Alternative Methods
for measuring
Primordial Non-Gaussianity

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Outline

- What is primordial non-Gaussianity?
- Traditional methods
- Squeezed limit methods
- Persistent homology

What is primordial non-Gaussianity

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Anything beyond this = non-Gaussianity.

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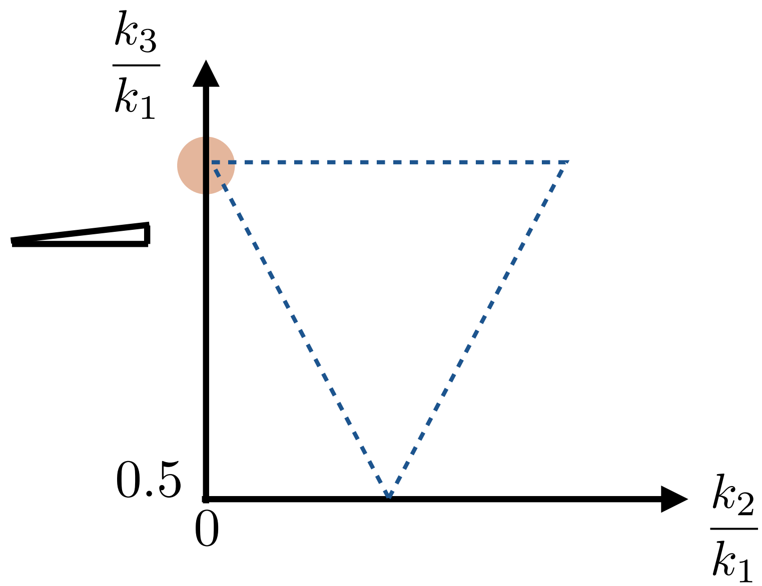

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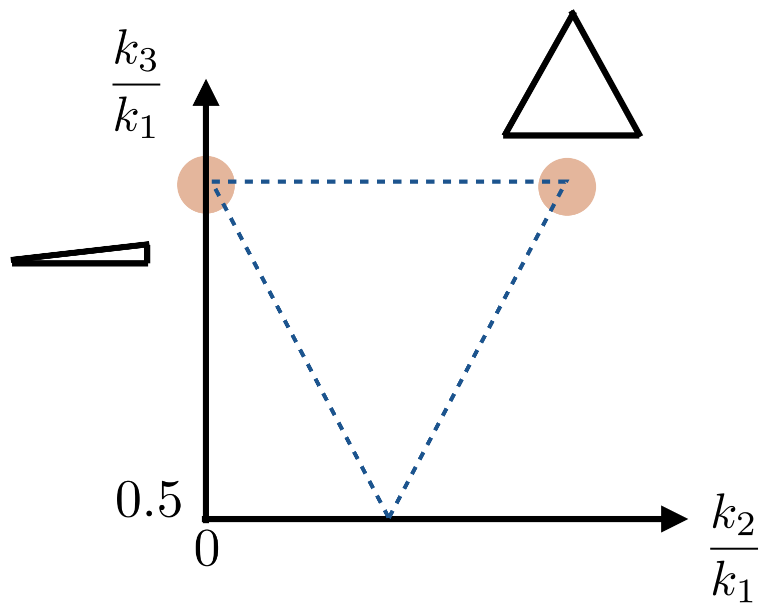

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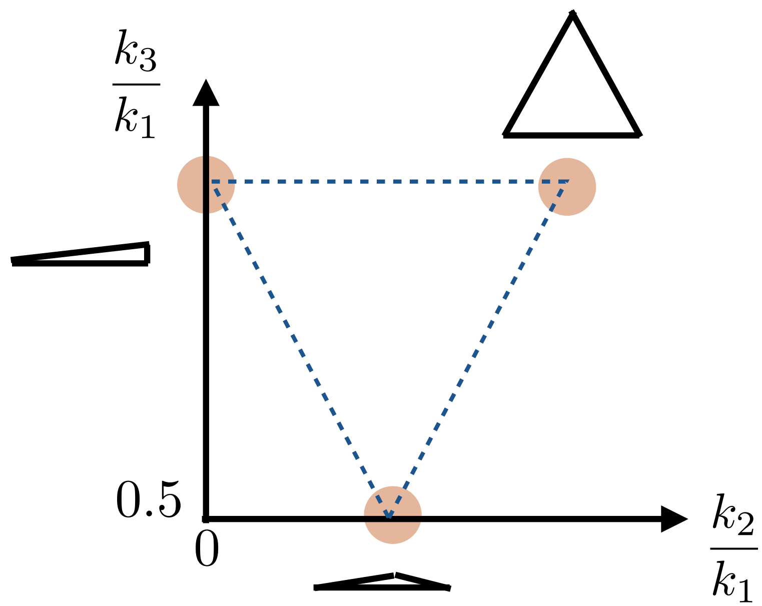
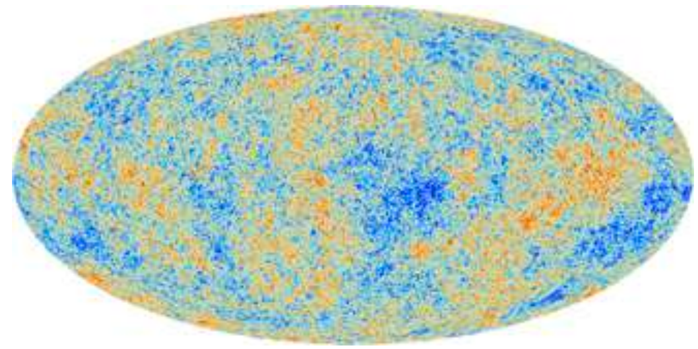


How can it be observed?

$$\left\langle \frac{\delta T}{\bar{T}} \frac{\delta T}{\bar{T}} \frac{\delta T}{\bar{T}} \right\rangle \longleftarrow$$



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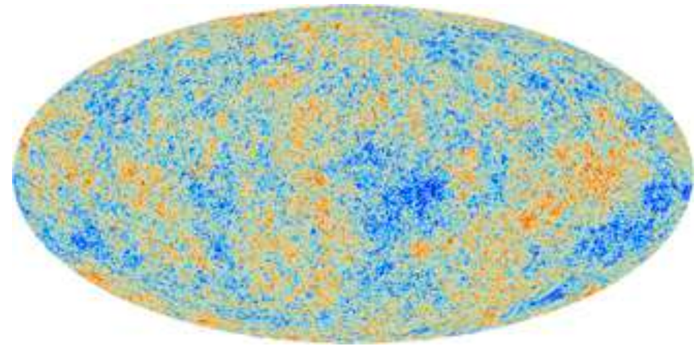
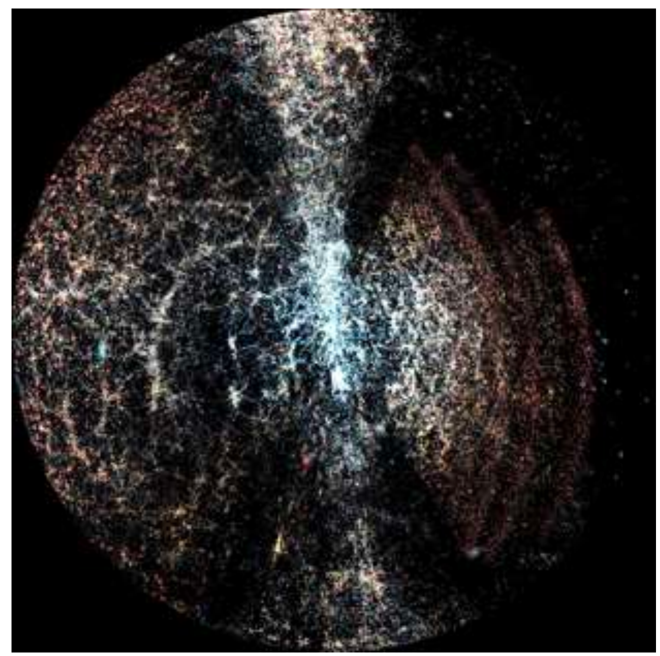


$$\Delta_g = \frac{n_g(z,\hat{n}) - \bar{n}_g(z)}{\bar{n}_g(z)}$$

$$\Delta_g \sim \delta_m$$



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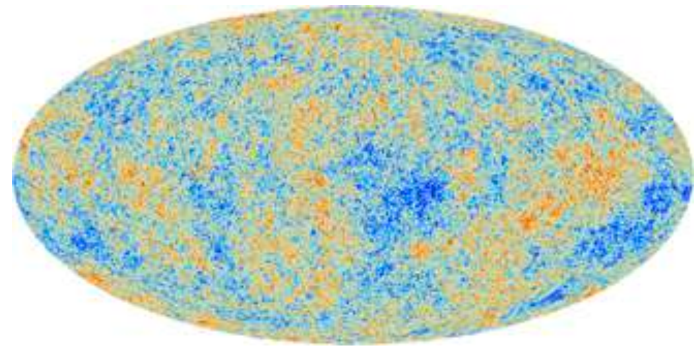

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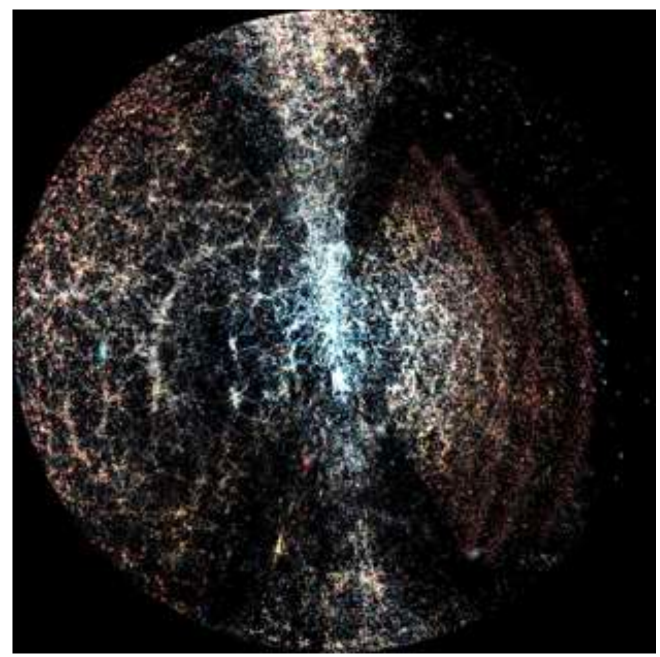

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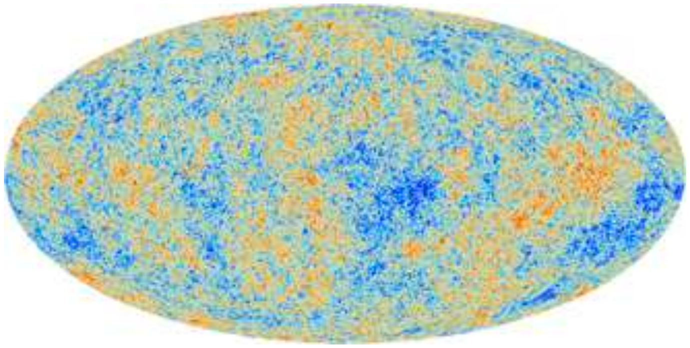

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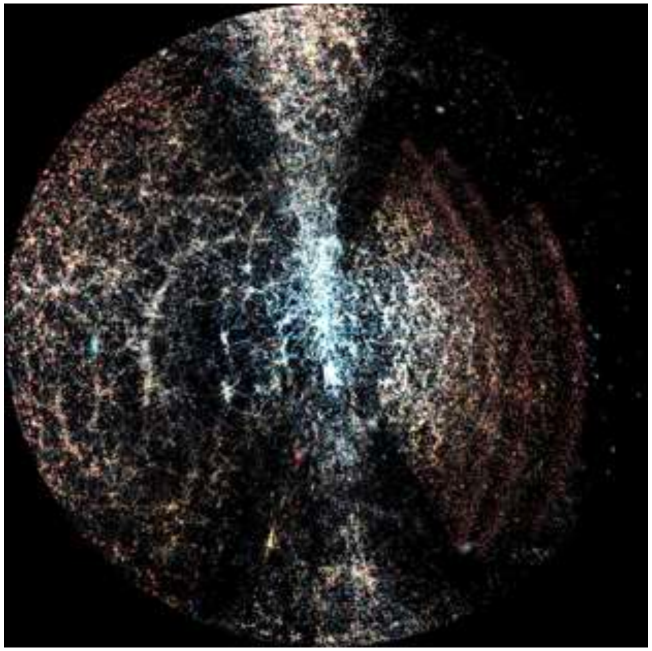

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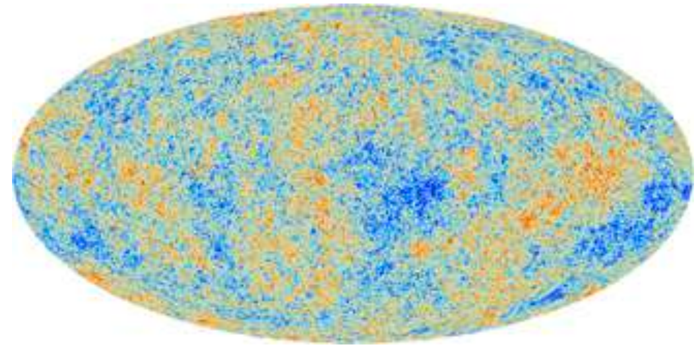

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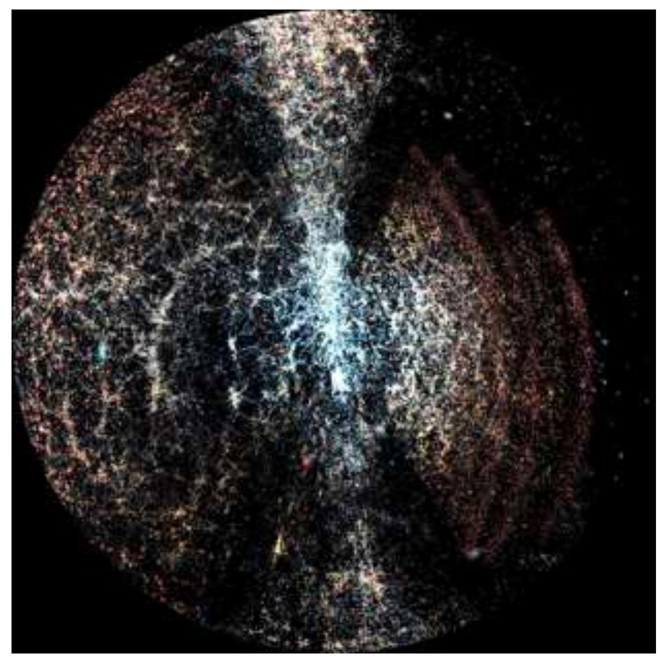
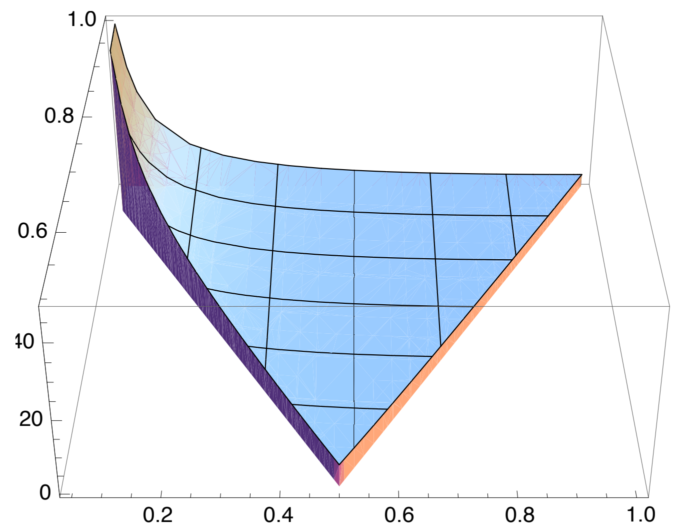


CMB Constraints

Consider a phenomenological model:

$$\zeta = \zeta_g + \frac{3}{5} f_{NL}^{local} \zeta_g^2$$

$$F^{local}(k_1, k_2, k_3) = -2\frac{3}{5} f_{NL}^{local} A^2 \frac{1}{k_1^3 k_2^3} + 3\ perms.$$



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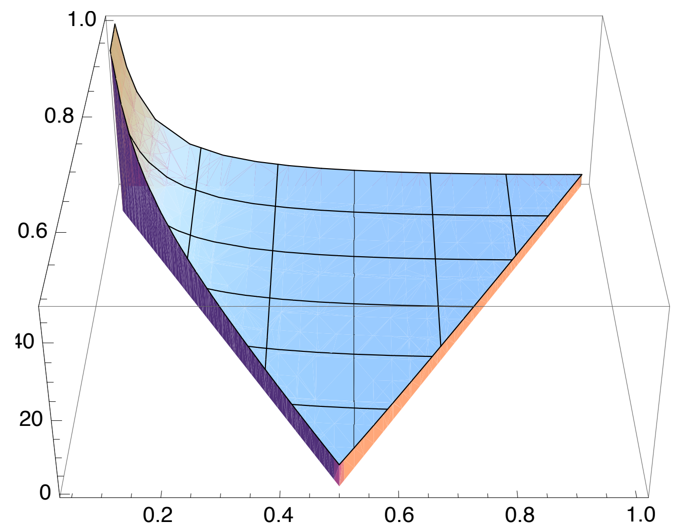

$$F^{local}(k_1, k_2, k_3) = -2\frac{3}{5} f_{NL}^{local} A^2 \frac{1}{k_1^3 k_2^3} + 3\ perms.$$

shape "Overlap"

$$F_1 \cdot F_2 \equiv \sum_{k_1, k_2, k_3} \frac{F_1(k_1, k_2, k_3) F_2(k_1, k_2, k_3)}{\sigma^2(k_1)\sigma^2(k_2)\sigma^2(k_3)}$$

Two shapes are "similar" if they have a cosine of order one.

Data is analyzed for simple shapes.

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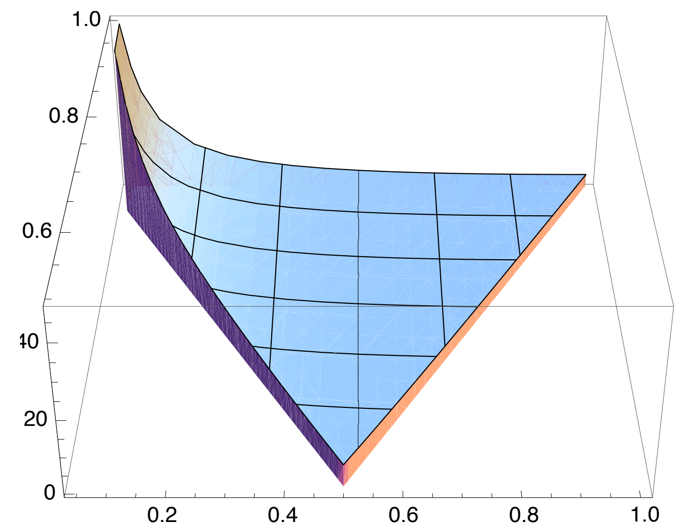

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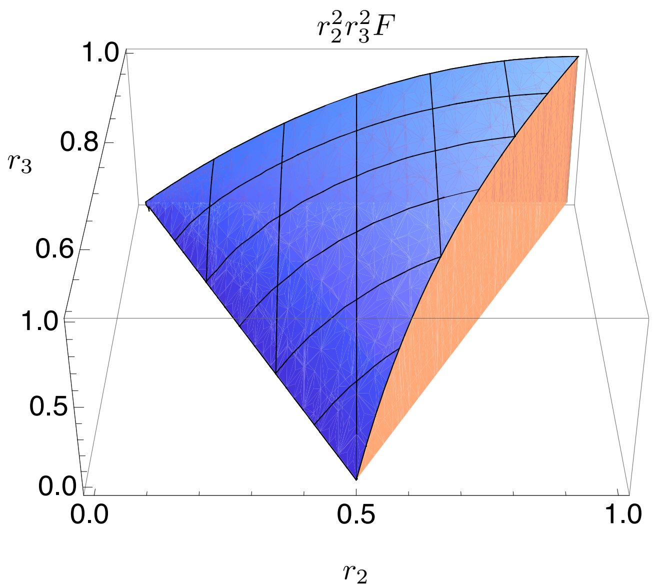
This shape is the one produced by multi-field models.

CMB Constraints

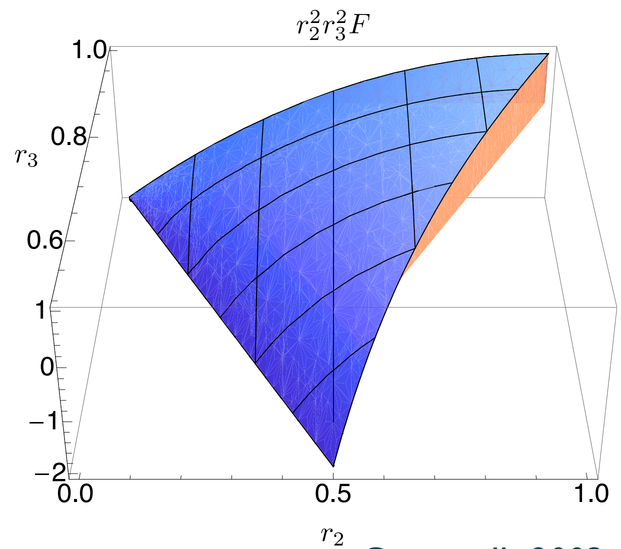
General single field models

Find templates that are like the NG produced by the 2 EFT operators

Equilateral

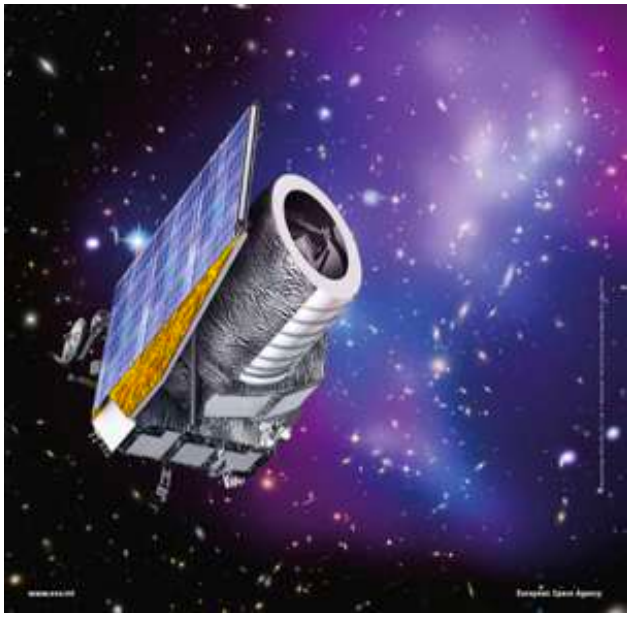


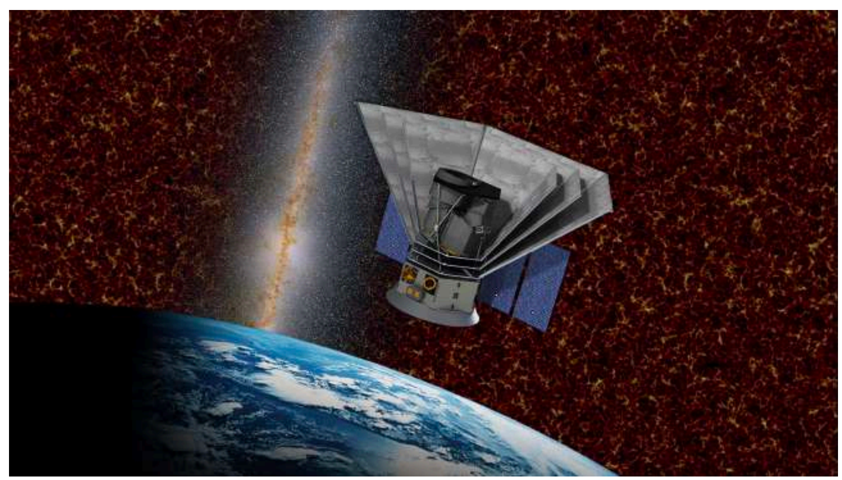
Orthogonal



Creminelli, 2003 [arXiv: astro-ph/0306122]
Cheung et. al., 2008 [arXiv: 0709.0293]
Senatore et. al., 2010 [arXiv: 09053746]

Future: LSS

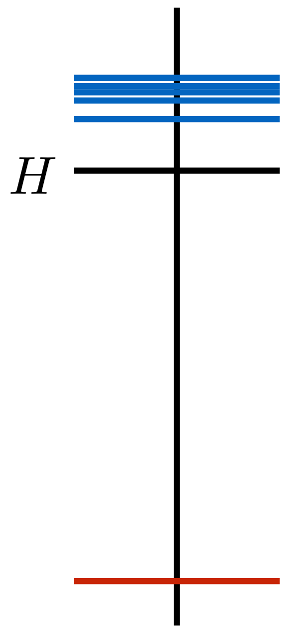




Squeezed limit information

The squeezed limit contains model independent information about the physics during inflation

Single field



$$B(q, k_1, k_2) \overset{q \to 0}{\sim} \frac{1}{q}$$

J. Maldacena, 2003

P. Creminelli, M. Zaldarriaga, 2004

P. Creminelli, G. D'Amico, M. Musso, JN, 2011

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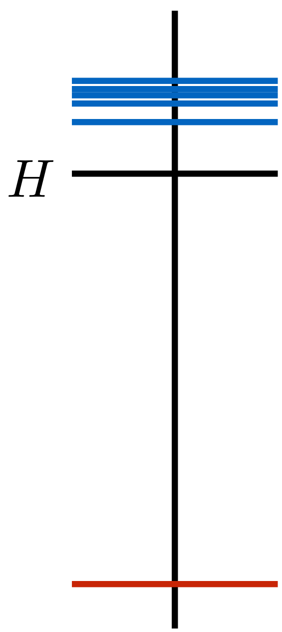
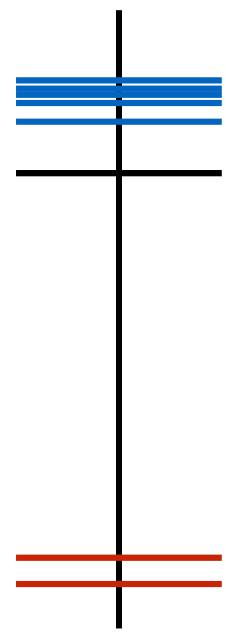


Multi field



$$B(q, k_1, k_2) \overset{q\to 0}{\sim} \quad \frac{1}{q} \qquad\qquad \frac{1}{q^3}$$

Squeezed limit information

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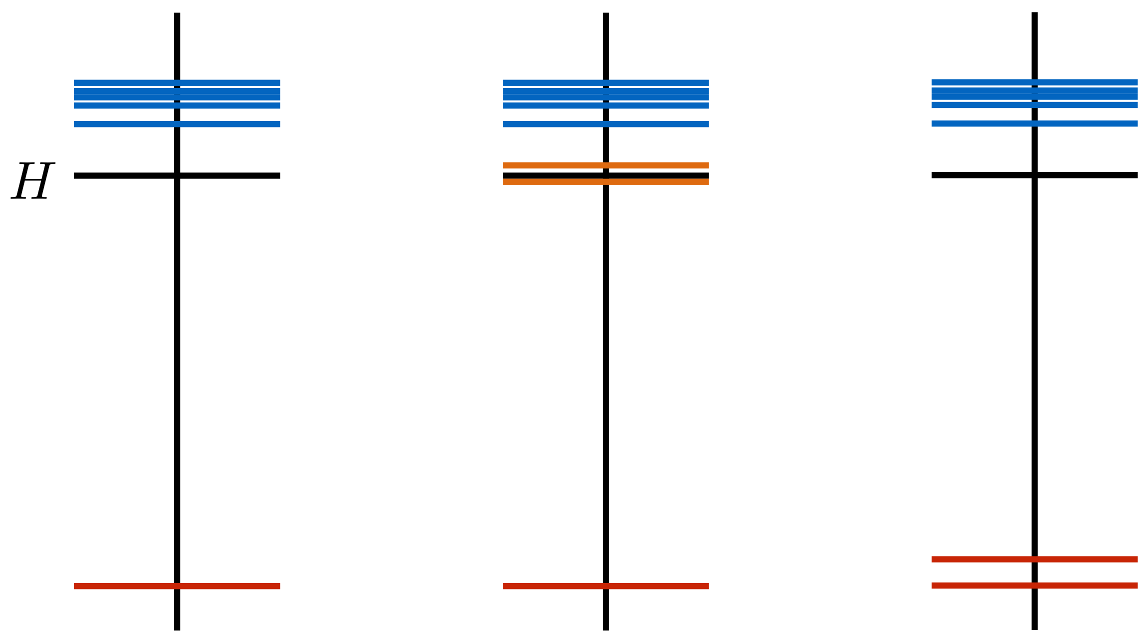
Single field | Multi field



$$B(q, k_1, k_2) \overset{q\to 0}{\sim} \quad \frac{1}{q} \qquad \frac{1}{q^\alpha} \quad 1 < \alpha < 3 \qquad \frac{1}{q^3}$$

X. Chen, J. Wang, 2009

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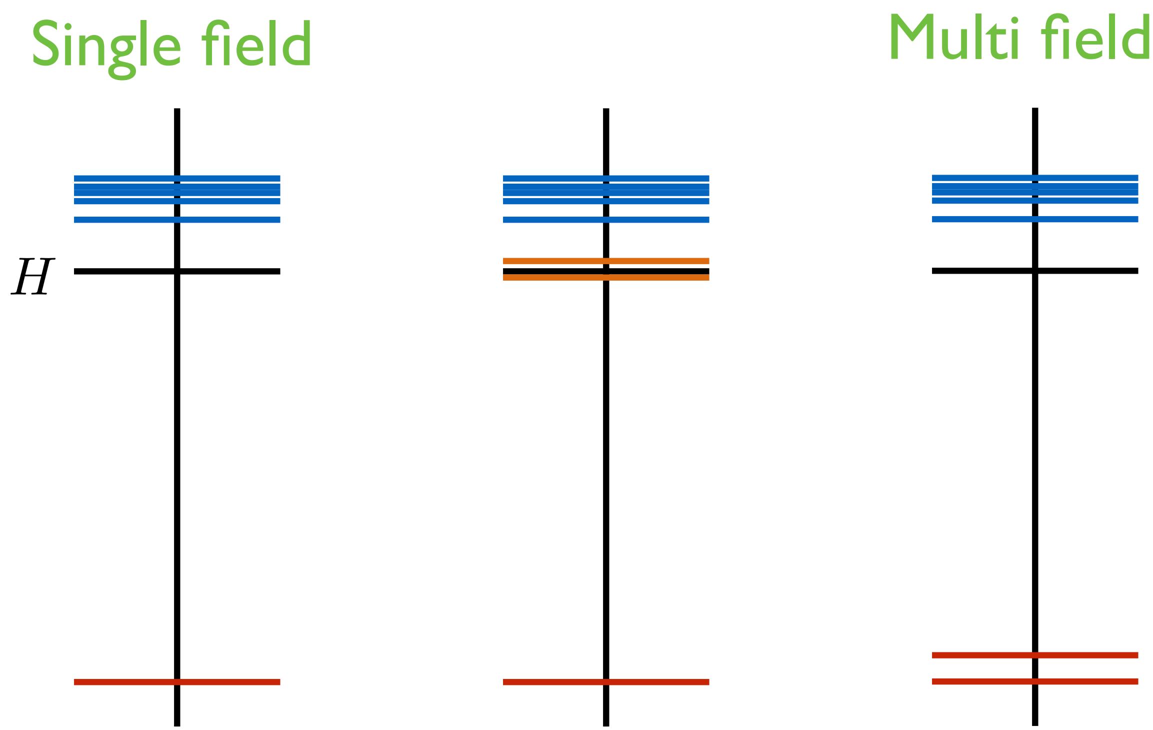

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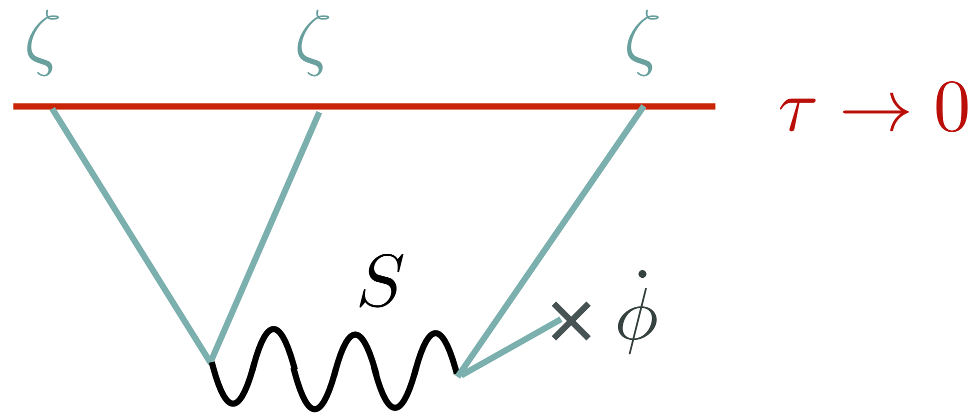
Figure Assassi, Baumann, Green, 2012

Other fields



J. Maldacena, N. Arkani-Hamed, 2015
H. Lee, D. Baumann, G. Pimentel, 2016
A. Riotto, A. Kehagias, 2017
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L. Bordin, P. Creminelli, A. Khlemintsky, L. Senatore 2018

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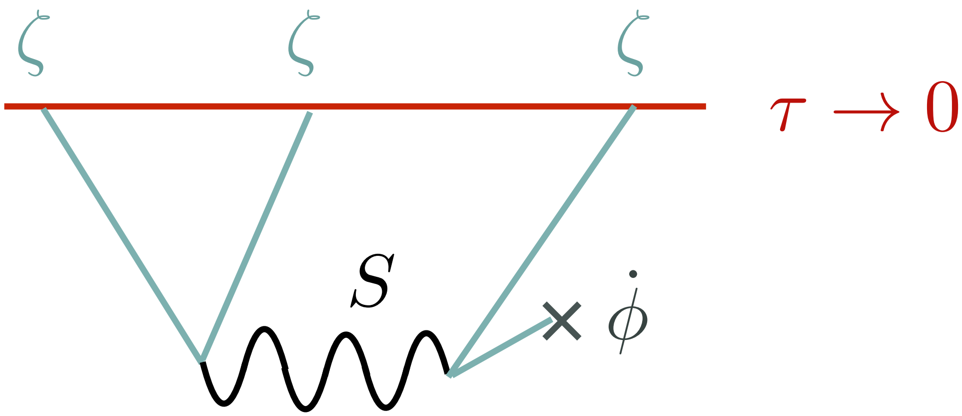

$$\langle \zeta(q)\zeta(k)\zeta(k)\rangle \sim e^{-\pi\mu}\left[e^{i\delta(\mu)}\left(\frac{q}{k}\right)^{\frac{3}{2}+i\mu} + e^{-i\delta(\mu)}\left(\frac{q}{k}\right)^{\frac{3}{2}-i\mu}\right] P_s(\cos\theta)$$

$$\mu = \sqrt{\frac{m^2}{H^2} - \left(s - \frac{1}{2}\right)}$$

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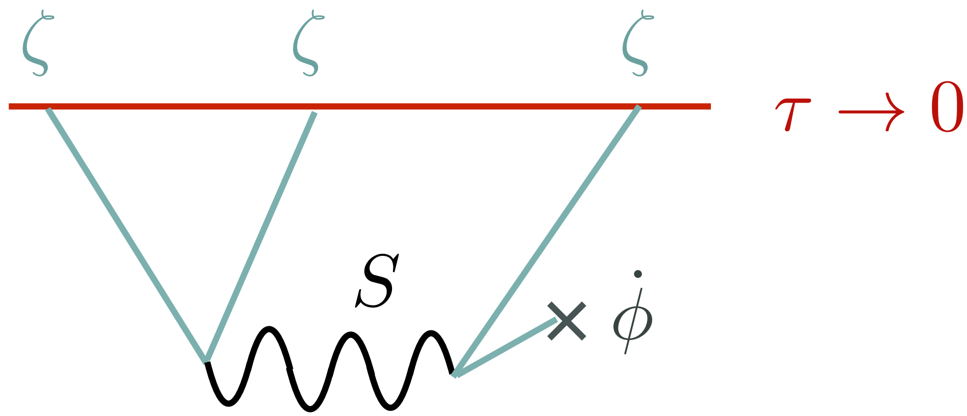

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Characteristic angle dependence

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EFT of Inflation

Even within single-field, PNG teaches about the effective Lagrangian

$$\mathcal{L} = -\frac{M_{\rm pl}^2 \dot{H}}{c_s^2}\left(\dot{\pi}^2 - \frac{c_s^2}{a^2}(\partial_i \pi)^2\right) + \frac{M_{\rm pl}^2 \dot{H}}{c_s^2}(1 - c_s^2)\frac{1}{a^2}\dot{\pi}(\partial_i \pi)^2 + \frac{M_{\rm pl}^2 \dot{H}}{c_s^2} A \dot{\pi}^3$$

Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore, 2007

Constrained through the "equilateral" and "orthogonal" templates.

Senatore, Smith, Zaldarriaga, 2009

Copious particle production during inflation produces large equilateral non-Gaussianity.

The Scale-dependent bias

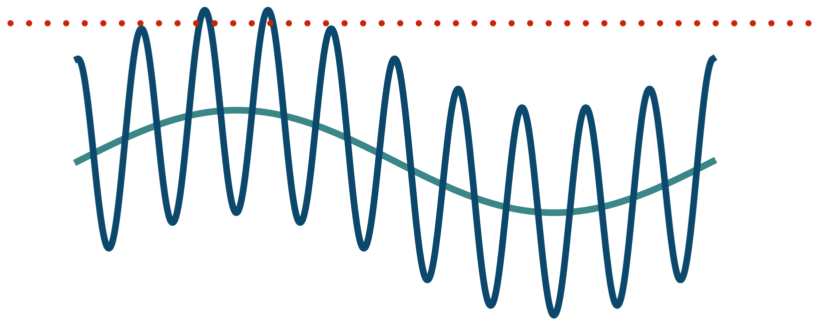
Bias is the connection of galaxies and matter $\delta_g = b\delta$

Dalal, et. al., 2008

For the local model: $\Phi = \Phi_g + f_{\rm NL}\Phi_g^2$

Matarrese, Verde, et. al., 2008

Slosar, et. al., 2008



$f_{\rm NL} = 0$

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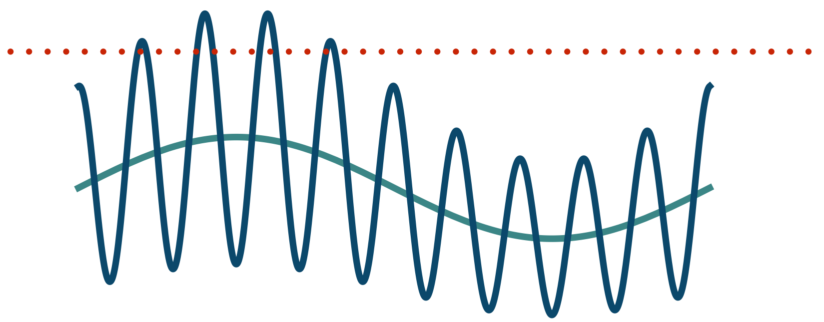

$f_{\rm NL} > 0$

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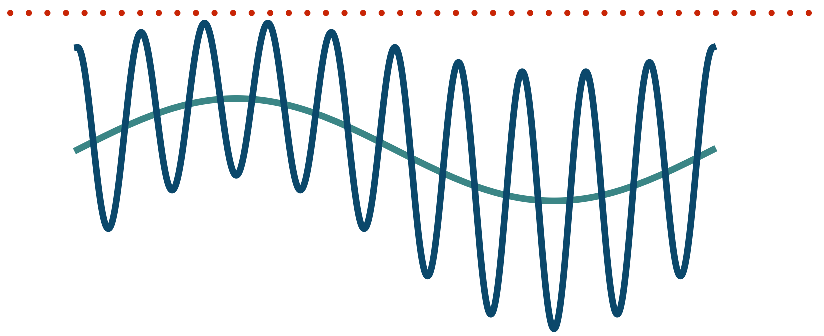


$f_{\rm NL} < 0$

There is a correlation between Φ and the number of galaxies Δ_g.

$$\langle \Delta_g \Delta_g \rangle \subset \langle \Phi\delta \rangle \sim \frac{1}{k^2}\langle \delta\delta \rangle$$

Sensitive to the squeezed limit!

No coupling with potential

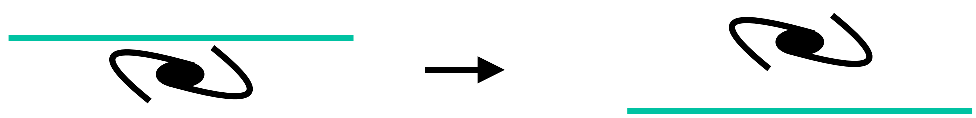
A homogeneus gravitational potential has no physical meaning

Peloso, Pietroni, 2013 Kehagias, Riotto, 2013 Creminelli, JN, Simonović, Vernizzi, 2014

No coupling with potential

A homogeneus gravitational potential has no physical meaning

$$\Phi \to 0$$



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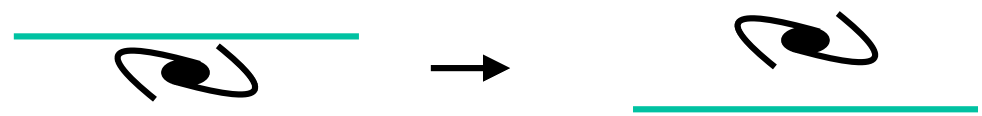
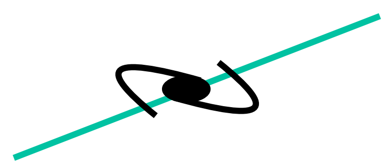


A homogeneus gravitational force can be set to zero by going to a freely falling frame



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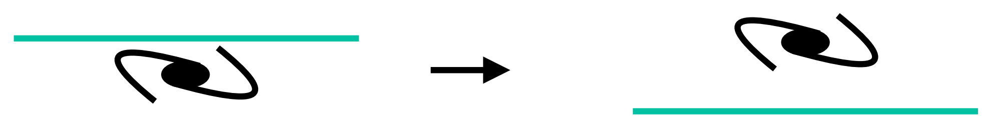

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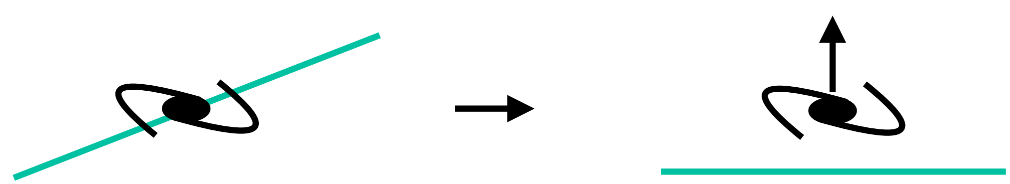
$$\nabla\Phi \to 0$$

$$\vec{V} \to \vec{V} - t\nabla\Phi$$



Peloso, Pietroni, 2013 Kehagias, Riotto, 2013 Creminelli, JN, Simonović, Vernizzi, 2014

Current constraints

CMB (68% CL)

Single-field EFT	Equilateral	$f_{\rm NL}^{equi} = -26 \pm 47$
	Orthogonal	$f_{\rm NL}^{orth} = -38 \pm 24$
Multi-field	Local	$f_{\rm NL}^{loc} = -0.9 \pm 5.1$

Planck collaboration 2019

Current constraints

LSS

(68% CL)

Single-field EFT	Equilateral	$f_{\rm NL}^{equi} = 2 \pm 212$
	Orthogonal	$f_{\rm NL}^{orth} = 126 \pm 72$
Multi-field	Local	$f_{\rm NL}^{loc} = -33 \pm 28$

D'Amico, Lewandowski, Senatore, Zhang, 2022

See also

Cabass, Ivanov, Philcox, Simonović, Zaldarriaga, 2022

Forecasts

Karagiannis et. al., 2018 — f_{NL}^{loc} — Doré et. al., 2014

	"Euclid-like"	"LSST inspired"	SPHEREx
P	~ 6	~ 1	~ 1
B	~ 6	~ 0.5	~ 0.2
$P+B$	~ 5	~ 0.5	

Single tracer ("Euclid-like", "LSST inspired") — Multi tracer (SPHEREx)

"High-z" P $\sigma(f_{\mathrm{NL}}^{loc}) \sim 0.8$ Castorina et. al., 2021

MegaMapper B $\sigma(f_{\mathrm{NL}}^{equi}) \sim 17$ $\sigma(f_{\mathrm{NL}}^{orth}) \sim 8$

Cabass et. al., 2022

Squeezed limit methods

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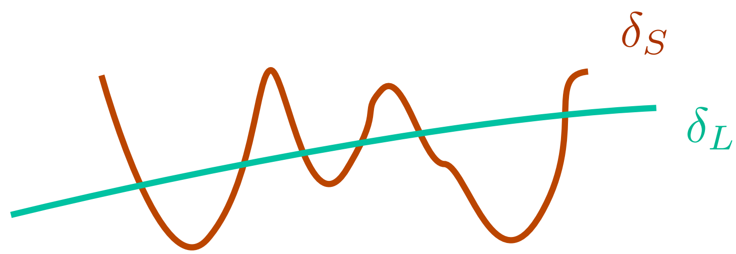
$$\langle \delta(\mathbf{q})\delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\rangle = (2\pi)^3\delta(\mathbf{q}+\mathbf{k}_1+\mathbf{k}_2)B(q,k_1,k_2)$$

We will be interested in the limit $q \ll k_1, k_2$



$$\langle \delta(\mathbf{q})\delta(\mathbf{k}_1)\dots\delta(\mathbf{k}_n)\rangle \overset{q\to 0}{=} \langle \delta(\mathbf{q})\langle\delta(\mathbf{k}_1)\dots\delta(\mathbf{k}_n)\rangle_{\delta_L}\rangle$$

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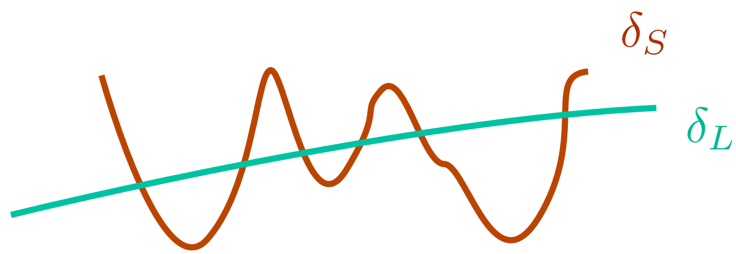


$$\langle\delta(\mathbf{q})\delta(\mathbf{k}_1)\ldots\delta(\mathbf{k}_n)\rangle \overset{q\to 0}{=} \langle\delta(\mathbf{q})\langle\delta(\mathbf{k}_1)\ldots\delta(\mathbf{k}_n)\rangle_{\delta_L}\rangle$$

Correlation between a 2-point function and the potential Φ

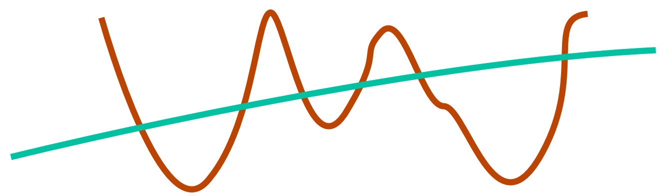
=

Local non-Gaussianity.

Squeezed limit methods

$$\langle \delta(\mathbf{q})\delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\rangle = (2\pi)^3\delta(\mathbf{q}+\mathbf{k}_1+\mathbf{k}_2)B(q,k_1,k_2)$$

We will be interested in the limit $q \ll k_1, k_2$



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Correlation between a 2-point function and the potential Φ

=

Local non-Gaussianity.

Can be pushed to very small scales for the short modes.

Squeezed limit methods

Modulation of the short modes by a long-wavelength mode:

$$\delta(\vec{k})\big|_{\Phi_L} = \delta(\vec{k})\big|_{\Phi_L=0} + \frac{(\vec{k}-\vec{q}).\vec{q}}{\vec{q}^2}\delta_L(\vec{q})\delta(\vec{k}-\vec{q})$$

$$+ \delta_L(\vec{q})\Delta_1(\vec{k}) + \hat{q}^i\hat{q}^j\delta(\vec{q})\hat{k}^i\hat{k}^j\Delta_\theta(\vec{k}) + ...$$

Biagetti, Calles, Castiblanco, Gonzalez, Noreña, 2022

Squeezed limit methods

Modulation of the short modes by a long-wavelength mode:

Equivalence principle.

$$\delta(\vec{k})\big|_{\Phi_L} = \delta(\vec{k})\big|_{\Phi_L=0} + \frac{(\vec{k}-\vec{q}).\vec{q}}{\vec{q}^2}\delta_L(\vec{q})\delta(\vec{k}-\vec{q})$$

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Dynamical response.

Biagetti, Calles, Castiblanco, Gonzalez, Noreña, 2022

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Modulation of the short modes by a long-wavelength mode:

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$$+ \delta_L(\vec{q})\Delta_1(\vec{k}) + \hat{q}^i\hat{q}^j\delta(\vec{q})\hat{k}^i\hat{k}^j\Delta_\theta(\vec{k}) + \dots$$

Dynamical response.

Biagetti, Calles, Castiblanco, Gonzalez, Noreña, 2022

Squeezed limit methods

Modulation of the short modes by a long-wavelength mode:

Equivalence principle.

Local NG

$$\delta(\vec{k})\big|_{\Phi_L} = \delta(\vec{k})\big|_{\Phi_L=0} + \frac{(\vec{k}-\vec{q}).\vec{q}}{\vec{q}^2}\delta_L(\vec{q})\delta(\vec{k}-\vec{q}) + \Phi_L(\vec{q})\Delta_{NG}(\vec{k})$$

$$+ \delta_L(\vec{q})\Delta_1(\vec{k}) + \hat{q}^i\hat{q}^j\delta(\vec{q})\hat{k}^i\hat{k}^j\Delta_\theta(\vec{k}) + \ldots$$

Dynamical response.

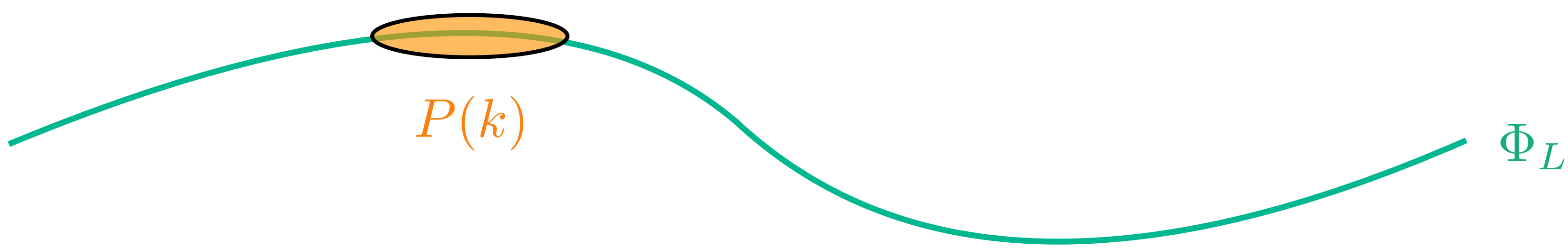
Simple covariance, easy to model, non-linear scales, complementary information.

Modulated power spectrum

Modulation of power spectrum

De Putter, 2018

$$P(k)\big|_{\Phi_L} = P(k)\big|_{\Phi_L=0} + \Phi_L(q) P_{\delta\Delta_{NG}}(k) + \delta_L(q) P_{\delta\Delta}(k) + \ldots$$



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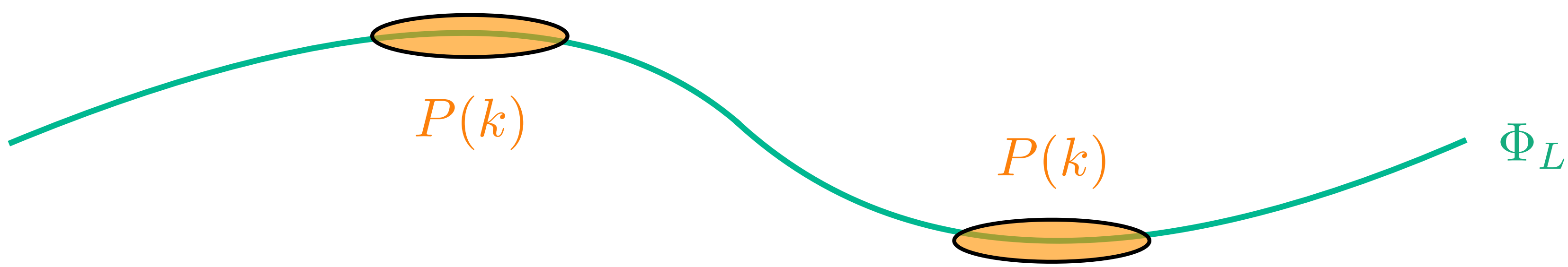

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$1/q^2$ modulation

Modulated power spectrum

Modulation of power spectrum

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↓

$1/q^2$ modulation

Tested in Quijote simulations.

Giri, Müchmeyer, Smith, 2023

$$0.006 < q < 0.047 \qquad 0.5 < k < 3.0$$

Modulated power spectrum

Modulation of power spectrum

De Putter, 2018

$$P(k)\big|_{\Phi_L} = P(k)\big|_{\Phi_L=0} + \Phi_L(q)P_{\delta\Delta_{NG}}(k) + \delta_L(q)P_{\delta\Delta}(k) + \ldots$$

↓

$1/q^2$ modulation

Tested in Quijote simulations.

Giri, Müchmeyer, Smith, 2023

$$0.006 < q < 0.047 \qquad 0.5 < k < 3.0$$

Powerful probe using matter field.

Due to shot noise, comparable with scale-dependent bias using halos (in Quijote).

Squeezed bispectrum

Leading orders in q

$$B(q,k_1,k_2) = P(q)P(k)\left[\frac{1}{q^2}R_{NG}(k_1) + \frac{\vec{k}_1.\vec{q}}{q^2} + R_1(k_1) + R_\theta(k_1)(\hat{k}_1.\hat{q})^2\right] + (1 \leftrightarrow 2)$$

Squeezed bispectrum

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Esposito, Hui, Scoccimarro, 2019

Goldstein, Esposito, et. al., 2022

Factor k dependence by averaging over it

$$B(q) \sim \sum_k B(q, k_1, k_2) \sim \frac{1}{q^2}a_{NG} + a_0 + a_1 q + ...$$

Squeezed bispectrum

Leading orders in q

$$B(q,k_1,k_2) = P(q)P(k)\left[\frac{1}{q^2}R_{NG}(k_1) + \frac{\vec{k}_1.\vec{q}}{q^2} + R_1(k_1) + R_\theta(k_1)(\hat{k}_1.\hat{q})^2\right] + (1 \leftrightarrow 2)$$

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Goldstein, Esposito, et. al., 2022

Factor k dependence by averaging over it

$$B(q) \sim \sum_k B(q,k_1,k_2) \sim \frac{1}{q^2}a_{NG} + a_0 + a_1 q + \ldots$$

Don't find substantial improvement over scale-dependent bias.

$$0.006 < q < 0.047 \qquad 0.5 < k < 3.0$$

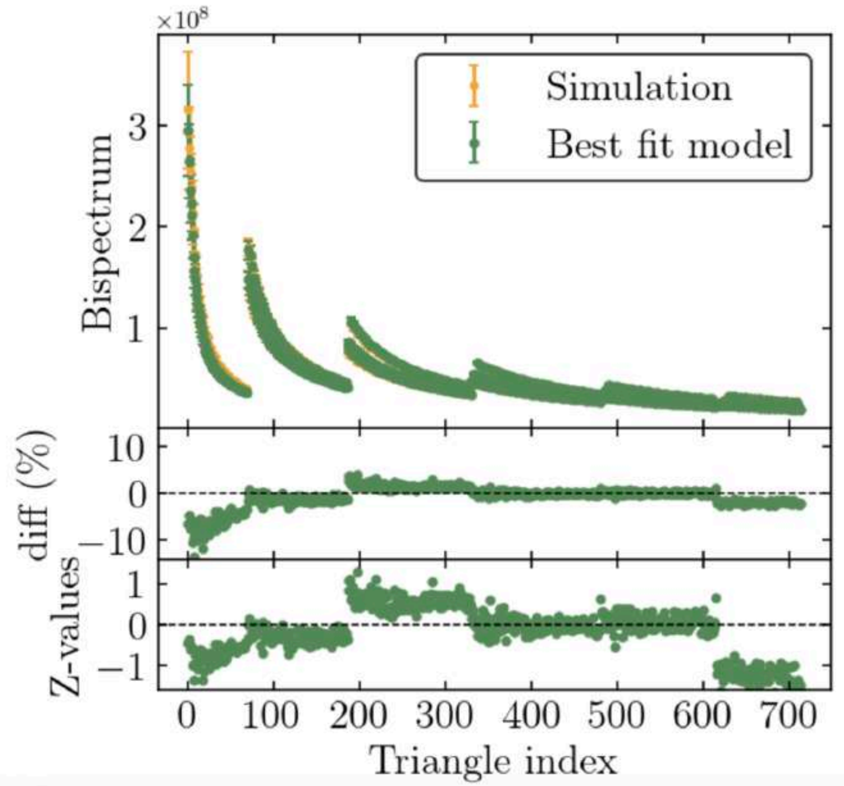
Very easy to model, simple covariance, GR effects.

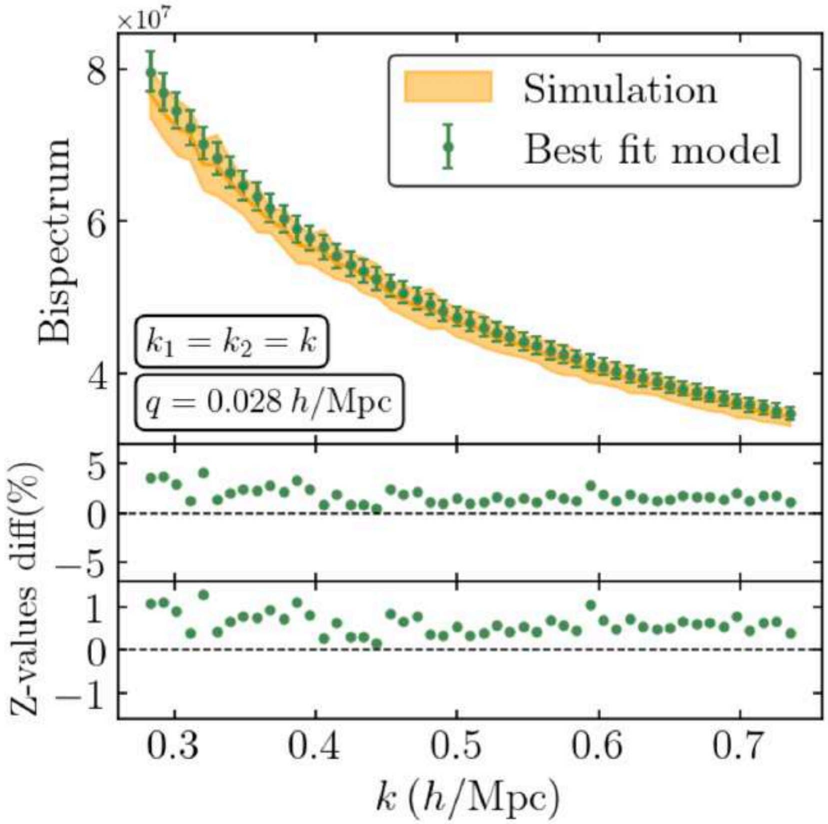
Biagetti, Calles, Castiblanco, Gonzalez, Noreña, 2022

Squeezed bispectrum

$$B(q,k_1,k_2) = P(q)P(k)\left[\frac{\vec{k}_1.\vec{q}}{q^2} + R_1(k_1) + R_\theta(k_1)(\hat{k}_1.\hat{q})^2\right] + (1 \leftrightarrow 2)$$

$$R_1(k) = \frac{10}{7} + S_1^1 k + S_2^1 k^2 + S_3^1 k^3 \qquad R_\theta(k) = \frac{4}{7} + S_1^\theta k + S_2^\theta k^2 + S_3^\theta k^3$$

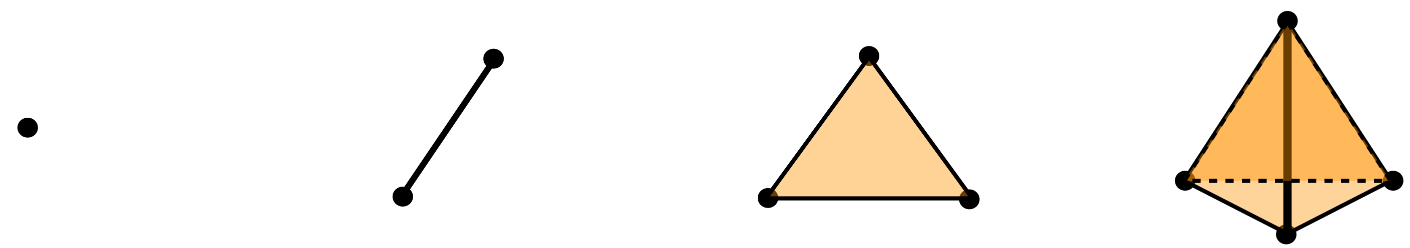




Persistent Homology

Persistent Homology

Use topological features to constrain NG



In 3D three homology groups

H_0 "Clusters" H_1 "Loops" H_2 "Voids"

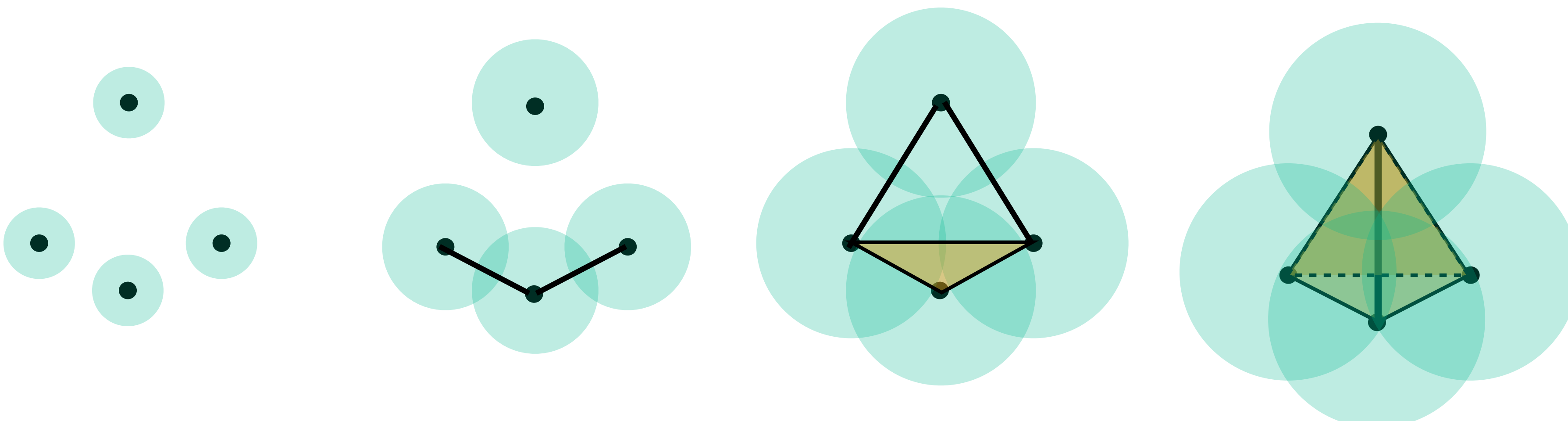
Build simplical complexes out of your point cloud. Thus have a systematic way of counting structures.

Cole, Shiu, 2017

Biegetti, Cole, Shiu, 2020

Persistent Homology

How to build the complex? Connect points if they are less than a certain distance apart.



Very sensitive to outliers. A single halo in a void can destroy it.

Choose a better "filtration", with a "scale" parameter.

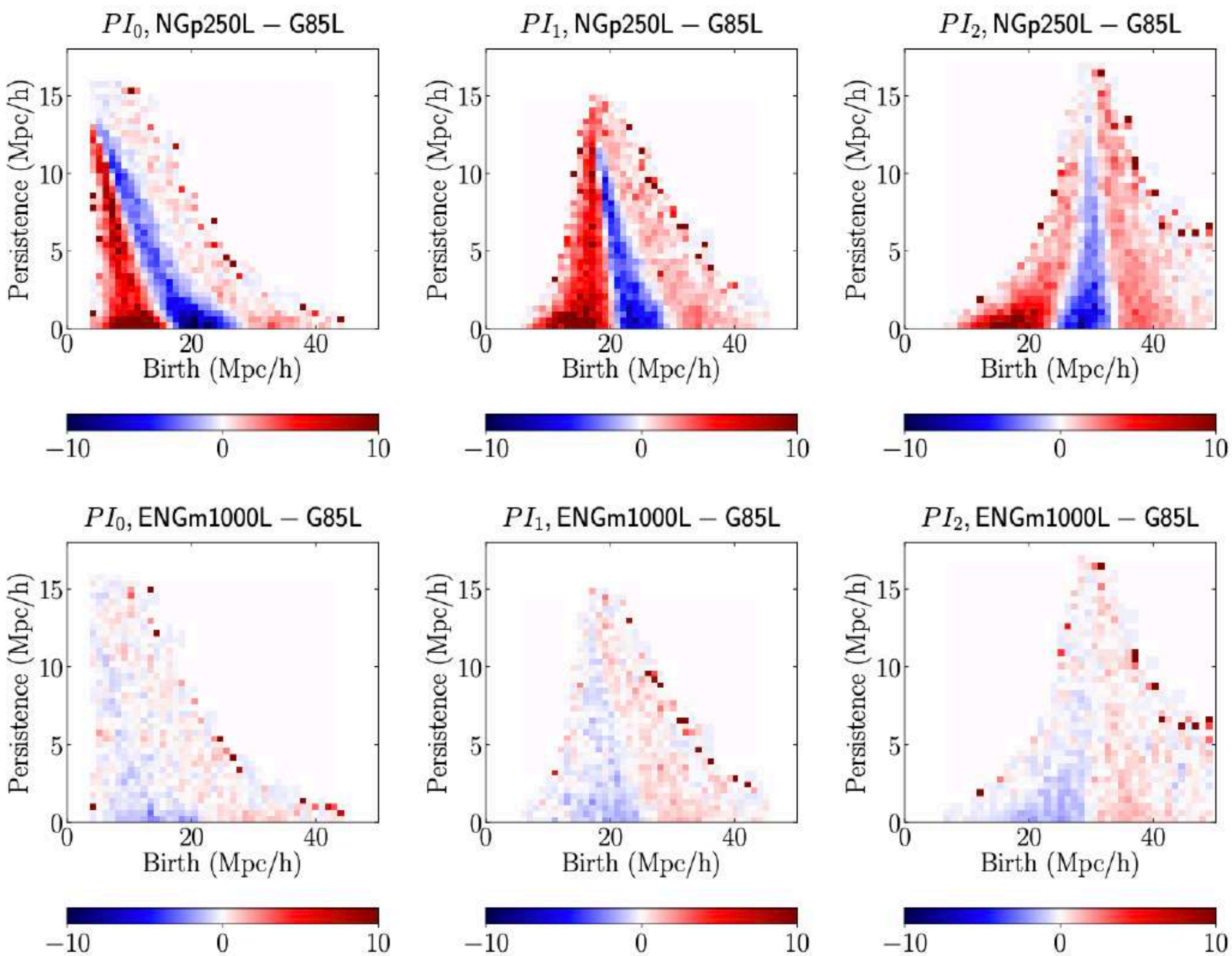
Thus, for each feature we have a birth and a death.

Cole, Shiu, 2017

Biegetti, Cole, Shiu, 2020

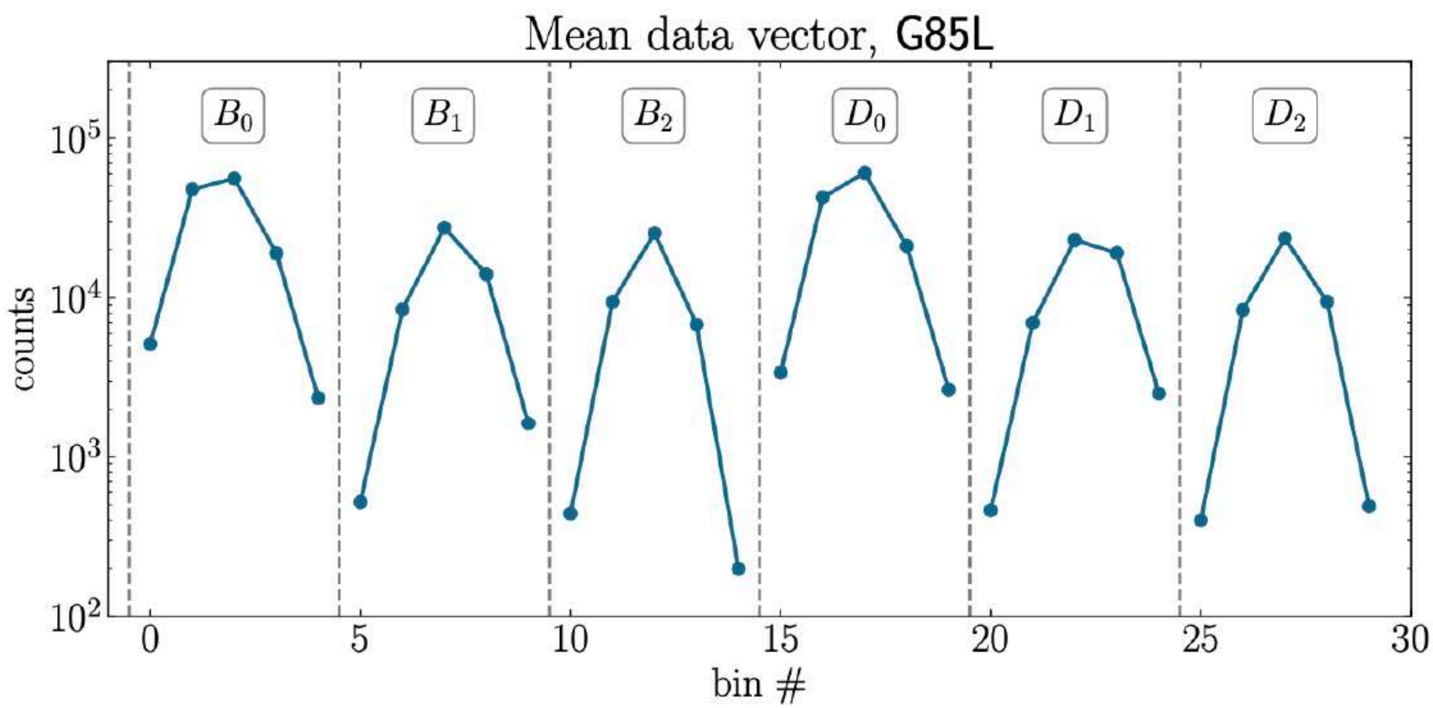
Persistent Homology

Plot weighted histograms of birth vs persistence (death - birth)



Persistent Homology

As data vectors choose a few bins of births and deaths



Persistent Homology

Use simulations to compute derivatives for a Fisher analysis

In redshift space, with different types of errors.

Spectroscopic

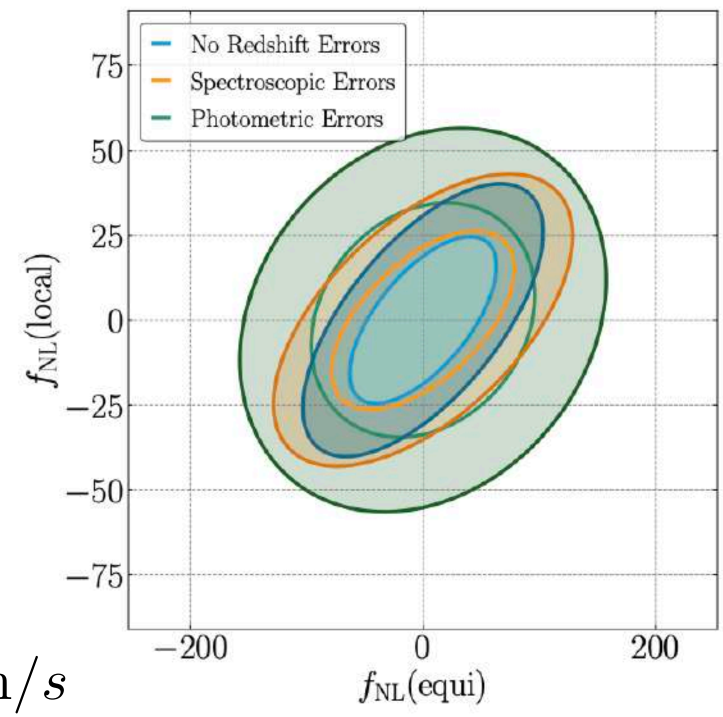
$$\vec{s} = \vec{x} + \frac{\vec{v}.\hat{n}}{\mathcal{H}}\hat{n} + \frac{\vec{v}_{spec}}{\mathcal{H}}\hat{n}$$

$$\sigma_{spec} \sim \sigma_v/5 \sim 30\ \mathrm{km}/s$$

Photometric

$$\vec{s} = \vec{x} + \frac{\vec{v}_{photo}}{\mathcal{H}}\hat{n}$$

$$\frac{\Delta z}{(1+z)} \sim 0.01 \rightarrow \sigma_{photo} \sim 3000\ \mathrm{km}/s$$



$$V \sim 1\ (\mathrm{Gpc}/h)^3$$

Conclusions

- Hard to get $\Delta f_{\rm NL}^{equi} \sim 1$ with standard techniques.
- Scale dependent bias still most promising probe for local PNG. With bispectrum to break degeneracies.
- Squeezed limit techniques could be refined to provide "many scale dependent biases".
- Persistence homology techniques can be used to greatly improve limits on equilateral PNG. But still need a lot of work.

THE

END