# Quantum field theory in curved spacetime and the CMB hemispherical power asymmetry

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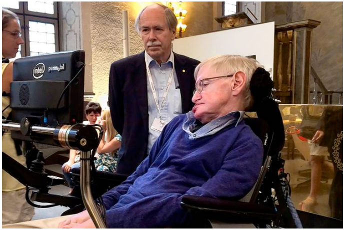
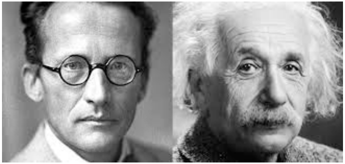
# Quantum Gravity and the three big questions

- What is the modification of GR towards high energy scales to avoid bigbang and black hole singularities?
- Quantum gravity, Unification! (Planck scale physics)
- Quantum field theory in curved spacetime (relevant at way below Planck scales).

# Open questions

- The heart of SQFT is S-matrix Unitarity and CPT invariance. In other words, quantum theory is governed by **discrete** spacetime transformations.
- The question of CPT invariance has worried generations of physicists. Also the question about particle description in the curved space-time.





# Time reversal in quantum theory

Thanks to the recent observations on quantum theory made by John Donoghue and Gabriele Menezes (2020,2021,2022)

*Factors of $i = \sqrt{-1}$, in quantum theory, play an important role. If we change $i \to -i$ everywhere in quantum theory, we would change the conventional arrow of causality without changing the physics.*

If we change $i \to -i$ everywhere in the standard quantum field theory we change the conventional notion of time from $-\infty < t < \infty$ to the unconventional one $\infty < t < -\infty$. Thus, the conventional and uncoventional Schrödinger equations are

$$i\frac{\partial|\psi_C\rangle}{\partial t} = \hat{H}|\psi_C\rangle, \quad -i\frac{\partial|\psi_U\rangle}{\partial t} = \hat{H}|\psi_U\rangle$$

**GR+QM? = QFT in curved spacetime $\implies$ Quantum Gravity**

which goes in parallel with

**Special Relativity+QM = QFT in Minkowski $\implies$ Standard Model**

In the standard QFT time is a parameter and its status should remain the same in quantum field theory in curved spacetime (QFTCS) as well.

# Merging of special relativity and quantum mechanics

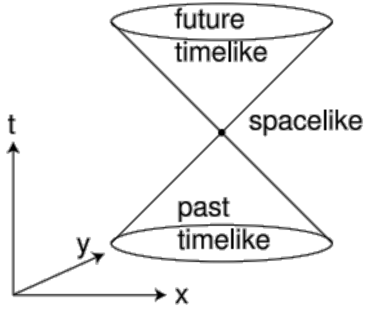



Figure: Source: Internet

Vanishing of commutator for spacelike distances

$$[\mathcal{O}(x), \mathcal{O}(y)] = 0, \quad (x-y)^2 > 0$$

Any new conditions for curved spacetime?

# Standard dS QFT

De Sitter spacetime in flat FLRW

$$ds^2 = -dt^2 + a(t)^2 d\mathbf{x}^2 \quad = \frac{1}{H^2\tau^2}\left(-d\tau^2 + d\mathbf{x}^2\right) .$$

We assume $H > 0$ and $\tau < 0$.

$$S_{KG} = \int \sqrt{-\bar{g}}\left[\phi\Box\phi\right]$$

Rescaling $\phi \to a\phi$ we get

$$S_{KG} = \int d\tau d^3x\phi\left[\partial_\tau^2 + \left(k^2 - \frac{2}{\tau^2}\right)\right]\phi .$$

Here $\tau = -\frac{1}{aH}$. In the limit $k^2 \gg a^2H^2$ we have Minkowski.

# $\mathcal{PT}$ symmetry in de Sitter spacetime

$$ds^2 = -dt^2 + a(t)^2 d\mathbf{x}^2 \quad = \frac{1}{H^2\tau^2}\left(-d\tau^2 + d\mathbf{x}^2\right) .$$

where $d\tau = \frac{dt}{a}$,

$$a(t) = e^{Ht}, \quad r_H = \left|\frac{1}{aH}\right|, \quad R = 12H^2 = \left(\frac{1}{a}\frac{da}{dt}\right)^2 = \text{const} .$$

The metric is $\mathcal{PT}$ symmetric.

$$t : -\infty \to \infty, H > 0 \implies \tau < 0, \quad t : \infty \to -\infty, H < 0 \implies \tau > 0.$$

Expanding Universe means $\tau : \pm\infty \to 0$.

# Direct-sum quantization in dS

- Let us take a massless field in de Sitter space. We split the field operator into two components

$$\hat{\phi}(\tau, \mathbf{x}) = \frac{1}{\sqrt{2}}\hat{\varphi}_{\rm I}(\tau, \mathbf{x}) \oplus \frac{1}{\sqrt{2}}\hat{\varphi}_{\rm II}(-\tau, -\mathbf{x}) = \frac{1}{\sqrt{2}}\begin{pmatrix} \hat{\varphi}_{\rm I}(\tau, \mathbf{x}) & 0 \\ 0 & \hat{\varphi}_{\rm II}(-\tau, -\mathbf{x}) \end{pmatrix}$$

corresponding to two vacua which form total direct-sum vacuum

$$|0\rangle = |0\rangle_{\rm I} \oplus |0\rangle_{\rm II} = \begin{pmatrix} |0\rangle_{\rm I} \\ |0\rangle_{\rm II} \end{pmatrix}$$

$$a_{\mathbf{k}}|0\rangle_{\rm I} = 0, \quad b_{\mathbf{k}}|0\rangle_{\rm II} = 0, \quad [\hat{\varphi}_{\rm I}(\tau, \mathbf{x}), \hat{\varphi}_{\rm II}(-\tau, -\mathbf{x})] = 0\,.$$

Quantum mechanically $\hat{\varphi}_{\rm I}(\tau, \mathbf{x})|0\rangle_{\rm I}$ is the postive energy component that propagate forward in time at $\mathbf{x}$ while $\hat{\varphi}_{\rm II}(-\tau, -\mathbf{x})|0\rangle_{\rm II}$ is the postive energy component that propagate backward in time at -**x**.

This is similar to what are known as super selection rules in AQFT proposed by Wick, Wightman and Wigner (1952).

# Direct-sum quantum field theory (DQFT)

$$\hat{\phi}_I(x) = \int \frac{d^3\vec{k}}{(2\pi)^{3/2}} \left[ \hat{a}_{\vec{k}} e^{ik\cdot x} + \hat{a}^{\dagger}_{\vec{k}} e^{-ik\cdot x} \right]$$

$$\hat{\phi}_{II}(-x) = \int \frac{d^3\vec{k}}{(2\pi)^{3/2}} \left[ \hat{b}_{\vec{k}} e^{-ik\cdot x} + \hat{a}^{\dagger}_{\vec{k}} e^{ik\cdot x} \right]$$

Now we define two vacuum

$$a_{\vec{k}}|0\rangle_I = 0, \quad b_{\vec{k}}|0\rangle_{II} = 0\,.$$

The total vacuum of Minkowski space now is

$$|0\rangle = |0\rangle_I \oplus |0\rangle_{II}\,.$$

And correspondingly the field operators are direct sum as well

$$\phi(x) = \frac{\hat{\phi}_I(x) \oplus \hat{\phi}_{II}(-x)}{\sqrt{2}}\,.$$

# Direct-sum quantum mechanics

*We divide position space into two parts, I and II, based on the parity operation $\mathcal{P}$. Subsequently, we assign time evolution, for states in spatial regions I and II. Finally we construct a single particle quantum mechanical state, in the whole position space, using the direct-sum of states in region I and II. Thus, the total Hilbert space $\mathcal{H}$ of single particle state becomes the direct-sum of the Hilbert spaces corresponding to the region I and II, respectively.*

Direct-sum Schrödinger equation

$$i\frac{\partial}{\partial t}\begin{pmatrix} |\psi_I\rangle \\ |\psi_{II}\rangle \end{pmatrix} = \begin{pmatrix} \hat{H} & 0 \\ 0 & -\hat{H} \end{pmatrix}\begin{pmatrix} |\psi_I\rangle \\ |\psi_{II}\rangle \end{pmatrix},$$

Direct-sum means

$$\hat{\mathcal{O}} = \hat{\mathcal{O}}_I \oplus \hat{\mathcal{O}}_{II} = \begin{pmatrix} \hat{\mathcal{O}}_I & 0 \\ 0 & \hat{\mathcal{O}}_{II} \end{pmatrix}, \quad |\psi\rangle = \frac{|\psi_I\rangle \oplus |\psi_{II}\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}}\begin{pmatrix} |\psi_I\rangle \\ |\psi_{II}\rangle \end{pmatrix}.$$

# Observer complementarity in de Sitter with DQFT

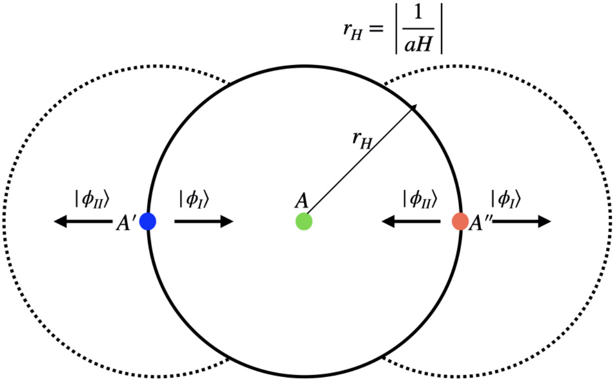



Figure: Points $A'$ and $A''$ are on the horizon of $A$ at opposite sides of the horizon, that are spacelike separated, i.e., at the angles $(\theta, \varphi)$ and $(\pi - \theta, \pi + \varphi)$ respectively. The dotted circles with the same $r_H$ represent the respective comoving horizons of $A'$ and $A''$.

# DQFT in Minkowski and Scattering

$$\left[\mathcal{O}_I\left(t,\,\mathbf{x}\right),\,\mathcal{O}_{II}\left(-t',\,-\mathbf{x}'\right)\right]=0\,.$$
$$S_T = S_I \oplus S_{II}$$

where

$$S_I = T_1\left\{e^{-i\int_{-\infty}^{\infty} H_{int}\ dt}\right\},\quad S_{II} = T_2\left\{e^{i\int_{\infty}^{-\infty} H_{int}\ dt}\right\}$$

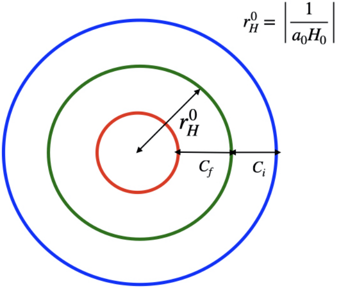
$\mathcal{CPT}$ means

$$\mathcal{A}_{N\to M}\left(p_i,-p_f\right)=\mathcal{A}_{M\to N}\left(-p_i,\,p_f\right)\ ,$$

The *in* states are those with zeroth component of momenta positve and the *out* states are those with the zeroth component of momenta negative.

$$\mathcal{A}=\frac{\mathcal{A}_{N\to M}\left(p_i,-p_f\right)+\mathcal{A}_{M\to N}\left(-p_i,\,p_f\right)}{2}$$
$$\mathcal{A}=\frac{\mathcal{A}^{\rm I}_{N\to M}\left(p_i,-p_f\right)+\mathcal{A}^{\rm II}_{N\to M}\left(-p_i,\,p_f\right)}{2}$$

# Scattering in dS DQFT




$$S_{dS} = S_{dSI} \oplus S_{dSII}$$

where

$$S_{dSI} = T_{dSI}\left\{ e^{-i\int_{-r_H^0 - C_i}^{-r_H^0 + C_f} H_{int}\, d\tau} \right\}, \quad S_{dSII} = T_{dSII}\left\{ e^{i\int_{r_H^0 + C_i}^{r_H^0 - C_f} H_{int}\, d\tau} \right\}$$

Figure: Scattering process schematic picture in expanding dS. $r_H^0$ is the scale at which the modes start to interact and undergo scattering.

# Hemispherical power asymmetry

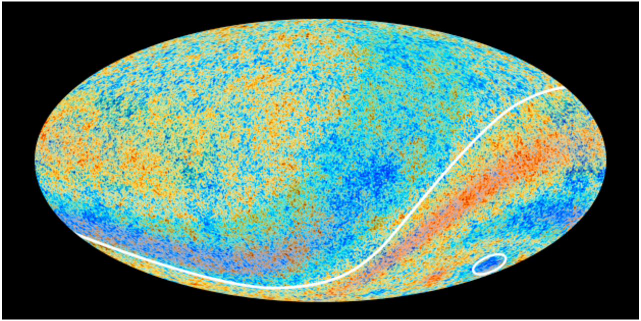


Figure: HPA breaks the isotropy of primordial fluctuations and CMB appears to be asymmetric with slightly higher temperatures in the north and slightly lower temperatures in the south.

# Quantization in quasi-de Sitter space-time: Single field inflation case

- In the inflationary space-time we quantize the Mukhanov-Sasaki variables $v = 2a\frac{\dot{\phi}}{H}\zeta$ and $u_{ij} = \frac{a}{2}h_{ij}$.
- Inflationary space-time is not $\mathcal{PT}$ symmetric like dS. Expectation is $\mathcal{PT}$ symmetry must be spontaneously broken at the quantum level.
- $$\hat{v}(\tau, \mathbf{x}) = \frac{1}{\sqrt{2}}\hat{v}_{\rm I}(\tau, \mathbf{x}) \oplus \frac{1}{\sqrt{2}}\hat{v}_{\rm II}(-\tau, -\mathbf{x}) \,.$$
  corresponding to $|0\rangle_{\rm qdS} = |0\rangle_{\rm qdS_I} \oplus |0\rangle_{\rm qdS_{II}}$.
- $\hat{v}_{\rm II}(-\tau, -\mathbf{x})$ is the fluctuation that goes backward in time. Logically, if the fluctuation propagates forward in time in a slow-roll background, the fluctuation that goes backward in time experience space-time as a "slow-climb". Therefore "quantum mechanically" we solve for $v_{\rm II,\, k}(\tau)$ following the time reversal operation

$$t \to -t \implies H \to -H, \quad \epsilon \to -\epsilon, \quad \eta \to -\eta \,.$$

# Hemispherical asymmetry of scalar power spectra

DQFT in the context of inflatio gives (as $\mathcal{CPT}$ symmetry gets spontaneously broken by the slow-roll)

$$A(k) = \frac{P_{\zeta 1} - P_{\zeta 2}}{4P_\zeta}$$

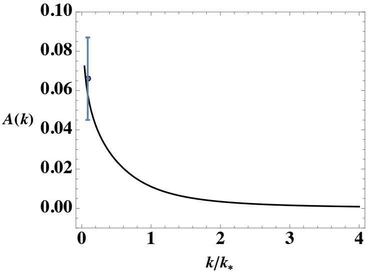



Figure: Here $k_* = a_* H_* = 0.05\mathrm{Mpc}^{-1}$ and we are within $|A| = 0.066 \pm 0.021$ for $k \lesssim 10^{-1} k_*$.

# Hemispherical asymmetry for tensor power spectra

Similarly, we get the power asymmetry of the tensor-power spectrum

$$T(k) = \frac{P_{h1} - P_{h2}}{4P_h}$$

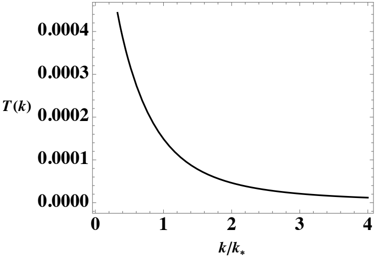
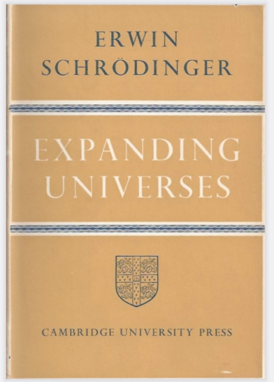



Figure: This plot is obtained for the case of Starobinsky and Higgs inflation.

# Conclusions

- QFTCS is unavoidable if we would like to formulate quantum gravity.
- Unitarity and Renormalizability are the torch lights to anchor quantum gravity research.
- Direct-sum QFT offers a unitary construction of QFT in dS. (See KSK, J. Marto, 2306.XXXX for QFT in the context of BH spacetime.)
- Application of DQFT in the context of inflation gives us robust predictions i.e., Hemispherical asymmetry of primordial power spectra arXiv: 2209.03928 which can be tested with future cosmological and gravitational wave probes.




Thank you very much for your attention.