Reconstructing the Dark Energy

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Introduction

$$H^2(a) = H_0^2\left[\Omega_{r,0}\left(\frac{a_0}{a}\right)^4 + \Omega_{m,0}\left(\frac{a_0}{a}\right)^3 + \Omega_\Lambda + \Omega_{k,0}\left(\frac{a_0}{a}\right)^2\right]$$

At this point, it is common knowledge that the LCDM model is considered the standard model in Cosmology.

LCDM is very good at explaining most observations and phenomena.

But as data becomes more abundant and precise it starts showing some problems, such as:

- LCDM has problems at small scales.
- Hubble and S8 tensions.
- Coincidence problem.
- Fine-tuning problem.
- Just to mention a few…

And also not to mention the unexplained nature of the Dark Sector (Matter and Energy).

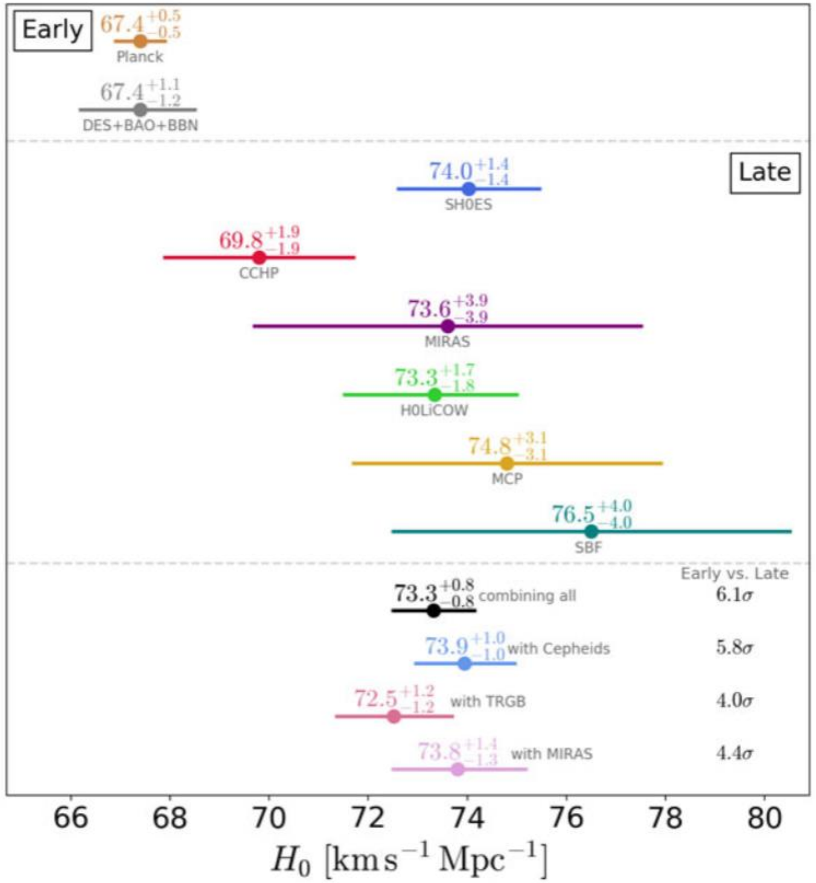


Image from:

Snowmass2021 - Letter of Interest Cosmology Intertwined II: The Hubble Constant Tension

What can we do?

Two inferences we can make are:
- There is something wrong with the data, or…
- There is something wrong with the model.

If the data is the problem, then one can only expect future better and bigger surveys to solve the problem.

If, on the other hand, the model or theory is the problem then we have several alternatives.

One of these alternatives is the focus of this work and can be generally referred to as **Reconstructions.**
They can be broadly classified in 3 types:
- Parametric
- Non-parametric
- Model-independent

Parametric reconstructions

In this kind of reconstruction, a parametric or functional form is proposed for the quantity. Among the widely employed parameterizations in the literature, two prominent ones are the wCDM and the CPL parameterizations.

To determine the values of these functional forms' parameters, cosmological observations and data are utilized for inference.

They can be used to study a particular type of behavior such as:

- Oscillations
- Exponential growth.
- Just to mention a few…

$$w(z) = w_0 + w_1 z$$
$$w(a) = w_0 + w_a(1-a) = w_0 + w_a \frac{z}{1+z}$$
$$w_a = \sum_{i=0}^{N} (1-a)^i w_i$$
$$\Omega_{DE} = A_1 + A_2 x + A_3 x^2 \text{ con } x = z+1$$
$$w(z) = \sum_{i=0}^{N} w_i z^i$$
$$w(z) = w_i + \frac{w_f - w_i}{1+e^{\frac{z-z_t}{\Delta}}}$$
$$w(\log a) = w_0 + w_1 \cos\left(A \log \frac{a}{a_c}\right)$$
$$w(a) = w_0 e^{a-1}$$
$$w(a) = w_0 a(1-\log(a))$$
$$w(a) = w_0 a e^{a-1}$$
$$w(a) = w_0 a(1+\sin(1-a))$$
$$w(a) = w_0 a(1+\arcsin(1-a))$$
…

Non-parametric reconstructions

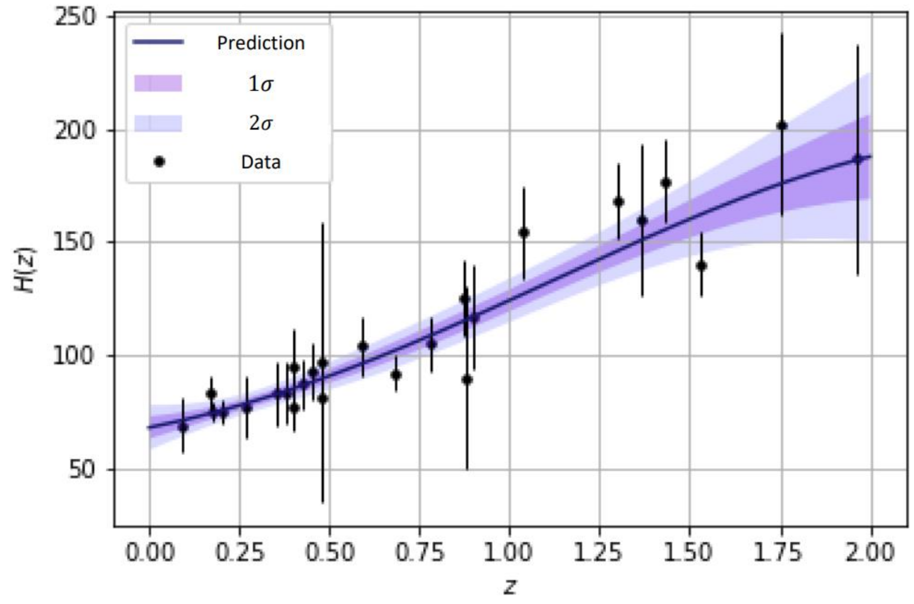
This approach to reconstructions does not assume a specific functional form or parametric model.

They typically rely on statistical and computational methods to analyze observational data.

Some examples are:

- Gaussian Process
- LOESS+Simex
- Neural Networks

Example of a non-parametric reconstruction of the Hubble parameter with a Gaussian Process.



Model-independent reconstructions

Unlike their non-parametric counterparts, these reconstructions do possess parameters and a functional form, although they are given much more freedom of shape and can be used to perform a model-selection procedure.

Some examples are:

- Bins reconstruction
- Spline reconstruction
- Fourier series
- Padé approximation

We will reserve a detailed discussion on these methods for later, as they hold a prominent position in this work.

Reconstructions used

Graduated DE

We performed a reconstruction of a physically motivated parameterization called "Graduated Dark Energy" (gDE).

Graduated dark energy: Observational hints of a spontaneous sign switch in the cosmological constant

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https://journals.aps.org/prd/abstract/10.1103/PhysRevD.101.063528

We have a DE density which can be allowed to change sign.

$$\frac{\rho_{\rm DE}}{\rho_{\rm c,0}} = \Omega_{\rm DE,0}\,\mathrm{sgn}[1-\Psi\ln a]\left|1-\Psi\ln a\right|^{\frac{1}{1-\lambda}}$$

Where $\Psi \equiv -3\gamma(\lambda-1)$

Which then leaves us with a Friedmann's equation:

$$\frac{H^2}{H_0^2} = \Omega_{\rm r,0}a^{-4} + \Omega_{\rm m,0}a^{-3} + \Omega_{\rm DE,0}\,\mathrm{sgn}[1-\Psi\ln a]\left|1-\Psi\ln a\right|^{\frac{1}{1-\lambda}}$$

The sign change occurs at a scale factor of:

$$a = a_* \equiv \mathrm{e}^{-\frac{1}{3}\frac{1}{\gamma(\lambda-1)}}$$

Nodal Reconstruction

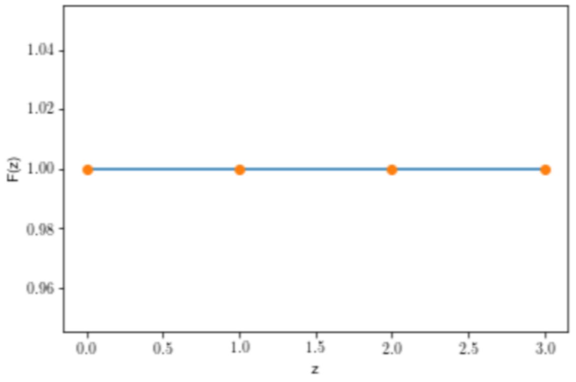
We use “nodes” which are then interpolated

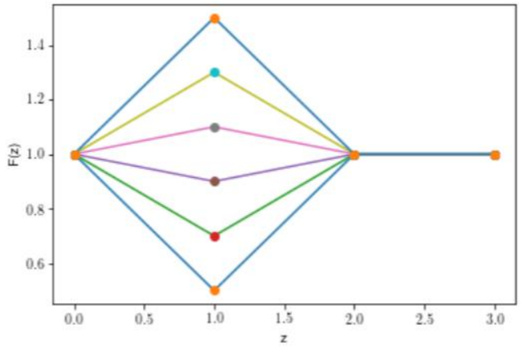
When using a straight line to interpolate:

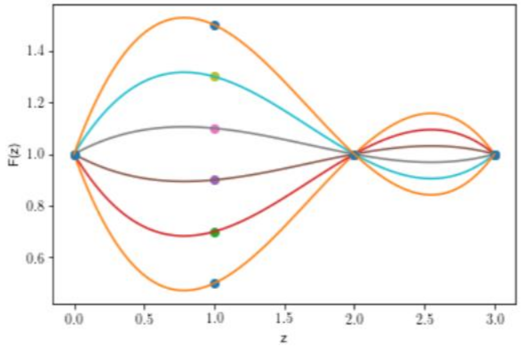
$$L_i(z) = \frac{w_{i+1} - w_i}{z_{i+1} - z_i}(z - z_i) + w_i, \quad z \in [z_i, z_{i+1}].$$

We can also use higher order polynomials to to the interpolations, such as the Cubic Spline interpolation. But the Cubic Spline could introduce unwanted noise to the reconstructed quantity.

Example with 4 nodes.







Bins reconstruction

Similar to the nodal one, but with the advantage of being easily differentiable.

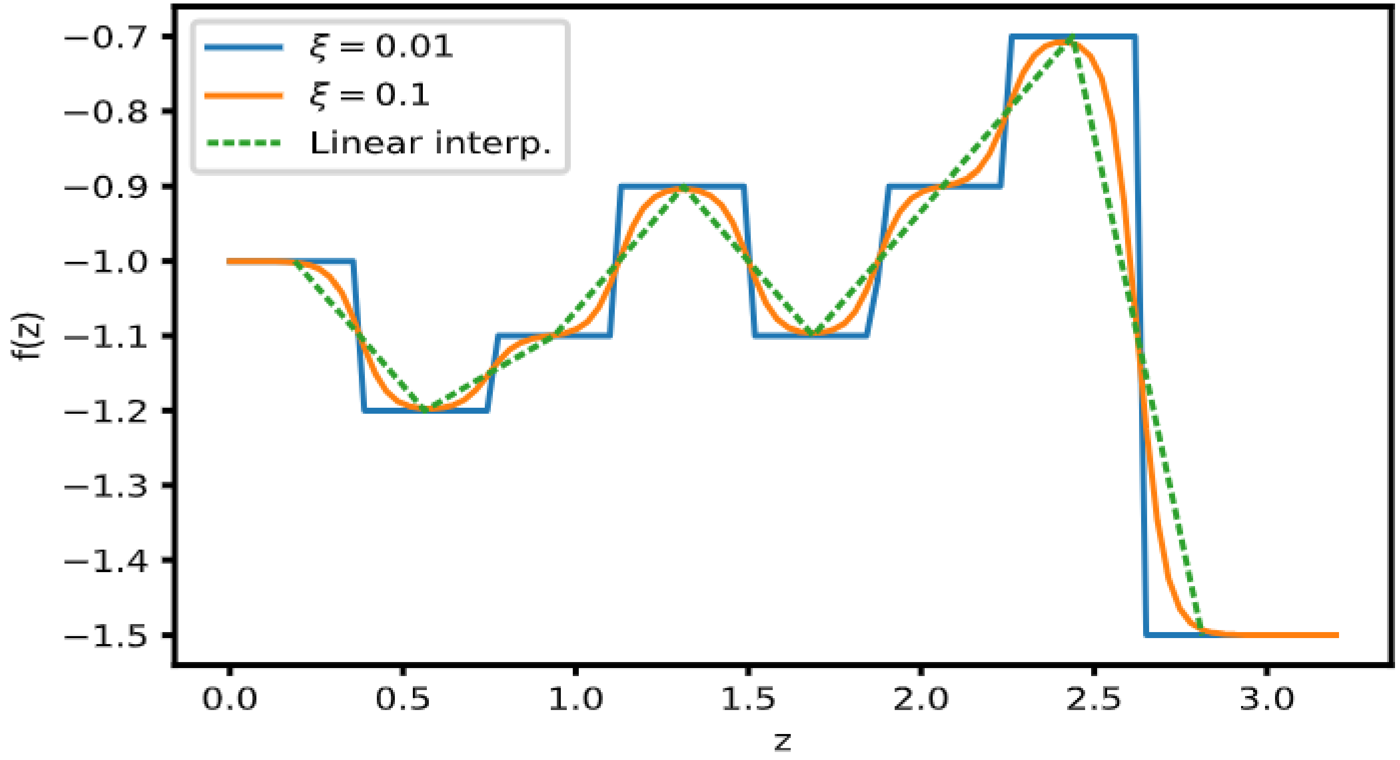
It uses step functions (bins) joined via hyperbolic tangents.

Its functional form is:

$$w(z) = w_1 + \sum_{i=1}^{N-1} \frac{w_{i+1} - w_i}{2} \left(1 + \tanh\left(\frac{z - z_i}{\xi}\right)\right)$$

The parameter ξ allows us to control the “smoothness” of the transition from one bin to another.

Using bins and nodes



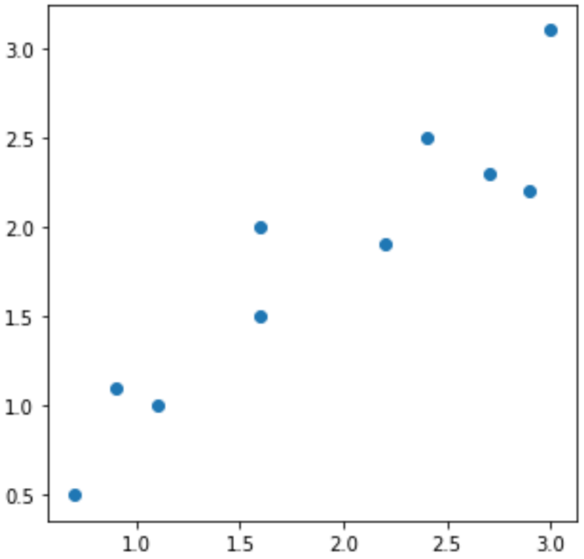
PCA

After conducting our reconstructions, a procedure called Principal Component Analysis (PCA) can be performed.

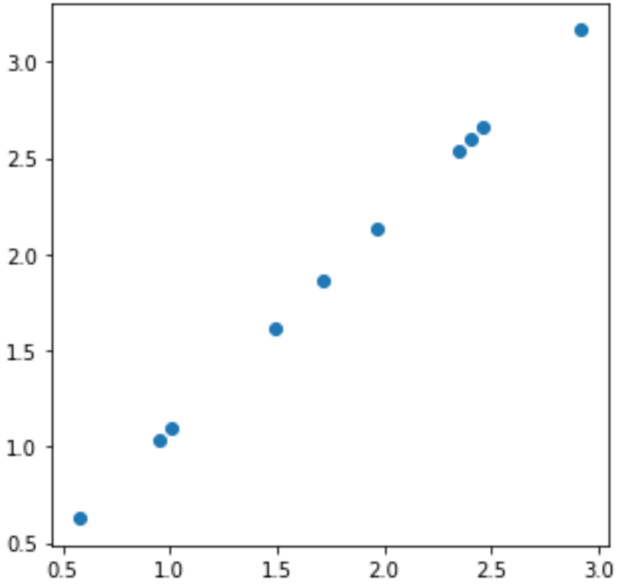
When it comes to data sets, PCA is used to eliminate 'noise,' extract 'hidden dynamics,' or even compress information by removing the least-contributing part to the total variance (e.g., in images).

We can obtain information about the constraint on our parameters, and therefore draw conclusions regarding the data used.

PCA



Eliminating 1 PC

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(876L, 1136L)
51
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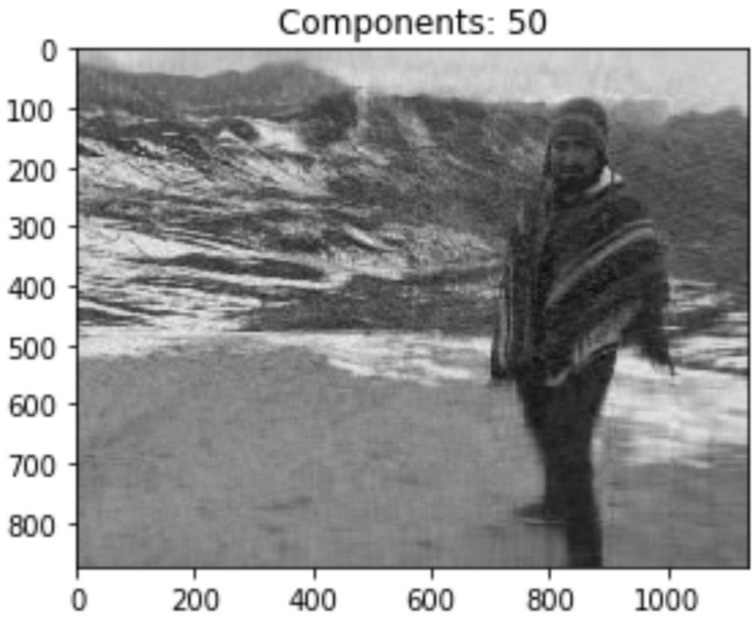
Eliminating 826 PCs



IDE model

$$\dot{\rho}_{\rm DM} + 3H(1 + w_{\rm eff,DM})\rho_{\rm DM} = 0$$

$$\dot{\rho}_{\rm DE} + 3H(1 + w_{\rm eff,DE})\rho_{\rm DE} = 0$$

$$w_{\rm eff,DM} = \frac{-Q}{3H\rho_{\rm DM}}, \quad w_{\rm eff,DE} = w_{\rm DE} + \frac{Q}{3H\rho_{\rm DE}}$$

$$\Pi_{\rm DM} = \frac{-Q}{3H\rho_{\rm c,0}} = -\Pi_{\rm DE}$$

$$\Rightarrow \left\{ \begin{aligned} \frac{\mathrm{d}(\rho_{\rm DM}/\rho_{\rm c,0})}{\mathrm{d}z} &= \frac{3}{1+z}\left(\frac{\rho_{\rm DM}}{\rho_{\rm c,0}} + \Pi_{\rm DM}\right), \\ \frac{\mathrm{d}(\rho_{\rm DE}/\rho_{\rm c,0})}{\mathrm{d}z} &= \frac{3}{1+z}\left[(1+w_{\rm DE})\frac{\rho_{\rm DE}}{\rho_{\rm c,0}} + \Pi_{\rm DE}\right] \end{aligned} \right.$$

$$\Downarrow$$

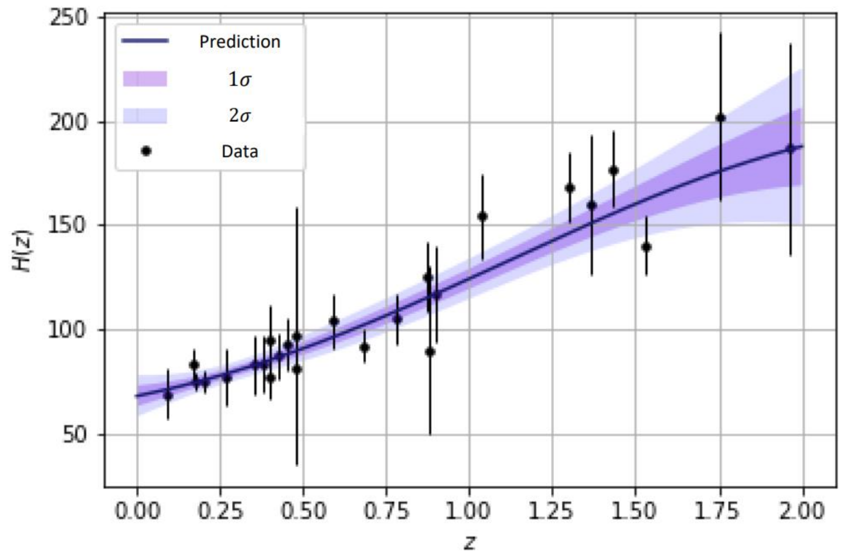
$$\frac{H^2(z)}{H_0^2} = \Omega_{b,0}(1+z)^3 + \frac{\rho_{\rm DM}(z)}{\rho_{\rm c,0}} + \frac{\rho_{\rm DE}(z)}{\rho_{\rm c,0}}$$

We also reconstructed the interaction kernel of an Interacting Dark Energy (IDE) model.

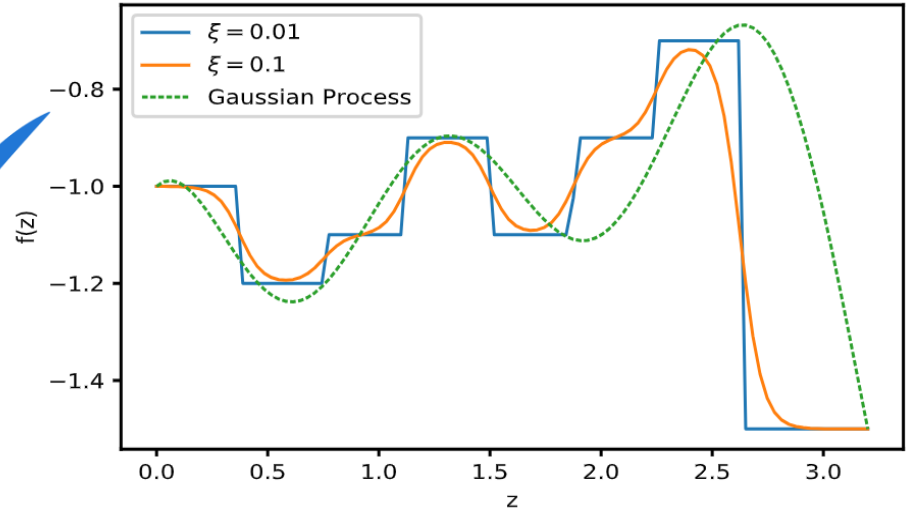
This type of models propose a phenomenological interaction between components of the Dark Sector.

Gaussian Process as an interpolation

For the IDE reconstruction we also used a Gaussian Process (GP) but in a different approach than the usual one.



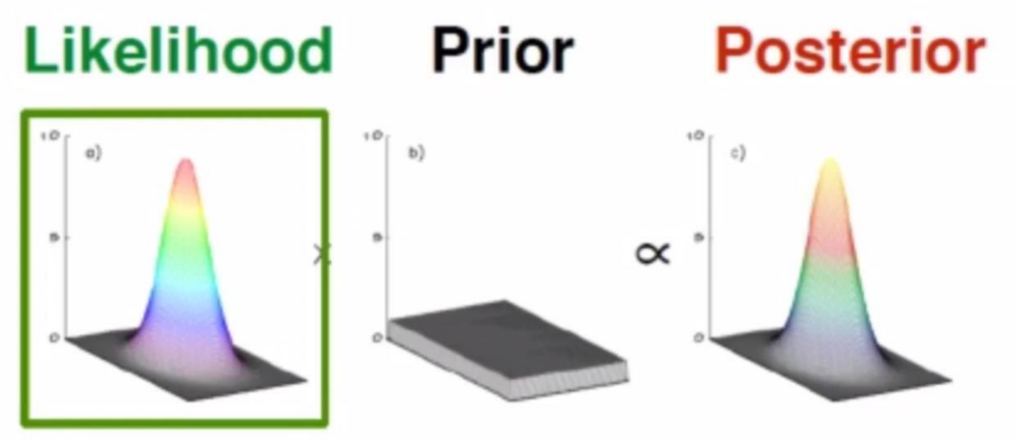
Our GP is used as an interpolation of nodes. This to perform model comparison with LCDM (and others).



Bayesian Statistics

To perform the parameter inference procedure we used Bayesian Statistics, which is a well-known statistical and numerical tool in Cosmology.

$$P(u, M|D) = \frac{\mathcal{L}(D|u, M)P(u, M)}{E(D|M)}$$



Cosmological parameter inference with Bayesian statistics

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[2] *Department of Astronomy and Texas Cosmology Center, University of Texas, Austin, TX, 78712-1083, U.S.A.*

[3] *Instituto de Ciencias Físicas, Universidad Nacional Autónoma de México, Apdo. Postal 48-3, 62251 Cuernavaca, Morelos, México.*

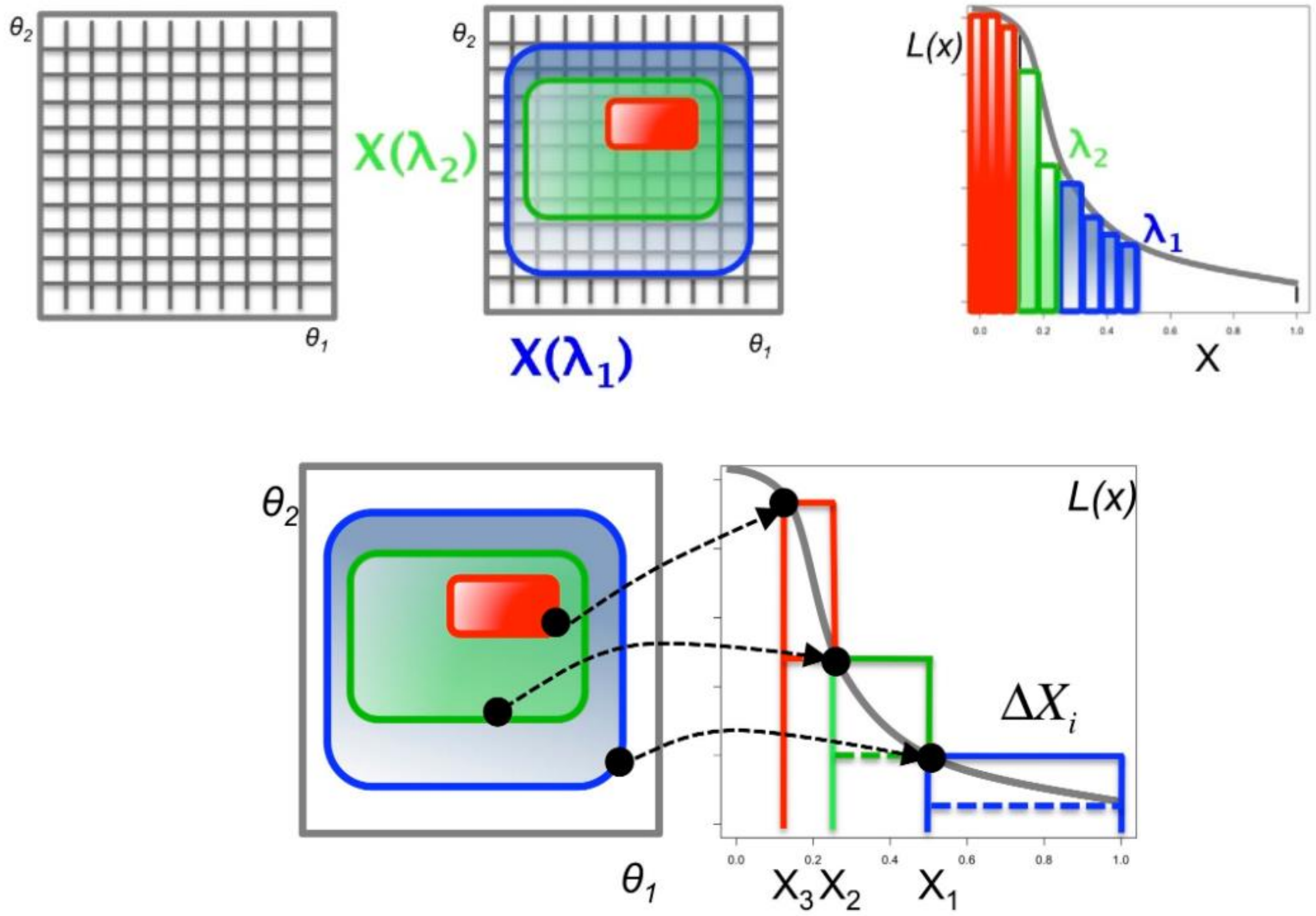
(Dated: March 28, 2019)

arXiv:1903.11127

Nested Sampling

Particularly we use the Nested-Sampling algorithm.

This approach to parameter inference provides us with the Bayesian Evidence, which tells us how good our model performs against LCDM taking into account the added “complexity” in our reconstructions.



Data and code

The datasets used for the reconstructions are:

(for a full explanation of the data used please refer to arXiv:2111.10457)

-Cosmic Chronometers.

-Pantheon's full catalogue of type 1a supernovae.

-SDSS BAO "low-redshift" data ($z<2$).

-Lyman-alpha "high-redshift" data ($z>2$).

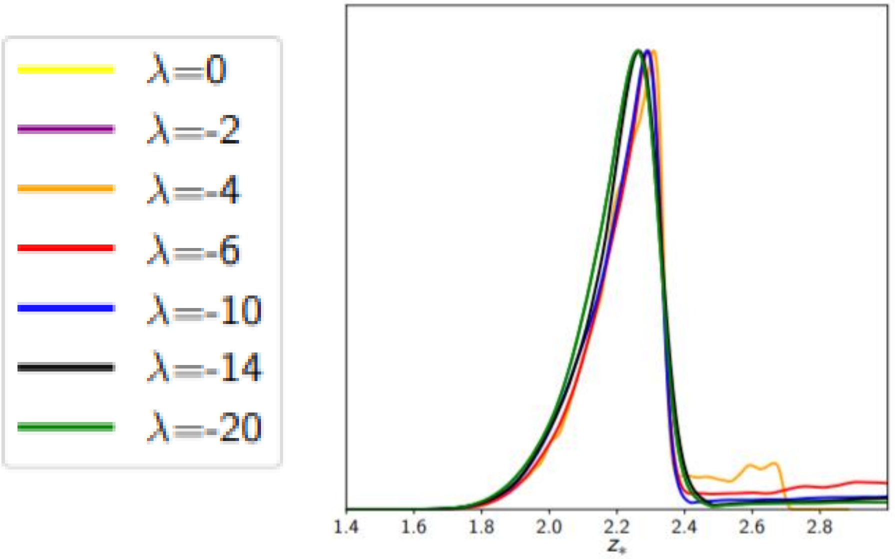
Every reconstruction was made using our own Bayesian Inference code

https://github.com/ja-vazquez/SimpleMC

RESULTS SO FAR…

Results (gDE)

$$a = a_* \equiv \mathrm{e}^{-\frac{1}{3}\frac{1}{\gamma(\lambda-1)}}$$



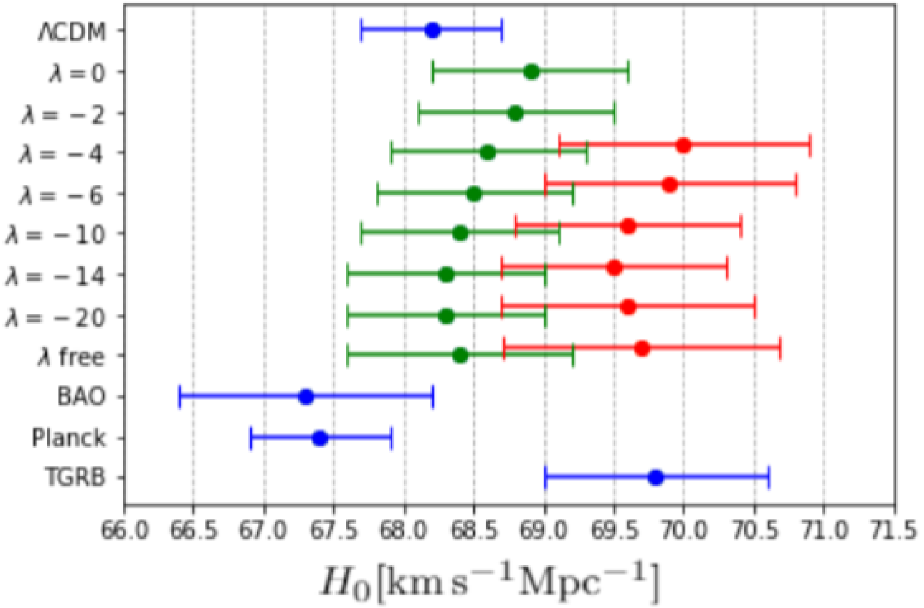
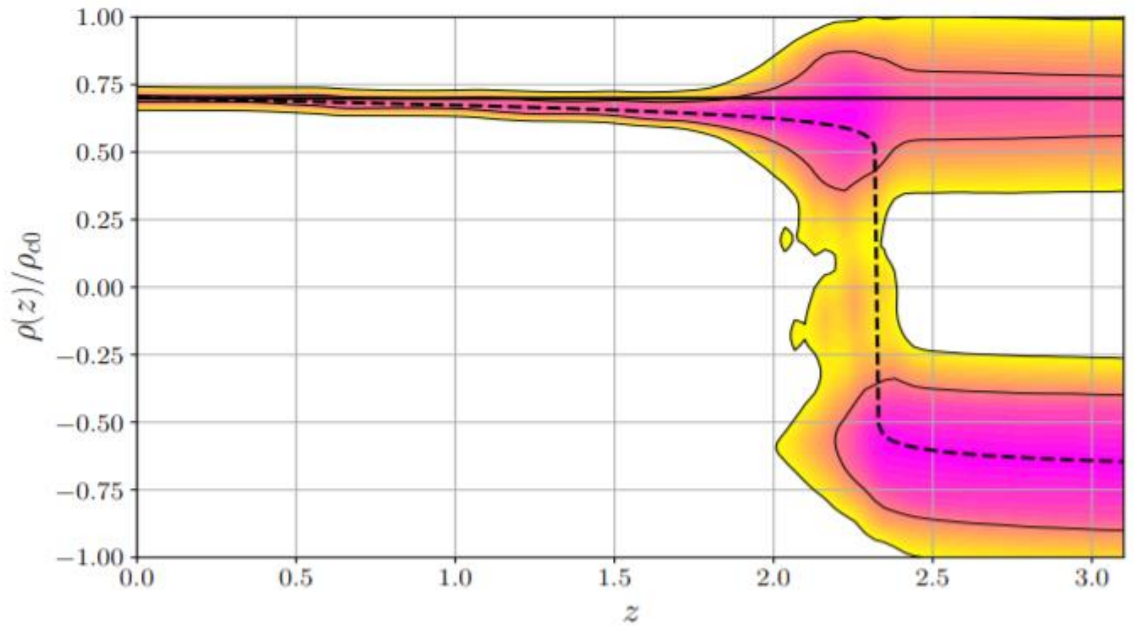
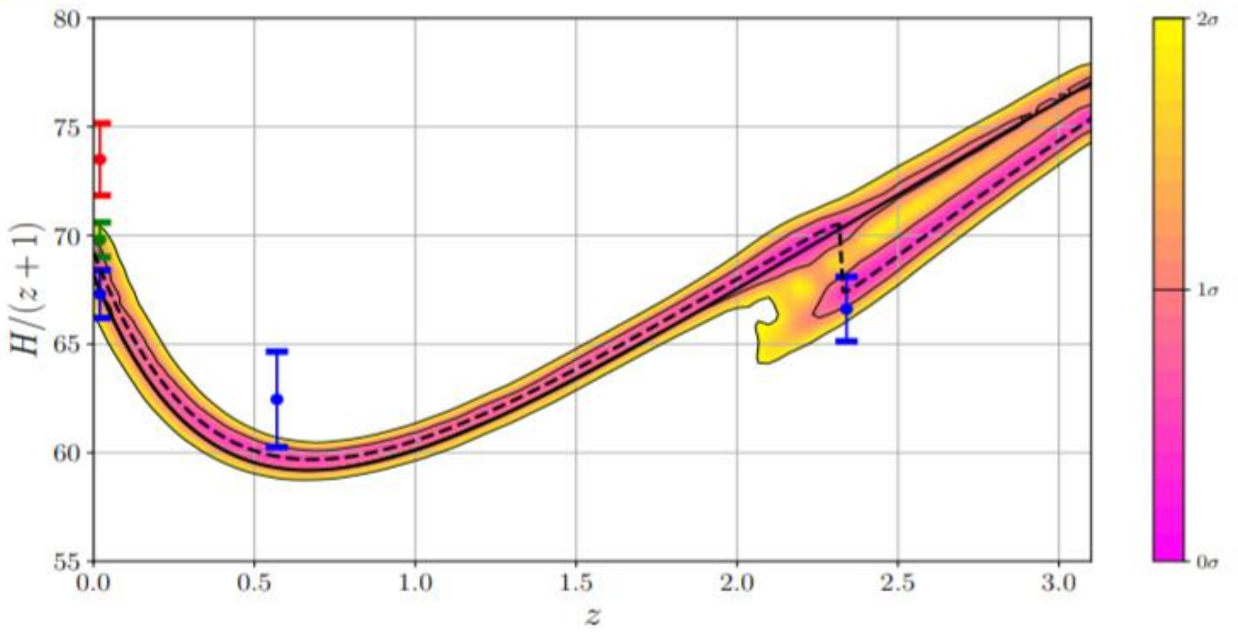


FIG. 4: Means values along with 1σ error bars from the 1D marginalised posterior distributions of $H_0[\mathrm{km\,s^{-1}Mpc^{-1}}]$. Green error bars are associated with the peak containing $\Psi \sim 0$ (ΛCDM), whereas red with the new peak stable at $\Psi \sim -0.86$.

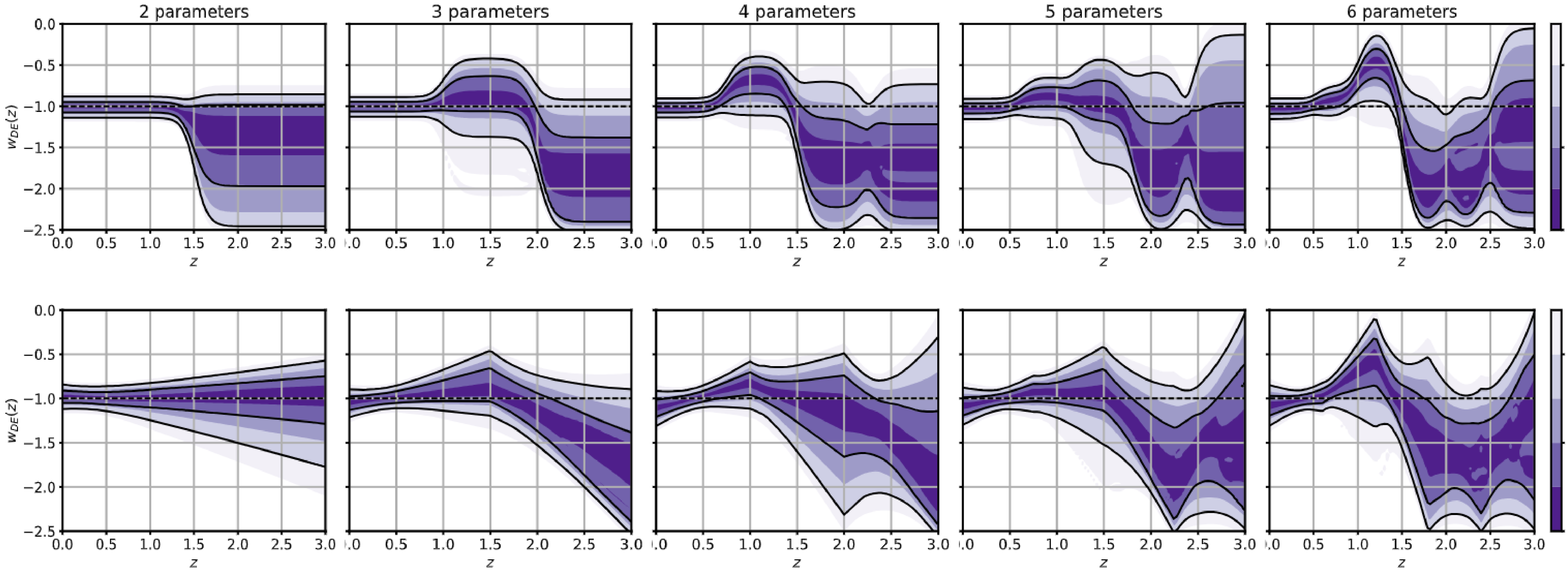
Graduated DE (gDE)

We found a sign change from positive to negative as the redshift increases. The switch occurs around z=2.34.

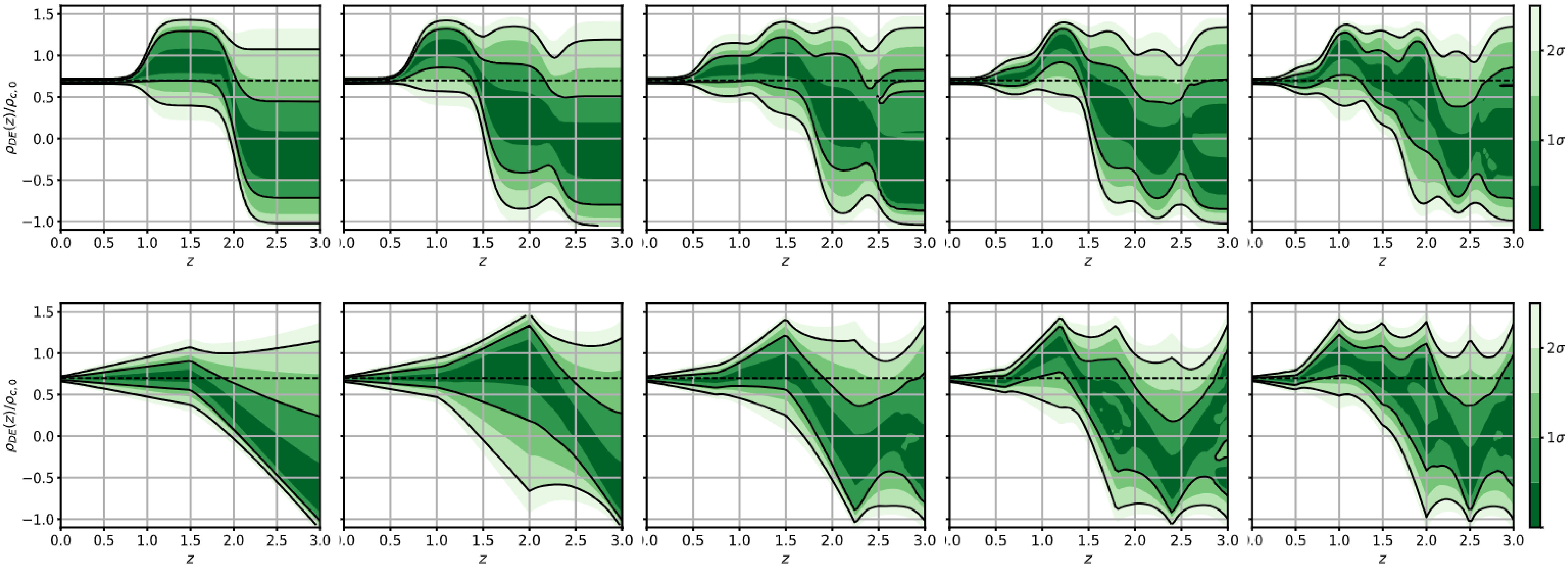




Results for the bins and nodes (EoS)

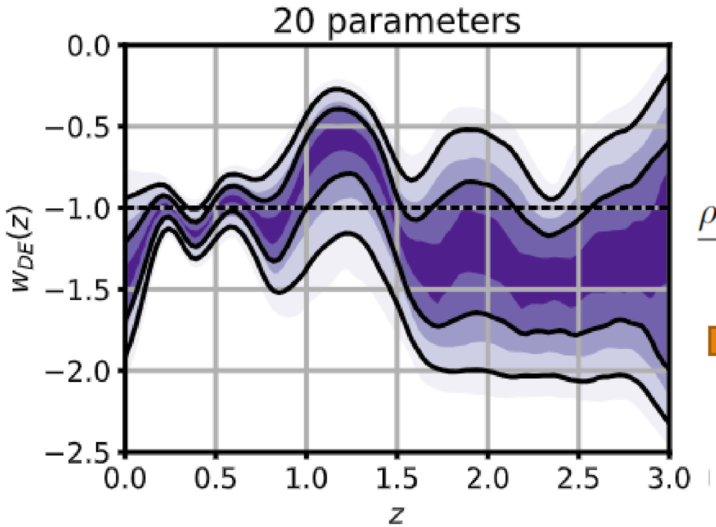


Results for the bins and nodes (density)

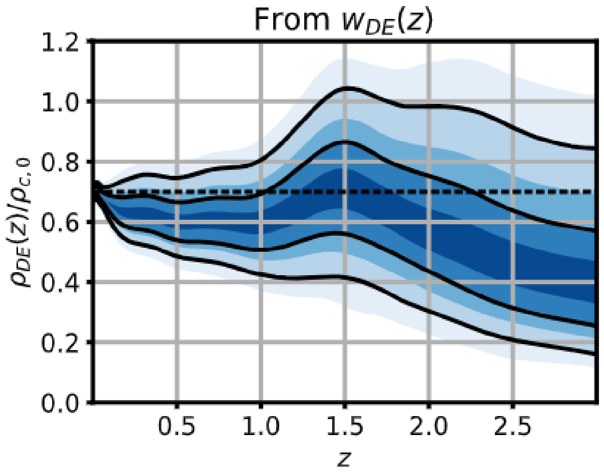


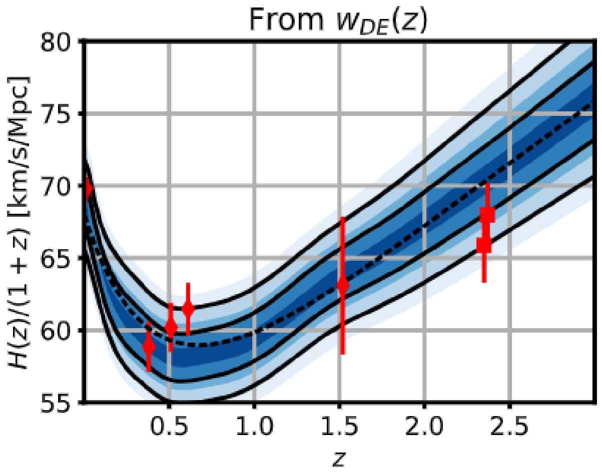
Results (20 bins)

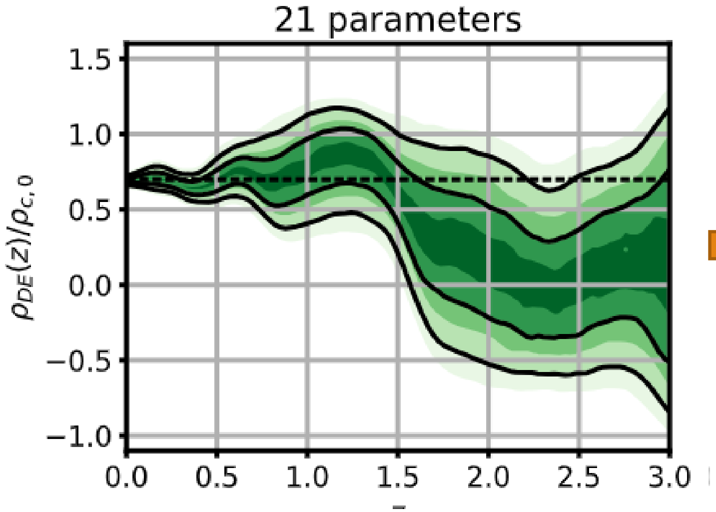
arXiv:2111.10457



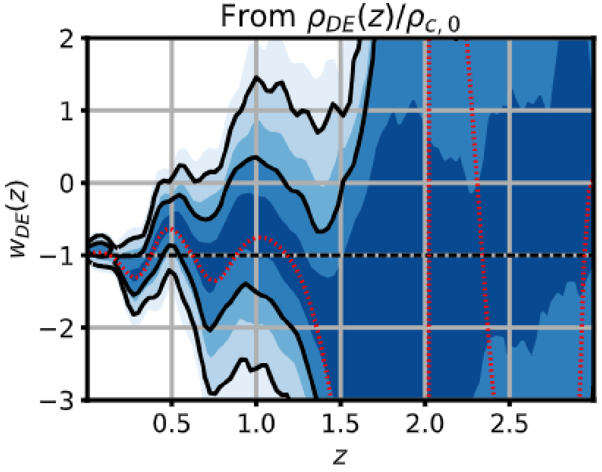
$$\frac{\rho_{\rm DE}(z)}{\rho_{c,0}} = \Omega_{\rm DE,0}\, e^{3\int_0^z \frac{dz'}{1+z'}(1+w_{\rm DE}(z'))}$$

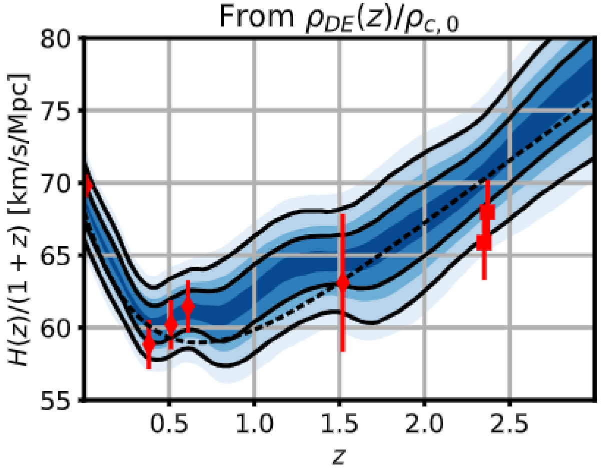






$$w_{\rm DE}(z) = -1 + \frac{1}{3}\frac{d\ln\rho_{\rm DE}(z)}{d\ln(1+z)}$$





Results (PCA)

The transitions occur after z=2.2

In this redshift the only available data is the Lyman-alpha BAO.

When performing PCA and eliminating the noisy part of our reconstruction we observe an "overestimation" of the information in this region.

We can conclude that Lyman-alpha data does not constrain our parameters.

This implies heavy systematic errors in the data used.

It is possible that most of the behavior found in this region is only due to noisy data.

We need more data in this region to discern real responses from noise.

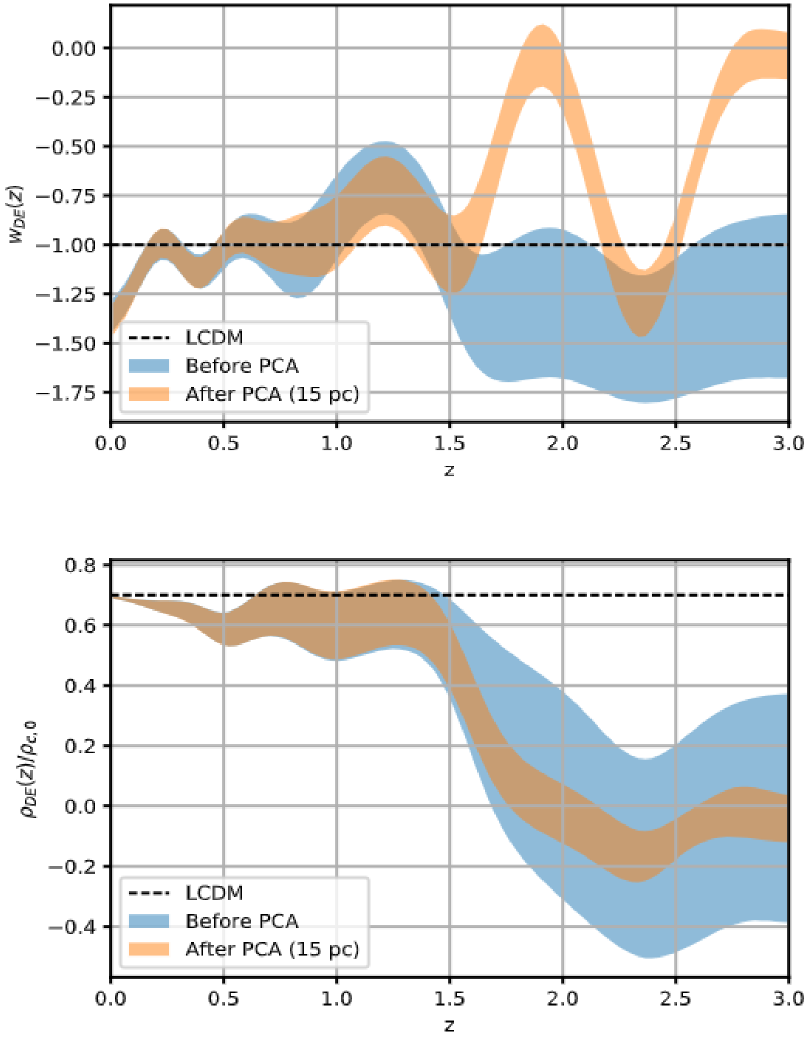


FIG. 7: Applying the principal component technique to our reconstructions of the EoS and the energy density with 20 bins. By eliminating 5 PCs (which add up to about 5% of the total variance for each reconstruction) we obtain the orange figures with slightly overestimated errors localized in $z > 1.5$.

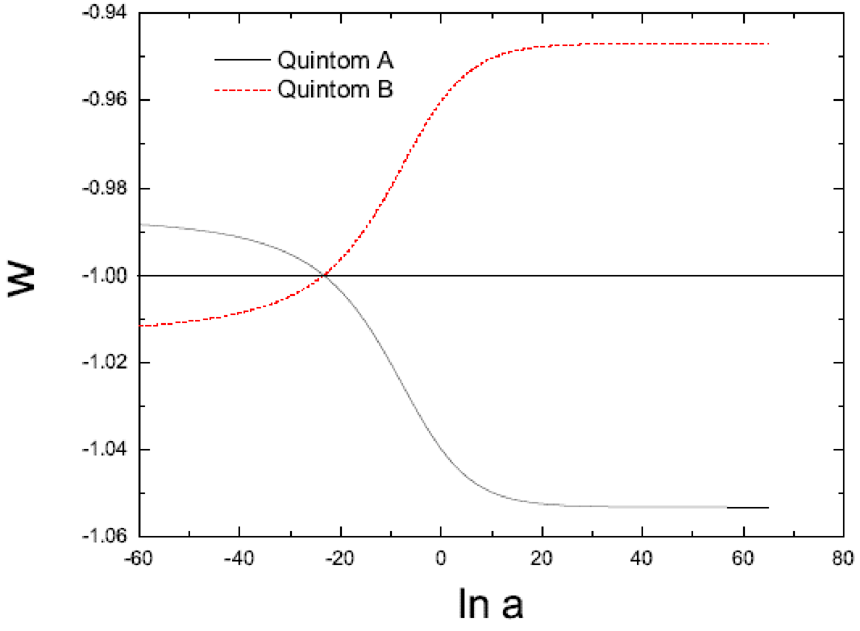
Results

Every reconstruction presents dynamic behavior, sometimes being preferred over LCDM by 2-sigma.

Also some type of oscillations are observed.

The EoS reconstructions present a possible crossing of the Phantom Divide Line in a manner similar to a Quintom Scalar Field. This transition is impossible for “simple” scalar fields.

The effective densities present possible transitions from positive to null or negative values, providing further motivation to study models (like gDE) that allow such a feature.

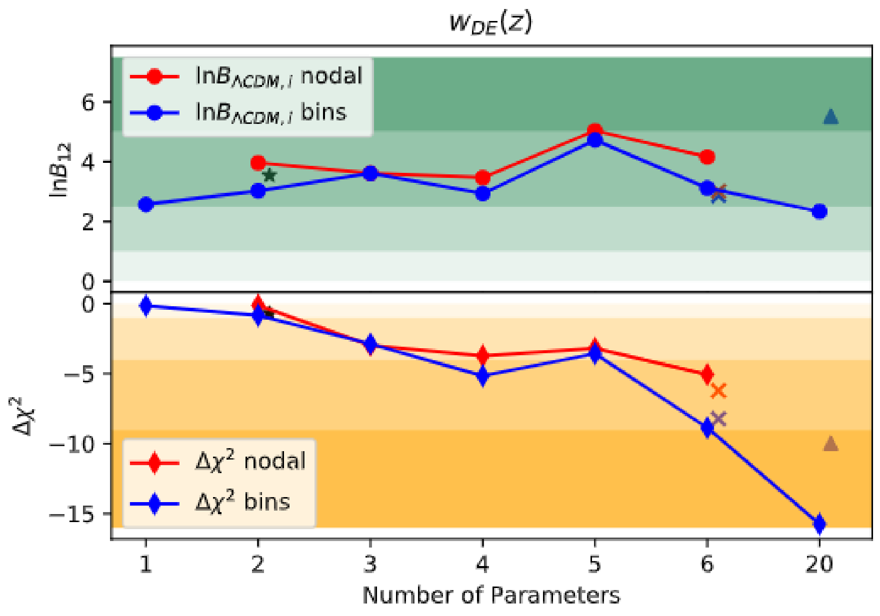


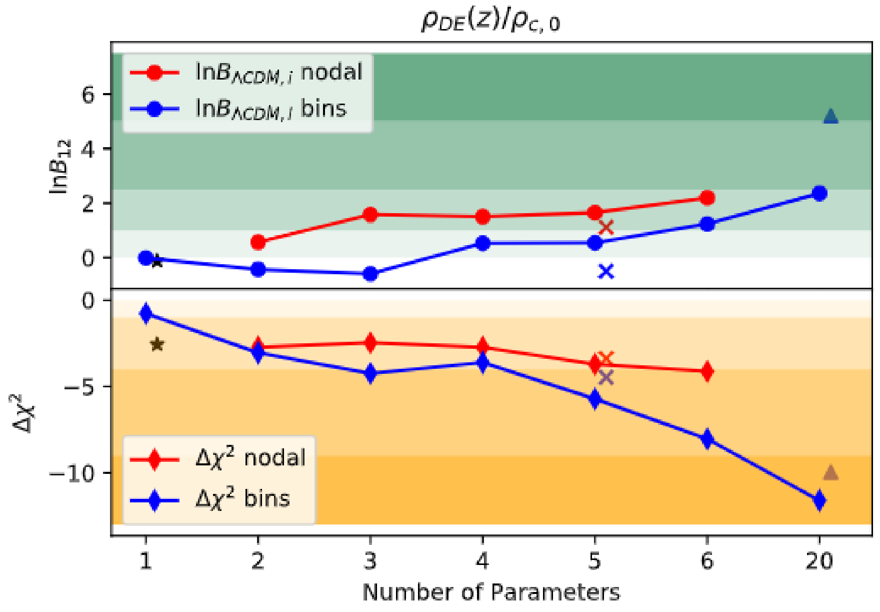
Results

Every reconstruction is worse than the LCDM model, although not by a long margin.

Even though our reconstructions present evidence against them when compared to LCDM, we see a clear improvement in the fitness to the data.

Finally, two of our reconstructions present a positive Bayes Factor, but the evidence in their favor is inconclusive at best.



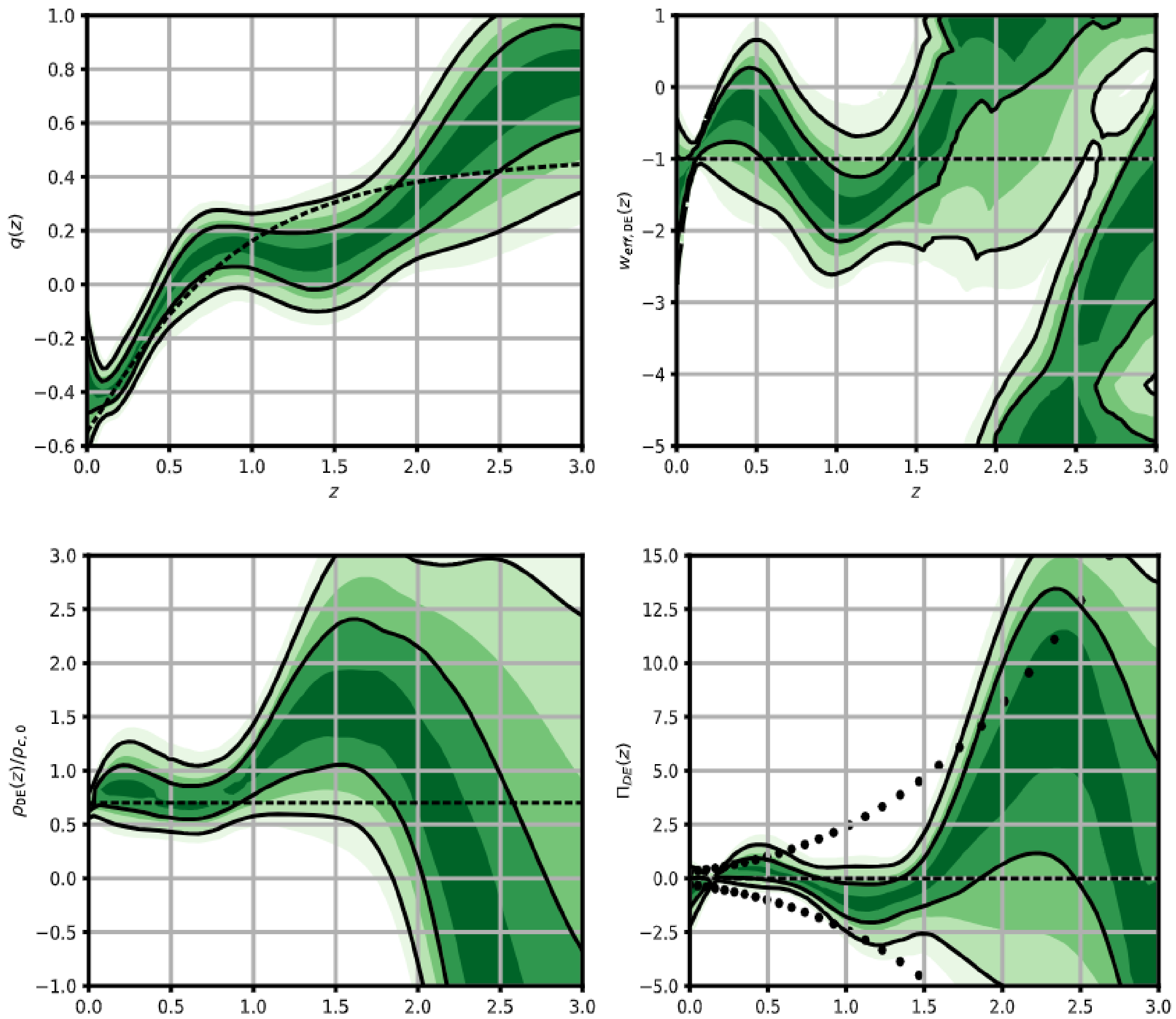


Results of IDE reconstruction

arxiv:2305.16290

Even though we are using an interactive DE we find a similar behavior: **negative density for DE**.

Also we find a possible oscillation in the interaction kernel, which implies several changes in the flux of energy from DE to DM.



Conclusions

-All reconstructions exhibit dependence on z (redshift).

-The reconstructions of the EoS show behavior similar to a Quintom-like dark energy.

-The reconstructions of the density parameter exhibit dynamic behavior. Its density may undergo a sign change at z>2 or a significant decrease.

-Bayesian evidence favors LCDM (Lambda Cold Dark Matter).

-The analysis conducted with PCA indicates that the data at z>2 do not tightly constrain our parameters, suggesting that the observed dynamics could be due to systematic errors (with the Lyman-alpha data in this region).

-When using an interacting Dark Sector we recover similar features such as oscillations and a possible sing-switching density for DE.

Future projects…

-Propose extensions to LCDM taking into account our findings so far.

-Study how a sign-switching density could alleviate the tensions of LCDM.

-Repeat the reconstructions when more data becomes available.

-Implement early-time data, not only background, such as Planck or ACT.

Thank you for your attention!