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QISS THE QUANTUM INFORMATION STRUCTURE OF SPACETIME

Primordial fluctuations from quadratic curvature terms

MAURICIO GAMONAL - PHD STUDENT - PENNSYLVANIA STATE UNIVERSITY

COSMOLOGY FROM HOME 2023 - PARALLEL TALK - ONLINE CONFERENCE

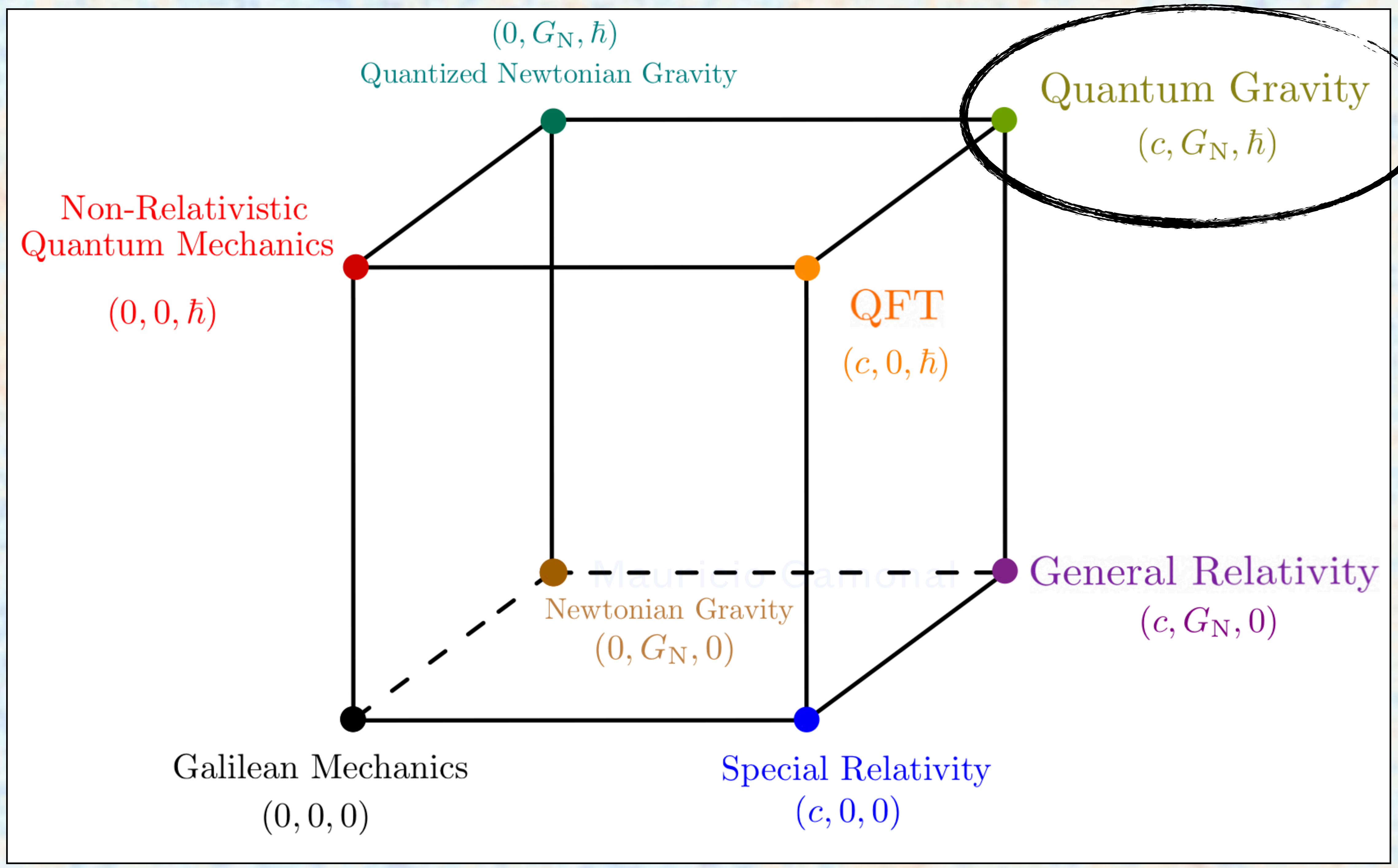
WORK IN PROGRESS: COLLABORATION WITH PROF. EUGENIO BIANCHI

MGAMONAL@PSU.EDU / EBIANCHI@PSU.EDU

Main Motivation

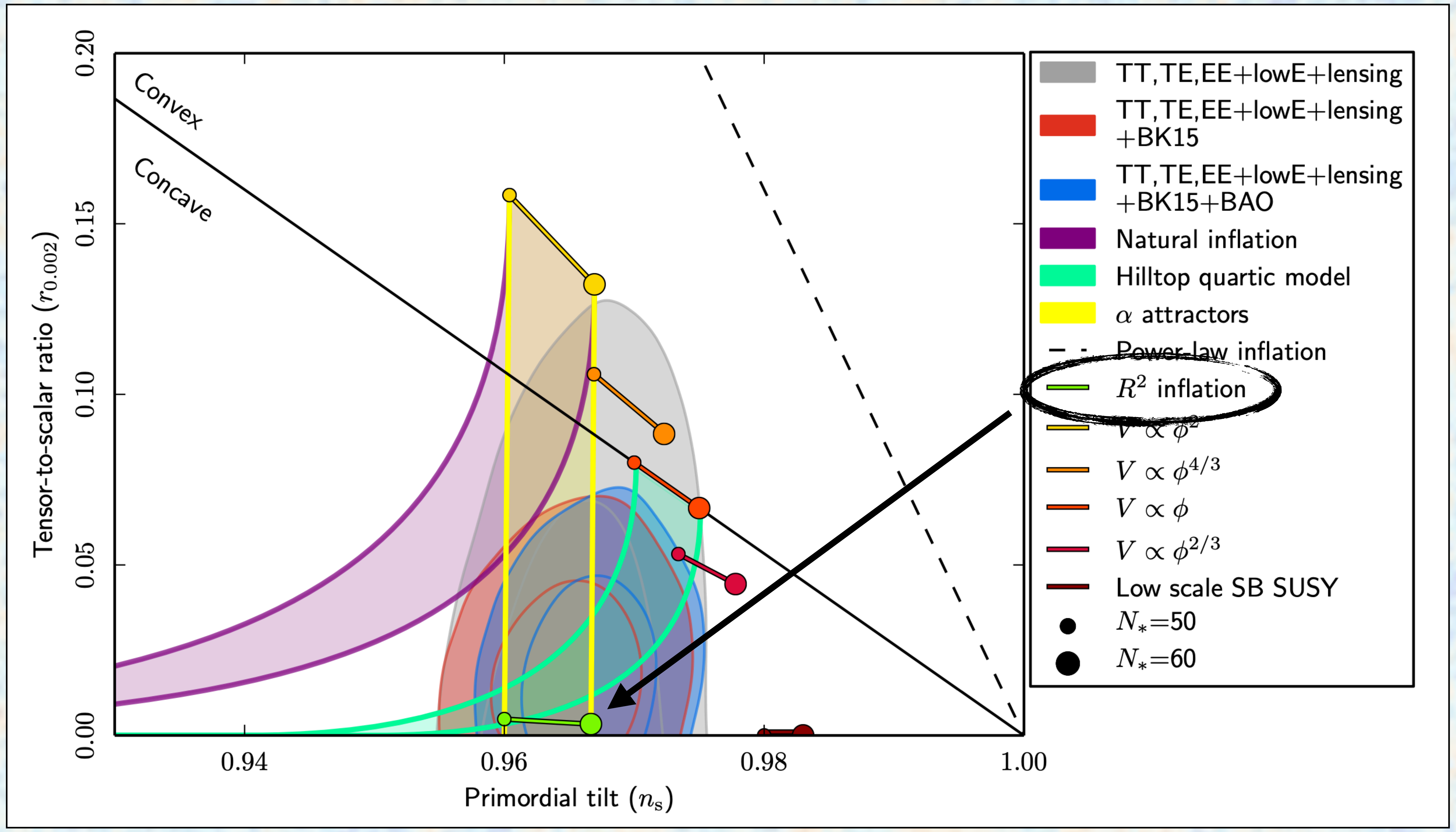
Looking for Quantum Gravity effects.
Primordial cosmology as a natural laboratory.

Quantum Gravity regime



[Bronstein '38]

"Inflationary models such as R^2, (...) and those having a potential with exponential tails provide good fits to Planck and BK15 data."



[Planck Collaboration '20]

Pushing the limits of General Relativity

How to extend the gravitational theory? : Organizational Principle of Derivatives

- Additional invariant terms are allowed by the symmetries of General Relativity [Lanczos '38, Stelle '77]
- Arguments from renormalization techniques [Utiyama & DeWitt '62, t'Hooft & Veltman '74, Weinberg '79]
- Arguments from Effective Field Theory methods [Donoghue '94, Burgess '03, Weinberg '08]

Most general action we can write up to four derivatives of the metric

$$S_g[g_{\mu\nu}] = \int \mathrm{d}^4x \sqrt{-g}\left(c_0 + c_2 R + c_4^{(1)} R^2 + c_4^{(2)} R_{\mu\nu}R^{\mu\nu} + c_4^{(3)} R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + \dots\right)$$

0 derivatives (c_0)

Up to 2 derivatives ($c_2 R$)

$$R \sim \partial\partial g$$

Up to 4 derivatives (R^2, $R_{\mu\nu}R^{\mu\nu}$, $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$)

$$R^2, R_{\mu\nu}R^{\mu\nu}, R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \sim \partial\partial\partial\partial g$$

Rewriting the most generic 4th order action

Gauss-Bonnet Theorem: χ is a surface term in 4D

$$\chi = \int d^4x\sqrt{-g}(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma})$$

Weyl Tensor in 4D satisfies:

$$W_{\lambda\mu\nu\kappa} = R_{\lambda\mu\nu\kappa} - \frac{1}{2}(g_{\lambda\nu}R_{\mu\kappa} - g_{\lambda\kappa}R_{\mu\nu} - g_{\mu\nu}R_{\lambda\kappa} + g_{\mu\kappa}R_{\lambda\nu}) + \frac{R}{6}(g_{\lambda\nu}g_{\mu\kappa} - g_{\lambda\kappa}g_{\mu\nu})$$

$$W_{\mu\nu\rho\sigma}W^{\mu\nu\rho\sigma} = \frac{1}{3}R^2 - 2R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$$

$$c_1R^2 + c_2R_{\mu\nu}R^{\mu\nu} + c_3R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \quad \rightarrow \quad c_1'R^2 + c_2'W_{\mu\nu\rho\sigma}W^{\mu\nu\rho\sigma}$$

Purely gravitational (primordial) cosmology

In this work we will consider an action of the form:

$$S_g[g_{\mu\nu}] = \frac{1}{16\pi G_{\rm N}} \int \mathrm{d}^4x \sqrt{-g} \left[\boxed{-2\Lambda + R} + \alpha R^2 + \beta W_{\mu\nu\rho\sigma} W^{\mu\nu\rho\sigma} \right]$$

[Hilbert '16]
[Einstein '17]

[Starobinsky '79]

[Weinberg '08, Anselmi et al. '20]
This work (2023)

Field equations:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} + \alpha H_{\mu\nu} + 4\beta B_{\mu\nu} = 8\pi G_{\rm N} \langle T_{\mu\nu} \rangle_{\rm ren}$$

$$H_{\mu\nu} \equiv 2R\, G_{\mu\nu} - 2\nabla_\mu \nabla_\nu R + 2\left(\nabla_\alpha \nabla^\alpha R + \frac{1}{4} R^2\right) g_{\mu\nu}$$

$$B^{\mu\nu} \equiv \nabla_\alpha \nabla_\beta W^{\mu\alpha\nu\beta} + \frac{1}{2} R_{\alpha\beta} W^{\mu\alpha\nu\beta}$$

Purely gravitational (primordial) cosmology

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[Hilbert '16]
[Einstein '17]

[Starobinsky '79]

[Weinberg '08, Anselmi et al. '20]
This work (2023)

What is new?

$$S \sim \int \mathrm{d}t \left(\frac{1}{2} m \dot{x}^2 - \frac{1}{2} m \omega^2 x^2 + \frac{1}{2} m \beta \ddot{x}^2 \right)$$

Different from Weinberg: No additional fields or degrees of freedom.

Different from Anselmi-Bianchi-Piva: No mention to purely virtual particles.

Our contribution

$$S \sim \int \mathrm{d}t \left(\frac{1}{2} m_{\mathrm{eff}} \dot{x}^2 - \frac{1}{2} m_{\mathrm{eff}} \omega_{\mathrm{eff}}^2 x^2 \right)$$

+ observational effects on the primordial power spectra

$$\mathcal{P}_h \sim \mathcal{P}_h (1 + \text{correction}) \qquad n_t \sim n_t (1 + \text{correction})$$

Cosmic inflation in the geometric framework

[Starobinsky '79; Guth '80; Linde '82, and others]

Few assumptions:
FLRW Background
$\Lambda = k = 0$

$$ds^2 = \bar{g}_{\mu\nu}\, dx^\mu\, dx^\nu = -dt^2 + a(t)^2 \delta_{ij}\, dx^i\, dx^j\,, \quad H(t) = \frac{\dot{a}(t)}{a(t)}$$

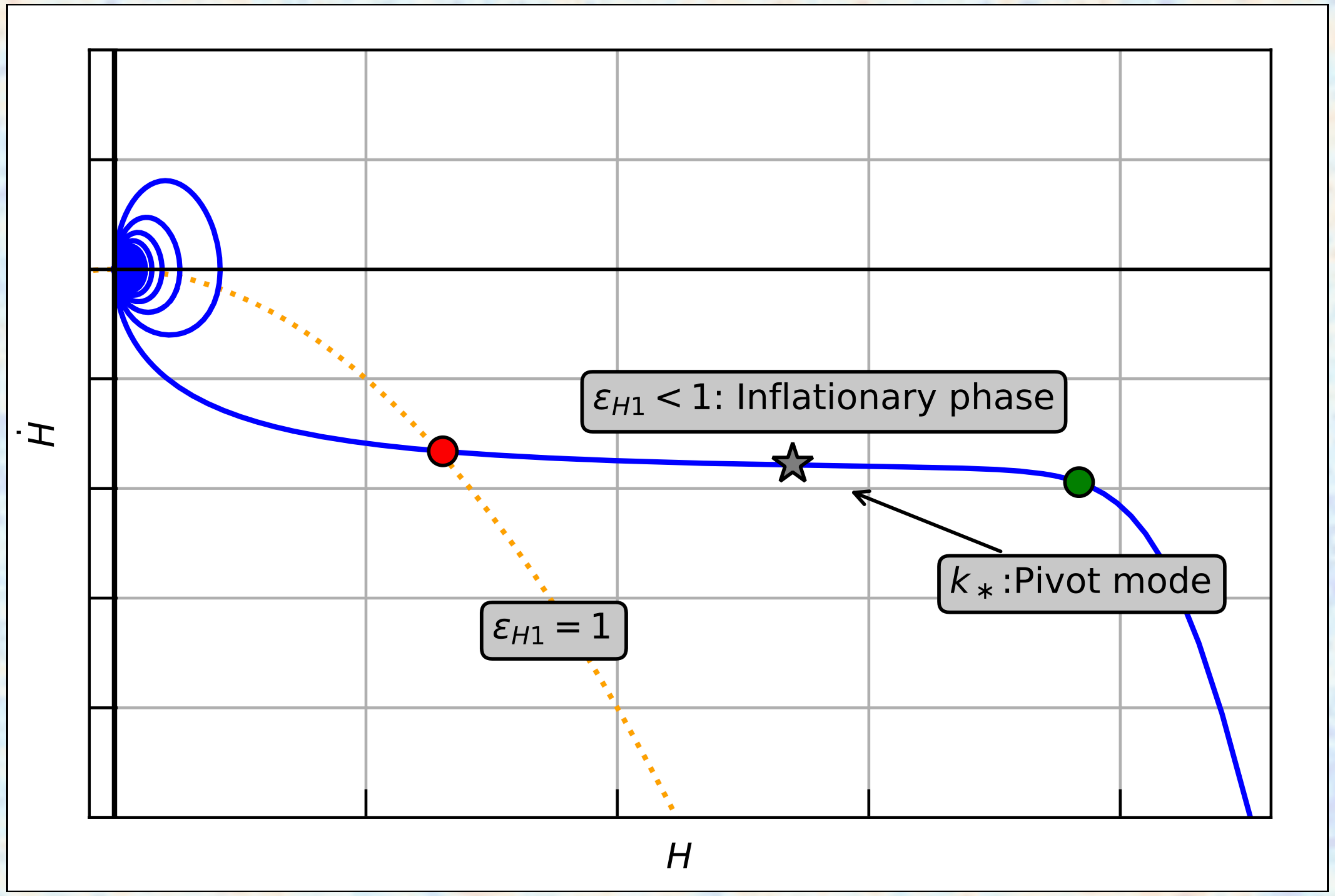
$$H^2(t) + 6\alpha\left(6H^2(t)\dot{H}(t) - \dot{H}^2 + 2H(t)\ddot{H}(t)\right) = 0$$

$$\epsilon_{H1}(t) = -\frac{\dot{H}(t)}{H^2(t)} \longrightarrow \ddot{a}(t) = a(t)H(t)^2\left(1 - \epsilon_{H1}(t)\right)$$

Inflationary phase occurs as long as $\ddot{a} > 0$, or $\epsilon_{H1}(t) < 1$

Standard requirement for amount of inflation ~ 60 e-folds

$$a(t) \simeq a_i e^N \;\rightarrow\; N_* \approx \frac{1}{2\epsilon_{H1*}}$$



Cosmological Perturbations for $\beta = 0$

[Kodama & Sasaki '84, Mukhanov, Feldman & Branderberger '92]

Different approach: Geometric formalism. No need to include a scalar field.

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu} \;\rightarrow\; S = S^{(0)}[\bar{g}_{\mu\nu}] + \cancel{\lambda S^{(1)}[\delta g_{\mu\nu}]} + \lambda^2 S^{(2)}[\delta g_{\mu\nu}]$$

Comoving gauge → scalar curvature perturbations ($\mathcal{R}$), and tensor perturbations ($h_{\times,+}$) are the only physical DOFs

$$S^{(2)}_{\text{scalar}}[\mathcal{R}] = \int \mathrm{d}t \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} a(t)^3 Z_S(t) \left(\frac{1}{2}\left|\dot{\mathcal{R}}(\mathbf{k},t)\right|^2 - \frac{1}{2}\left(\frac{\mathbf{k}}{a}\right)^2 |\mathcal{R}(\mathbf{k},t)|^2 \right)$$

$$S^{(2)}_{\text{tensor}}[h_\times, h_+] = \sum_{\sigma=\times,+} \int \mathrm{d}t \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} a(t)^3 Z_T(t) \left(\frac{1}{2}\left|\dot{h}_\sigma(\mathbf{k},t)\right|^2 - \frac{1}{2}\left(\frac{\mathbf{k}}{a}\right)^2 |h_\sigma(\mathbf{k},t)|^2 \right)$$

$$r \sim \frac{12}{N_*^2}$$

$$n_s \sim 1 - \frac{2}{N_*}$$

$$\mathcal{P}_\mathcal{R}(k) = \frac{1}{18\pi} \frac{\hbar G_\mathrm{N}}{c^3 \alpha} N_*^2 \left(1 - \frac{1+2C}{N_*} + \left(-\frac{2}{N_*}\right) \log\left(\frac{k}{k_*}\right) + \mathcal{O}(N_*^{-2}) \right)$$

$$\mathcal{P}_h(k) = \frac{2}{3\pi} \frac{\hbar G_\mathrm{N}}{c^3 \alpha} \left(1 - \frac{3}{2N_*} + \left(-\frac{3}{2N_*^2}\right) \log\left(\frac{k}{k_*}\right) + \mathcal{O}(N_*^{-3}) \right)$$

Preliminary Results: Cosmological Perturbations for $\beta \neq 0$

What happens when $\beta \neq 0$? First challenge: Tensor fluctuations

$$S^{(2)}_{\rm tensor}[h_\times, h_+] = \sum_\sigma \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\left[\frac{1}{2}A(k,t)\,\dot{h}_\sigma(\mathbf{k},t)^2 - \frac{1}{2}B(k,t)\left(\frac{k}{a(t)}\right)^2 h_\sigma(\mathbf{k},t)^2 + \frac{1}{2}\beta\,\Gamma(t)\ddot{h}_\sigma(\mathbf{k},t)^2\right]$$

$$\ddot{x}^2 \text{ in the action } \rightarrow \ddddot{x} \text{ in the EoM}$$

Can we extract a physically relevant solution?

Dealing with higher order time derivatives

Example: Abraham-Lorentz force (radiation reaction)

$$m\ddot{x} = \underbrace{\frac{\mu_0 q^2}{6\pi c}}_{\lambda} \dddot{x} + F_{\text{ext}} \longrightarrow$$

Reduced equation [Landau & Lifshitz '75]

$$m\ddot{x} = F_{\text{ext}} + \mathcal{O}(\lambda) \;\rightarrow\; \dddot{x} = \frac{1}{m}\dot{F}_{\text{ext}} + \mathcal{O}(\lambda)$$

$$m\ddot{x} = \underbrace{F_{\text{ext}} - \frac{\lambda}{m}\dot{F}_{\text{ext}}}_{F_{\text{eff}}} + \mathcal{O}(\lambda^2)$$

Experimental verification of the Landau–Lifshitz equation

C F Nielsen[1,*], J B Justesen[1], A H Sørensen[1], U I Uggerhøj[1], R Holtzapple[2] and CERN NA63

[1] Department of Physics and Astronomy, Aarhus University, 8000 Aarhus, Denmark
[2] Department of Physics, California Polytechnic State University, San Luis Obispo, CA 93407, United States of America
[*] Author to whom any correspondence should be addressed.

E-mail: christianfn@phys.au.dk

Keywords: radiation reaction, strong fields, classical electrodynamics

"The Lorentz force has been shown to be insufficient in describing the dynamics and resulting radiation spectra in our crystal-based strong-field experiments. This deficiency is remedied by theoretical predictions resting on the Landau-Lifshitz equation, when adjusted through the substitution model accounting for the photon recoil in the radiation spectra from planar channeled 50 GeV positrons"

[Nielsen et al. '21]

Method : Reduction of Order

[Inspired by Parker & Simon '93]

Illustrative example: Harmonic oscillator with acceleration term:

$$S[x(t)] = \int \mathrm{d}t \left(\frac{1}{2} m_0 \dot{x}^2 - \frac{1}{2} m_0 \omega_0^2 x^2 + \frac{1}{2} m_0 \beta \ddot{x}^2 \right) \to \ddot{x} + \omega_0^2\, x - \beta \ddddot{x} = 0$$

$$\ddot{x}(x) = -\omega_0^2\, x(t) + \mathcal{O}(\beta\omega_0^2) \longrightarrow S[x(t)] = \int \mathrm{d}t \left(\frac{1}{2} m_0 \dot{x}^2 - \frac{1}{2} m_0 \omega_{\text{eff}}^2 x^2 \right) \qquad \omega_{\text{eff}}^2 = \omega_0^2 (1 - \beta\omega_0^2) + \mathcal{O}((\beta\omega_0^2)^2)$$

Exact solution

$$x(t) = A_1 e^{\Lambda_+ t} + A_2 e^{-\Lambda_+ t} + B_1 e^{\Lambda_- t} + B_2 e^{-\Lambda_- t}$$

$$\Lambda_\pm = \omega_0 \sqrt{\frac{1 \pm \sqrt{1 + 4\beta\omega_0^2}}{2\beta\omega_0^2}} = \begin{cases} \Lambda_+ & = \frac{\omega_0}{\sqrt{\beta\omega_0^2}} + \mathcal{O}\left(\sqrt{\beta\omega_0^2}\right) \\ \Lambda_- & = i\,\omega_0 (1 - \frac{1}{2}\beta\omega_0^2) + \mathcal{O}((\beta\omega_0^2)^2) \end{cases}$$

Reduced solution

$$x(t) = A\cos(\omega_{\text{eff}} t + B) \qquad \omega_{\text{eff}} = \omega_0 \left(1 - \frac{1}{2}\beta\omega_0^2 \right)$$

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$$\ddot{x}(x) = -\omega_0^2\, x(t) + \mathcal{O}\left(\beta \omega_0^2\right) \longrightarrow S[x(t)] = \int \mathrm{d}t \left(\frac{1}{2} m_0 \dot{x}^2 - \frac{1}{2} m_0 \omega_{\rm eff}^2 x^2 \right)$$

$$\omega_{\rm eff}^2 = \omega_0^2 (1 - \beta \omega_0^2) + \mathcal{O}((\beta \omega_0^2)^2)$$

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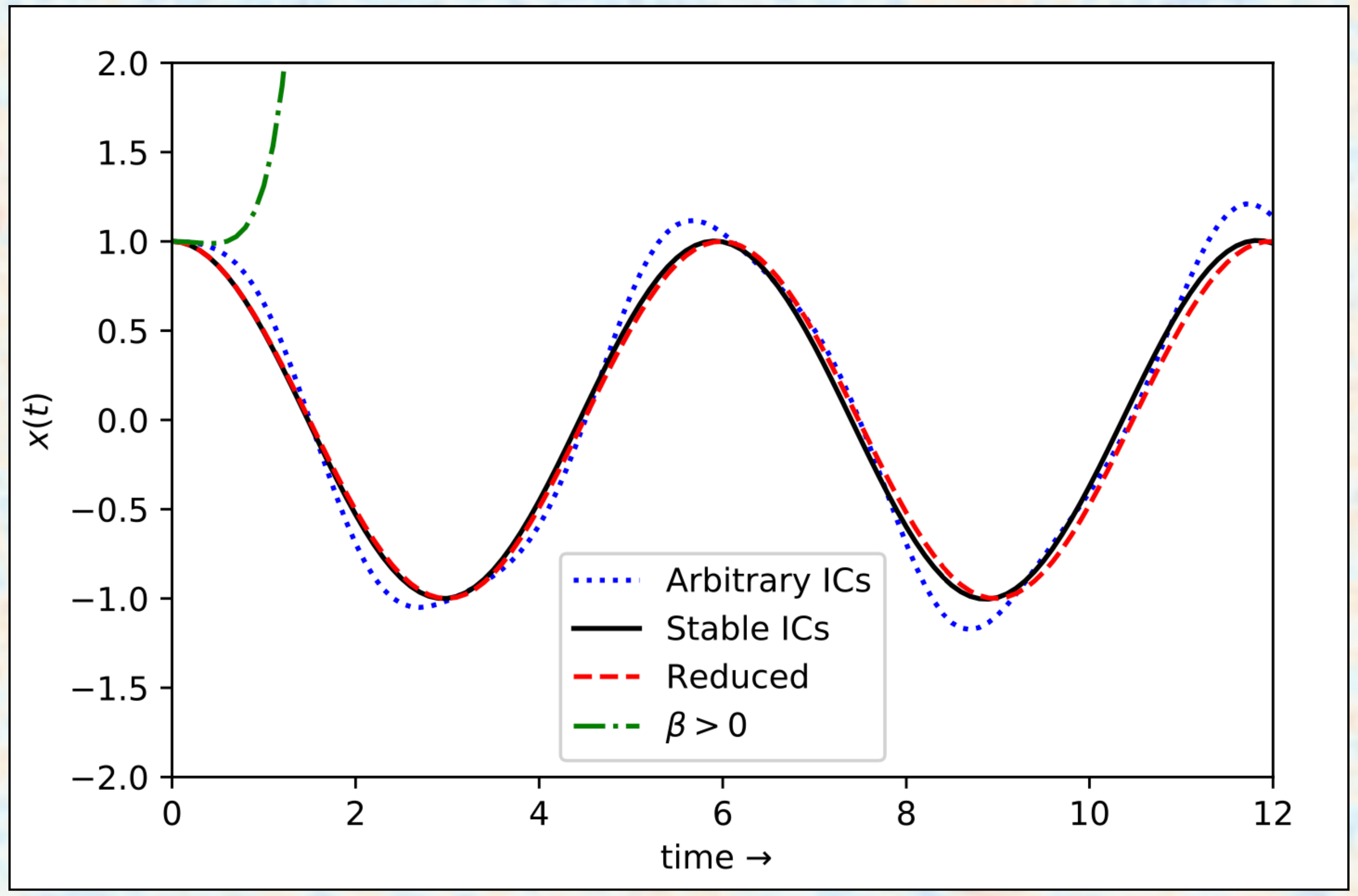
Stable initial conditions:

$$\ddot{x}(0) = \ddot{x}_{\rm red}(0), \;\; \dddot{x}(0) = \dddot{x}_{\rm red}(0)$$

$$\beta = -0.1, \, \omega = 1$$



Preliminary Results - $\alpha \neq 0, \beta \neq 0$

After reduction of order, we obtain:

$$S^{(2)}_{\text{tensor}}[h_\times, h_+] = \sum_{\sigma=\times,+} \int \mathrm{d}t \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} a(t)^3 Z_T(t) \left(\frac{1}{2} \left| \dot{h}_\sigma(\mathbf{k}, t) \right|^2 - \frac{c_s^2(t)}{2} \left(\frac{\mathbf{k}}{a} \right)^2 |h_\sigma(\mathbf{k}, t)|^2 \right)$$

$$c_s(t) = 1 + \left(\frac{\beta}{\alpha}\right) f_1(t) = 1 - \frac{1}{6}\left(\frac{\beta}{\alpha}\right) + \frac{5}{12}\left(\frac{\beta}{\alpha}\right)\epsilon_{H1*} + \mathcal{O}(\epsilon_*^2)$$

$$Z_T(t) = Z_T^{(\beta=0)}(t) + \left(\frac{\beta}{\alpha}\right)\left[f_2(t) - \frac{1}{4\pi G_\mathrm{N}} \left(\frac{k}{a(t)} \right)^2 \right]$$

$f_1(t), f_2(t)$: Functions of $H(t), \dot{H}(t), \ldots$

Some additional challenges: Dealing with c_s^2, and $(k/a)^2$

Primordial Power Spectra - $\alpha \neq 0, \beta \neq 0$

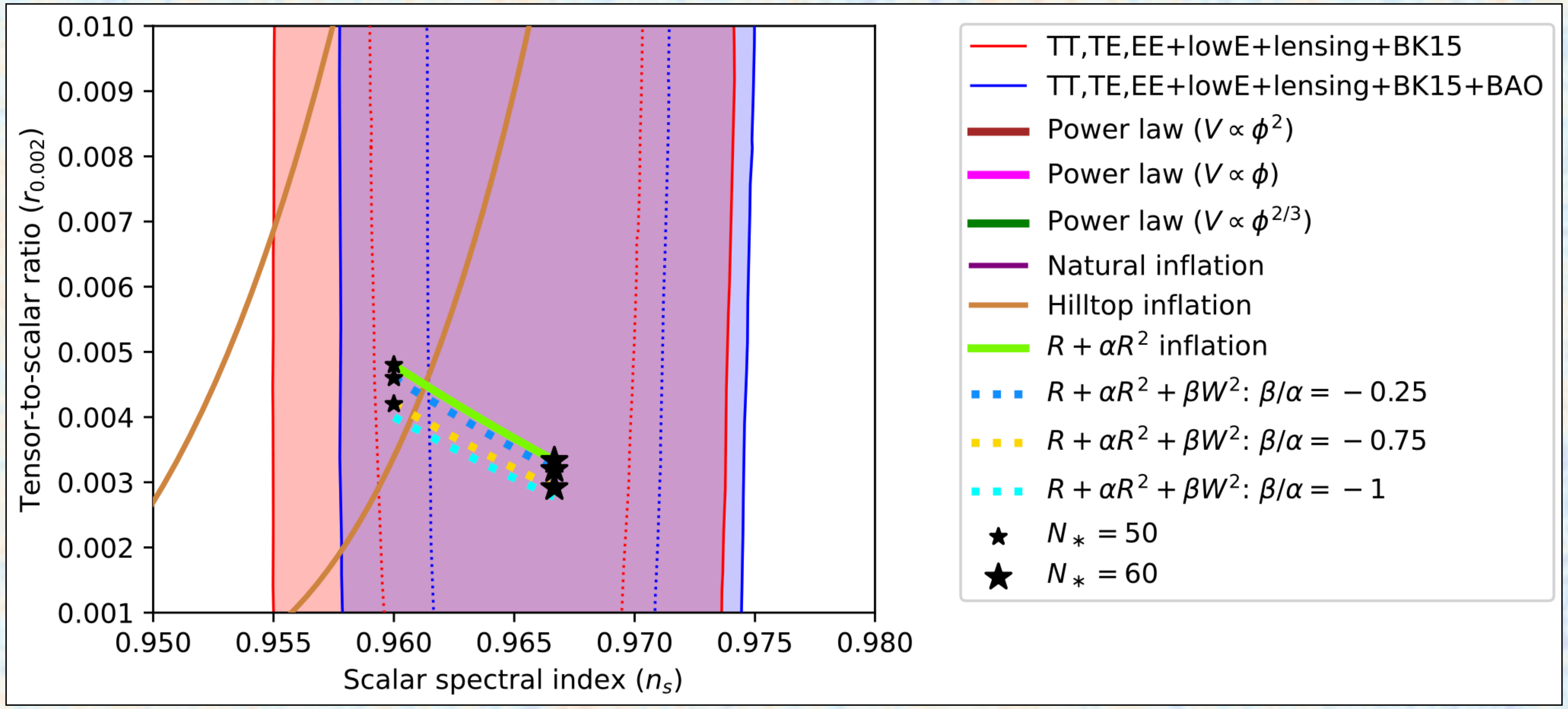
$$\mathcal{P}_\mathcal{R} = \mathcal{P}_\mathcal{R}^{(\beta=0)}\left[1 + \mathcal{O}(N_*^{-1})\right]$$

$$n_{s*} = n_{s*}^{(\beta=0)} = 1 - \frac{2}{N_*} + \mathcal{O}(N_*^{-2})$$

$$\mathcal{P}_h = \frac{2}{3\pi}\frac{\hbar G_\mathrm{N}}{c^3 \alpha}\left[1 + \frac{1}{6}\left(\frac{\beta}{\alpha}\right) + \mathcal{O}(N_*^{-1})\right]$$

$$r_* = \frac{12}{N_*^2}\left[1 + \frac{1}{6}\left(\frac{\beta}{\alpha}\right) + \mathcal{O}(N_*^{-1})\right]$$

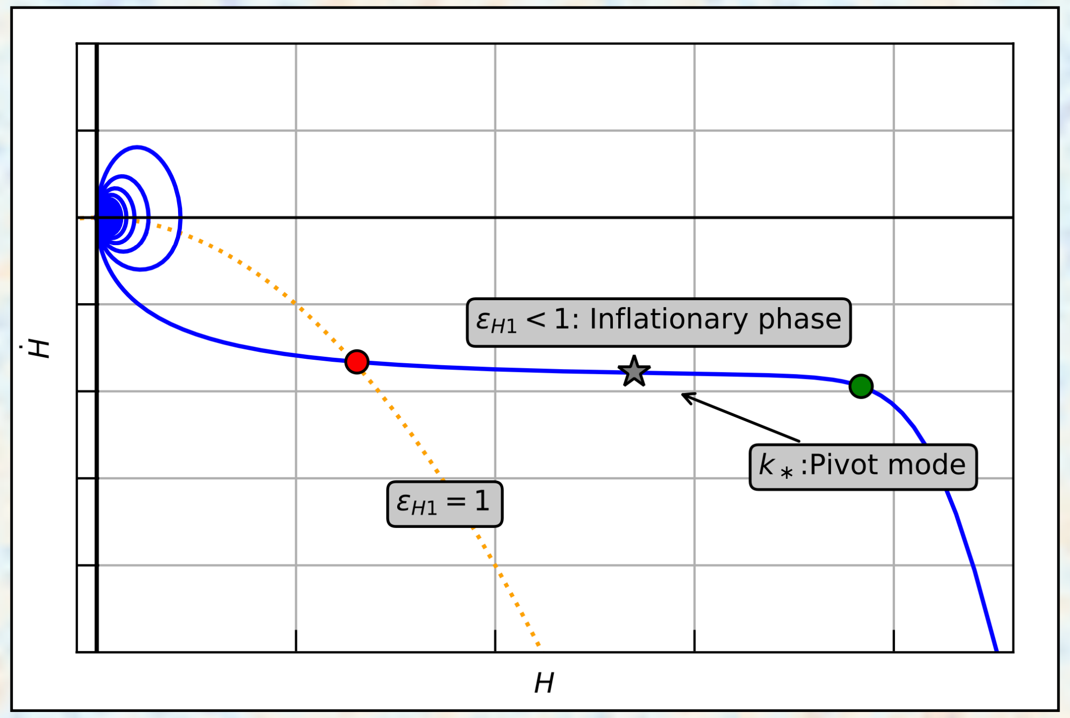
$$n_{t*} = -\frac{3}{2N_*^2}\left[1 + \frac{1}{6}\left(\frac{\beta}{\alpha}\right) + \mathcal{O}(N_*^{-1})\right]$$

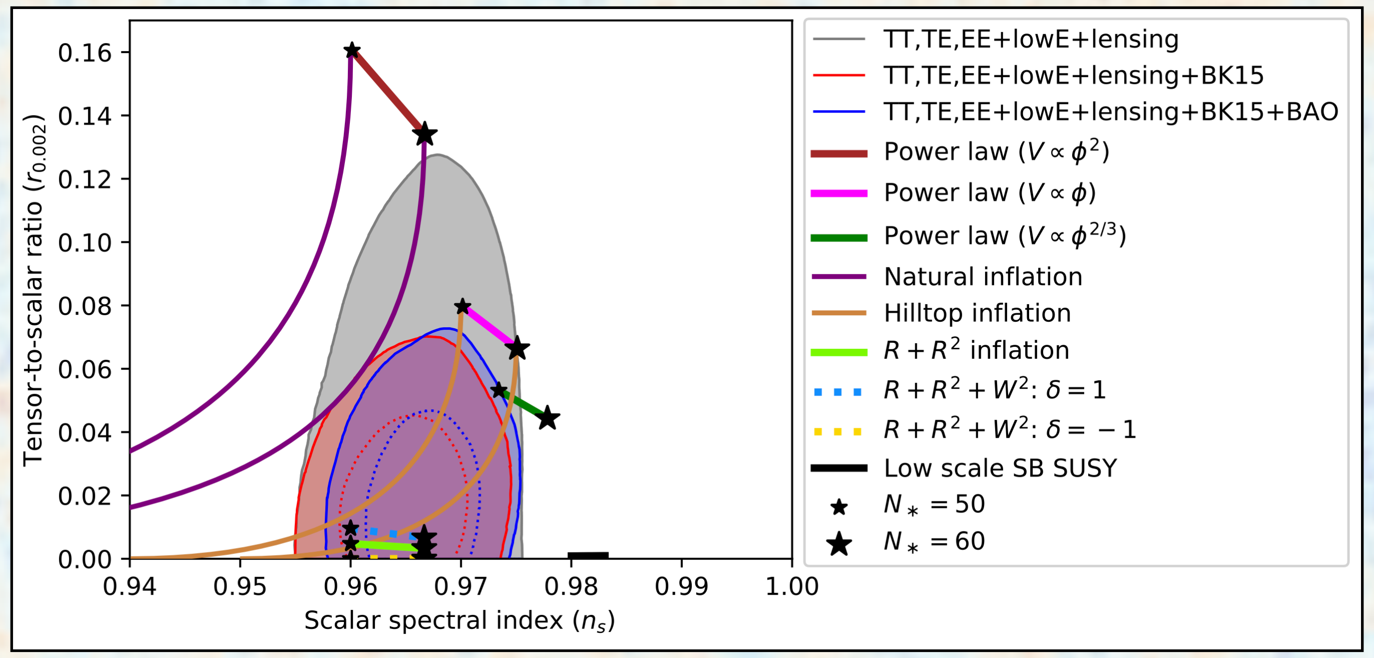


Self-elaborated plot, based on [Planck '20]

Conclusions

- Pure gravity + higher order curvature terms trigger cosmic inflation.
- We expect deviations from the Starobinsky model if a **Weyl-squared** term is considered.
- Cosmological perturbations from the new contribution are studied by **reducing the order** of the equations, extracting the physically relevant solutions.
- Corrections to the primordial power spectrum of tensor fluctuations and tensor tilt.





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