10^{-6}$
$s_{00} = 0$

Solutions



Solutions



4.25
4.2
4.15
4.1
4.05
4.
$2\pi^2\Delta_h^2/(16\pi G H^2)$
−0.02
−0.01
0.
0.01
0.02
s_{00}
4.00006
4.00004
4.00002
4.
3.99998
3.99996
3.99994
−0.00001
0.
0.00001

Solutions