Structure formation in an anisotropic universe: Eulerian perturbation theory[#1]

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Motivations

- Despite the great observational efforts there is still doubt about the nature of dark energy.
- Our best model so far is ΛCDM, but it has some tension.
- It is of interest to test the assumtions made on ΛCDM.
- Testing the cosmological principle could be a good avenue to solve some tensions.

The approximations

Main objective

1. To study the LSS in the context of an homogeneous but anisotropic bianchi I metric.

The main assumptions

1. We work in the Newton approximation.
2. We assume a small shear approximation.
3. We assume a small field big spatial derivative approximation.

The background

Bianchi I metric

$$ds^2 = a^2(\eta)[-d\eta^2 + \gamma_{ij}(\eta)dx^i dx^j].$$

The geometric shear

$$\sigma_{ij} = \frac{1}{2}\gamma'_{ij}\,, \quad \sigma^2 \equiv \sigma^{ij}\sigma_{ij}.$$

Fluid continuity equations

$$\rho'_m + 3\mathcal{H}\rho_m = 0\,,$$
$$\rho'_{de} + 3\mathcal{H}(\rho_{de} + p_{de}) = -\sigma_{ij}\pi^{ij}$$

Einstein equations

$$3\mathcal{H}^2 - \frac{1}{2}\sigma^2 = (\rho_m + \rho_{de})a^2\,,$$
$$(\sigma^i{}_j)' + 2\mathcal{H}\sigma^i{}_j = a^2\pi^i{}_j.$$

Perturbation theory

The perturbed Bianchi I metric

$$ds^2 = a^2 \left[-(1+2\phi)d\eta^2 + 2\omega_i dx^i d\eta + (\gamma_{ij} + h_{ij})\, dx^i dx^j \right] .$$

The perturbations

$$h_{ij} \equiv -2\psi \left(\gamma_{ij} + \frac{\sigma_{ij}}{\mathcal{H}} \right) + 2\tau_{ij} ,$$

$$\partial^i \omega_i = \tau^i_{\ i} = 0 = \partial^i \tau_{ij} .$$

The Newtonian limit

$$k^2 \phi = -\frac{3}{2} a^2 H^2 \delta ,$$

$$v_i = -\frac{k_i \phi}{aH} .$$

The order of perturbations

	Variable	n in $\mathcal{O}(\epsilon^n)$
Derivatives	∂_i / aH	$-1/2$
Fluid	ρ, δ, θ	0
	v	1
	v_i	$1/2$
Metric	ϕ, ψ	1
	ω_i	$3/2$
	τ_{ij}	1

The continuity equations for the matter perturbations

the continuity equations

$$\nabla_\mu[\rho_m(1+\delta)u^\mu] = 0\,,$$
$$u^\nu \nabla_\nu u^\mu = 0\,.$$

Continuity and Euler equations

$$\delta' + \theta(1+\delta) + v^i\partial_i\delta = 0\,,$$
$$\theta' + \mathcal{H}\theta + \partial_i(v^j\partial_j v^i) + \frac{3}{2}\Omega_m\mathcal{H}^2\delta = -2\sigma^{ij}\partial_i v_j\,.$$

Linear solutions

Linear matter contrast equation

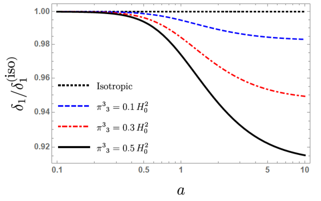
$$\delta_1''(\eta,\mathbf{k}) + \mathcal{H}\delta_1'(\eta,\mathbf{k}) - \frac{3}{2}\mathcal{H}^2\Omega_m\delta_1(\eta,\mathbf{k}) = -2\sigma_{\|}(\eta,\mathbf{k})\delta_1'(\eta,\mathbf{k})\,.$$

Linear solution

$$\delta_1(a,\mathbf{k}) = D_+(a)\left(1 + Q_{ij}(a)\hat{k}^i\hat{k}^j\right)\delta_l(\mathbf{k})\,,$$

$$Q_{ij}(a) \equiv -\frac{2}{D_+(a)}\int_{a_i}^{a} ds\,\mathcal{G}(a,s)\frac{\sigma_{ij}(s)\dot{D}_+(s)}{s^2 H(s)}\,.$$

First order ratio



Non-linear solutions

Non-linear equations

$$\delta_2'(\eta,\mathbf{k}) + \theta_2(\eta,\mathbf{k}) = -\alpha[\theta_1,\delta_1]\,,$$
$$\theta_2'(\eta,\mathbf{k}) + \left[\mathcal{H} + 2\sigma_{\parallel}(\eta,\mathbf{k})\right]\theta_2(\eta,\mathbf{k}) + \frac{3}{2}\mathcal{H}^2\Omega_m\delta_2(\eta,\mathbf{k}) = -\beta[\theta_1,\theta_1]\,.$$

Solutions split

$$\delta_1 = \delta_1^{\text{(iso)}} + \Delta_1\,,$$
$$\delta_2 = \delta_2^{\text{(iso)}} + \Delta_2\,.$$

Anisotropic contribution to the non-linear solution

$$\Delta_2(a,\mathbf{k}) = \int \frac{d^3q}{(2\pi)^3} F_2^{\sigma}(a,\mathbf{q},\mathbf{k}-\mathbf{q})\delta_l(\mathbf{q})\delta_l(\mathbf{k}-\mathbf{q})\,.$$

Summary

- We computed the Euler and continuity equations to the matter perturbations in an anasitropic background.
- We solved the equations to linear and non-linear order.
- We showed that a small deviation from isotropy can leave a signature in the matter power spectrum.
- However, the signal must be disentangled from the quadrupole in the redshift-space distortions.