# Rotating Scalar Field & Formation of Bose Stars

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# Motivation

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- The formation of Bose Stars in the kinetic regime, where $mvR \gg 1$ and $mv^2\tau_{gr} \gg 1$, has been studied by Levkov et al. (PRL, 2018), Also by Chen et. al.(PRD, 2021) for ULDM. However, the initial collapsing cloud did not have any intrinsic angular momentum.
- Since collapsing clouds due to the gravitational shearing, tidal torque,etc can have angular-momentum, we believe that it is important to study the star formation in presence of angular momentum.
- Our aim:
    - To analyse the effect of angular momentum on Bose Star formation in presence of self-interaction.
    - To study the properties of (formed) Bose Stars.

# Introduction

# Basic Equations

- In the Non-relativistic limit and for high mean occupation number, the system can be represented by a classical scalar field, $\psi(\boldsymbol{r}, t)$, whose time evolution can be described by the Gross-Pitaevskii-Poisson(GPP) equations,

$$i\,\frac{\partial}{\partial \tilde{t}}\,\tilde{\psi} = -\frac{1}{2}\,\tilde{\nabla}^2\,\tilde{\psi} + \tilde{\Phi}\,\tilde{\psi} + \tilde{g}\,|\tilde{\psi}|^2\,\tilde{\psi}, \tag{1}$$

$$\tilde{\nabla}^2\,\tilde{\Phi} = |\tilde{\psi}|^2 - \tilde{n}, \tag{2}$$

- The quantities are rescaled as $r = (1/mv_0)\,\tilde{r}$, $t = (1/m\,v_0^2)\,\tilde{t}$, $\Phi = v_0^2\,\tilde{\Phi}$, $\psi = \left(v_0^2\sqrt{m/4\pi G}\right)\tilde{\psi}$ and $g = (4\pi\,G/v_0^2)\,\tilde{g}$. Here, $v_0$ is the reference velocity and $m$ is the mass of the DM particle.

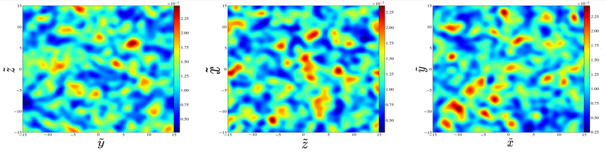
# Initial Conditions

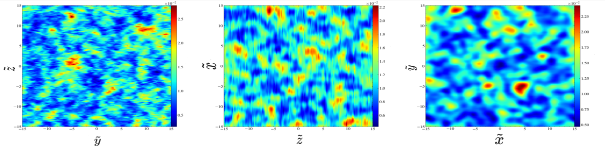
We construct initial wave function $\tilde{\psi}(\tilde{\boldsymbol{r}}, \tilde{t} = 0)$ for GPP equations as follows

- First consider a function, $\phi = \phi_1 \times \phi_2$, where, $\phi_1 \propto e^{-\tilde{\boldsymbol{r}}^2/2}$ and $\phi_2 \propto e^{i\,l\,\tan^{-1}(\tilde{y}/\tilde{x})}$.
- Next, $\phi$ is transformed into momentum space and multiplied with random phase $e^{i\alpha_p}$, where $\alpha_p$ is a random number between 0 to $2\pi$ to obtain $\phi_p$.
- Inverse Fourier transforming $\phi_p$ and normalizing will give the initial wave function $\tilde{\psi}(\tilde{\boldsymbol{r}}, \tilde{t} = 0)$.

Here, we note that our prescription is equivalent to that of Levkov et. al. 2018, when $\phi_2 = 1$. We have also tried other forms of $\phi_2 = 1$ to introduce angular momentum in the system. Changing the angular momentum prescription does not affect the final results.

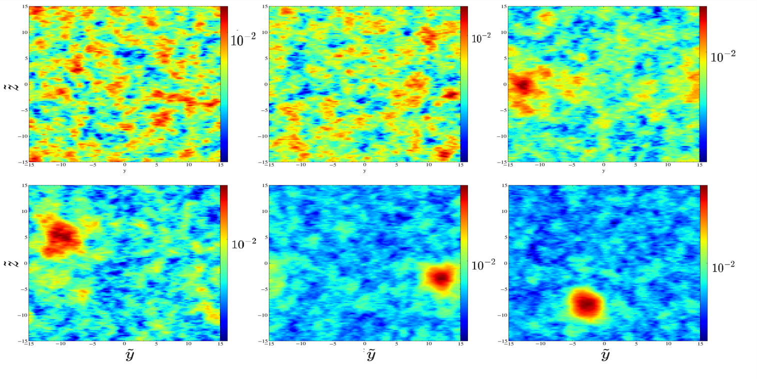
# Initial Conditions





The initial snapshots of density profiles of $|\tilde{\psi}|^2$ at the initial time $(\tilde{t} = 0)$ in $\tilde{y}\tilde{z}$, $\tilde{z}\tilde{x}$ and $\tilde{x}\tilde{y}$ planes. The top panel from left to right describes the initial cloud density for $\tilde{\mathcal{L}}_{\text{tot}} = 0$. The bottom panel respectively correspond to $\tilde{\mathcal{L}}_{\text{tot}} = 5$ case.

# Evolution of Rotating Scalar field and formation of Bose Star




Snapshots of a cloud's $|\tilde{\psi}|^2$ at different times in the $\tilde{z}\tilde{y}$-plane are shown. The initial cloud has a total-angular momentum $\tilde{\mathcal{L}}_{\rm tot} = 5$ and no self-interaction $\tilde{g} = 0$. The gravitational condensation time for this case is around 15600. The upper panel plots correspond to times $\tilde{t} = 0, , 0.5\tau_{gr}, , 1\tau_{gr}$, while the bottom plots correspond to $1.15\tilde{\tau}gr, , 1.15\tilde{\tau}gr, 1.8\tilde{\tau}_{gr}$.

# Upper Bound on Coupling Constant

- The star formation process in the presence of self-gravity and self-coupling may lead to an upper bound on $\tilde{g}$ and the size of the star $\tilde{L}_B$. Replacing all the spatial derivatives in GPP equations with $1/\tilde{L}$, and for stationary state $i\frac{\partial\tilde{\psi}}{\partial t} \sim \omega\tilde{\psi}$. we get,

$$\omega\tilde{\psi} = \left[\frac{1}{2\tilde{L}_B^4} - \left(|\tilde{\psi}|^2 - \tilde{n}\right) + \tilde{g}\frac{|\tilde{\psi}|^2}{\tilde{L}_B^2}\right]\psi\tilde{L}_B^2. \quad (3)$$

- Formation of a gravitationally bound structure requires that the following condition need to be satisfied,

$$|\tilde{g}\tilde{L}_B^{-2}| < 1 \quad (4)$$

# Estimation on size of Bound state

- Estimate for the size of the star can be obtained from Eqn (3),

$$\tilde{L}_B^2 = -\frac{\omega - \tilde{g}|\tilde{\psi}|^2}{2|\tilde{\psi}|^2} + \sqrt{\frac{(\omega - \tilde{g}|\tilde{\psi}|^2)^2 + 2|\tilde{\psi}|^2}{4|\tilde{\psi}|^4}}, \tag{5}$$

- This implies that the star formed with attractive self-interaction ( $\tilde{g} < 0$ & $\tilde{\omega} < 0$) are more compact in comparison with no self-interaction case ( $\tilde{g} = 0$ & $\tilde{\omega} < 0$) and when self-interaction is repulsive ( $\tilde{g} > 0$ & $\tilde{\omega} < 0$), stars are less compact in comparision with no self-intraction case.

# Methodology

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- The GPP equations were solved using a sixth-order pseudo-spectral method by modifying the publicly available AxioNyx code.
- We have plotted vorticity magnitude at various times to see the evolution of angular momentum in the system.
- For $\tilde{g} = 0$ and $l = 0$, our results our results are consistent with Levkov et. al (PRL, 2018),
- To ensure that the star formation process have begun, we follow two checks:
- Tracking the evolution of maximum amplitude/density of the system, and after a certain time, $\tau_{gr}$, the density of the system starts rising.
- The second test is about studying the power spectrum $F(t, \omega)$ of $\psi(x, t)$ defined as a Fourier image of the correlator,

$$F(\omega, t) = \int \frac{dt_1}{2\pi} d^3\boldsymbol{x}\, \psi^*(t, \boldsymbol{x})\, \psi(t + t_1, \boldsymbol{x})\, e^{i\omega t_1 - t_1^2/\tau_1^2}. \qquad (6)$$

Around the time when the gravitational condensation happens, $F(\omega, t)$ starts developing a peak at $\omega_s < 0$ which indicates the formation of a gravitationally bound state.

# Results

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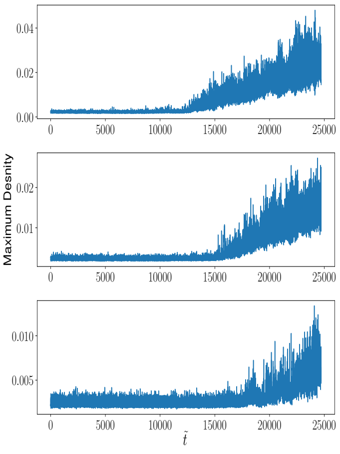
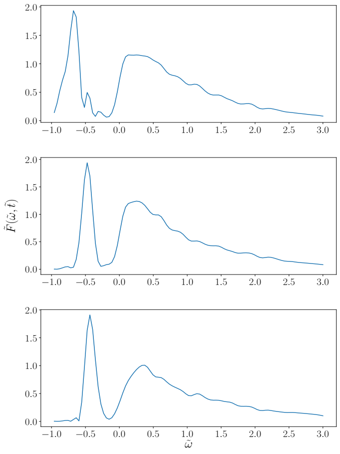
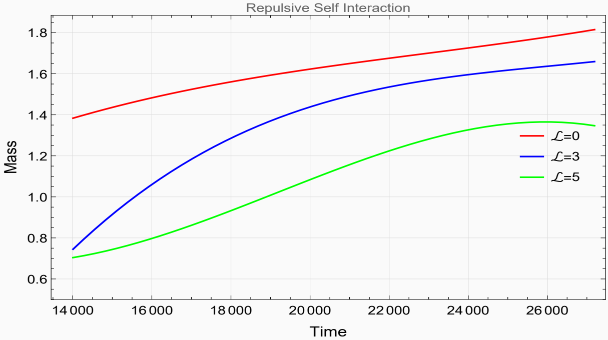


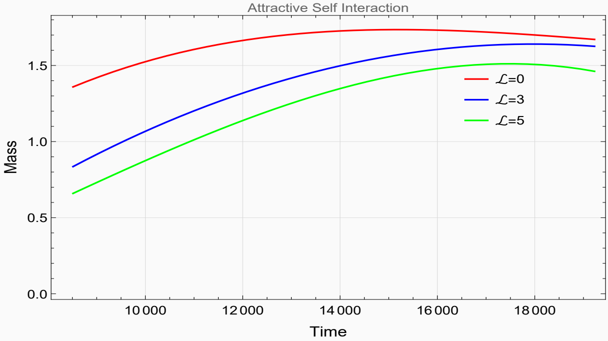

(left)Evolution of Maximum density for repulsive self interaction ($\tilde{g} = 4.54$)
(right) $F(\omega, \tau)$ for repulsive self interaction ($\tilde{g} = 4.54$)

# Results



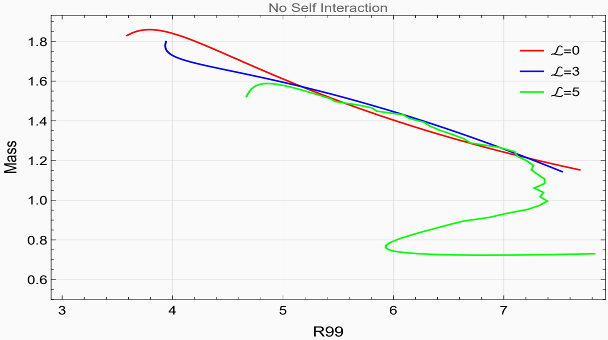

Evolution of Mass with repulsive self interaction for different values of angular momentum

# Results



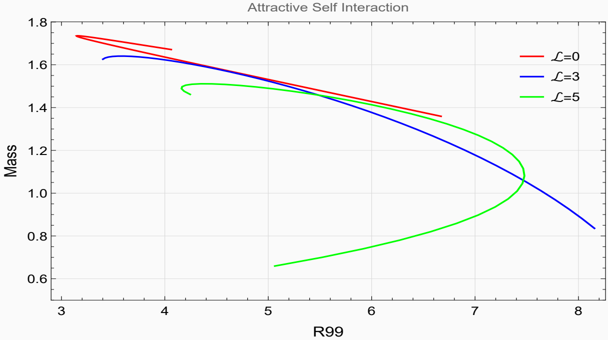

Evolution of Mass with attractive self interaction for different values of angular momentum

# Results




Mass-radius relation with no self interaction for different values of angular momentum

# Results




Mass-radius relation with attractive self interaction for different values of angular momentum

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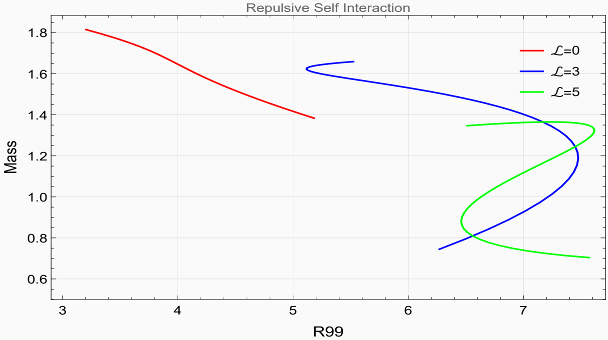


Mass-radius relation with repulsive self interaction for different values of angular momentum

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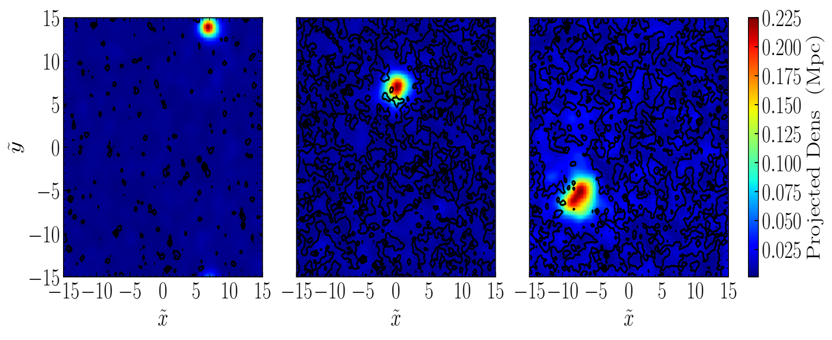
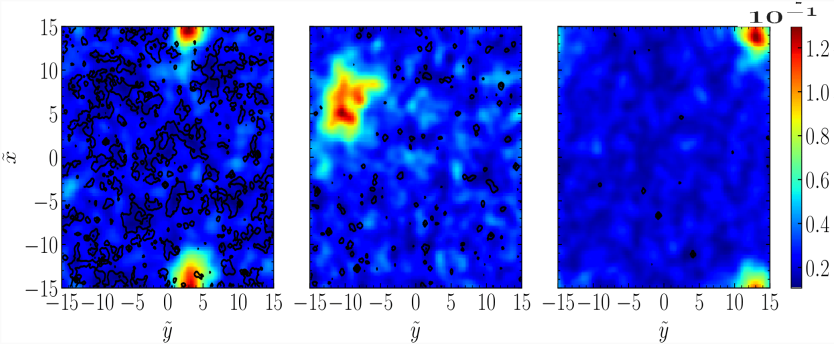



Vorticity magnitude plotted over Density for attractive self interaction.

# Results




Evolution of Vorticity magnitude plotted over Density for repulsive self interaction with angular moemntum =5.0

# Summary

- We have put a upper bound on the value of self-interaction coefficient, which is also consistent with our numerical results.
- The introduction of angular momentum in initial cloud, affects the condensation time, compactness, mass,etc of the formed stars.
- For the formation of rotating bose stars, one must have repulsive self interaction in the initial cloud.