The Big Bang in the Lab: Simulating the Early Universe with Bose-Einstein Condensates

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Canadian Institute for Theoretical Astrophysics

[in prep]
Related work :
JHEP 07 (2018) 014
JHEP 10 (2019) 174

Cosmology from Home 2023

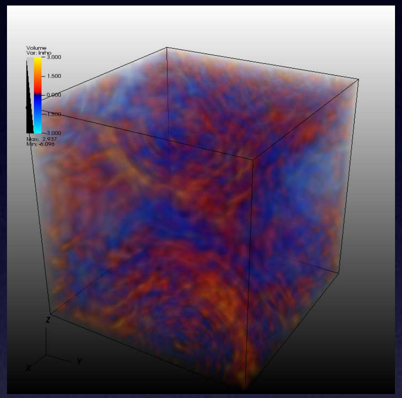
Overview

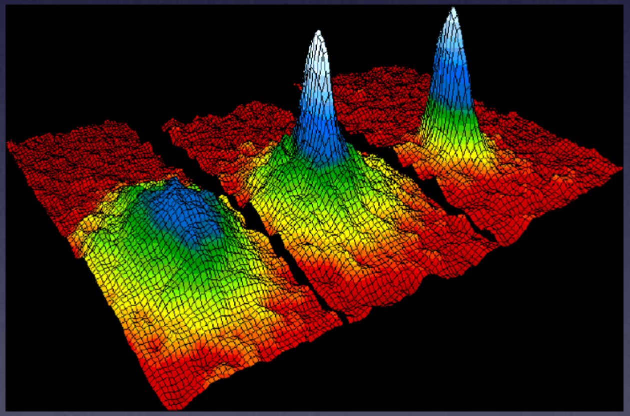
A. Inflationary Cosmology Review
   - Epochs of nonlinearity
   - Preheating
   - Phase transitions (vacuum decay)

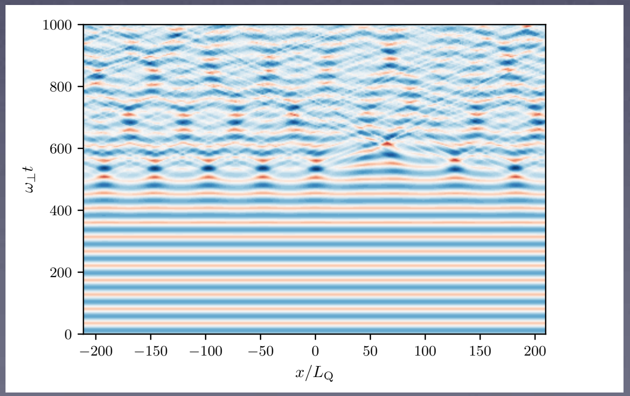
B. Analog BEC Dynamics
   - Mapping between cosmology and cold atoms

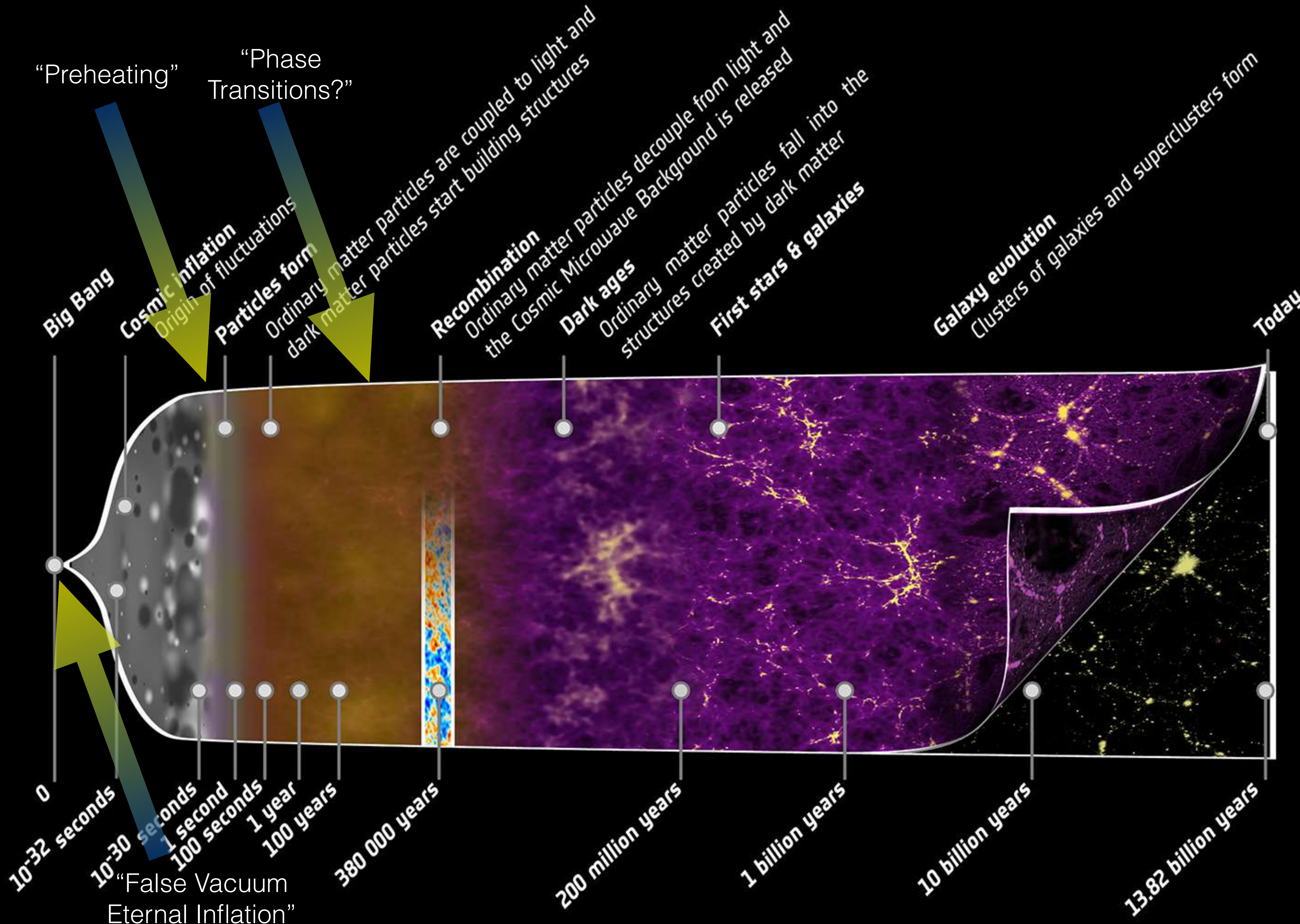
C. Preheating (end of inflation) in cold atom systems

D. A little vacuum decay mixed in

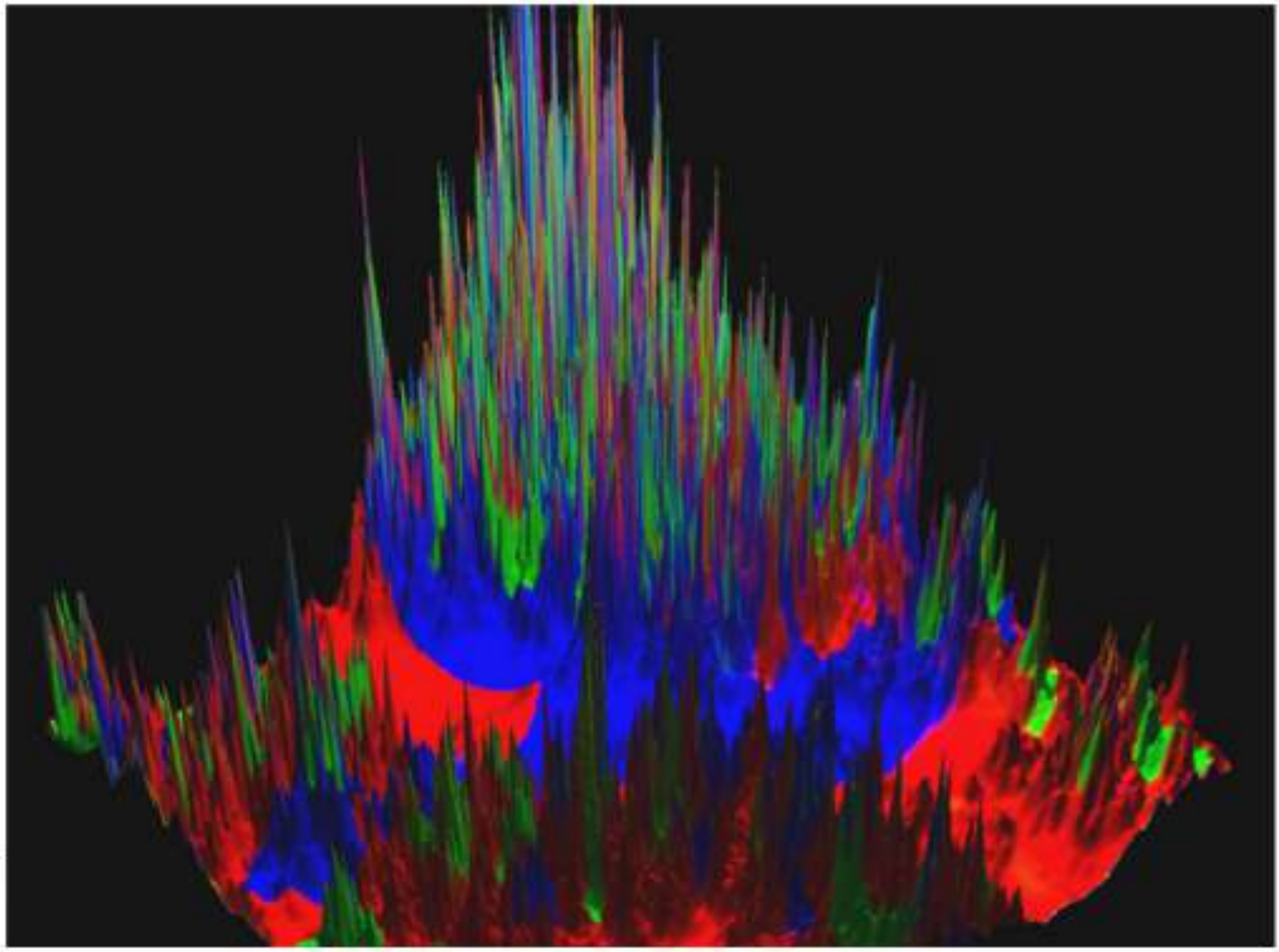


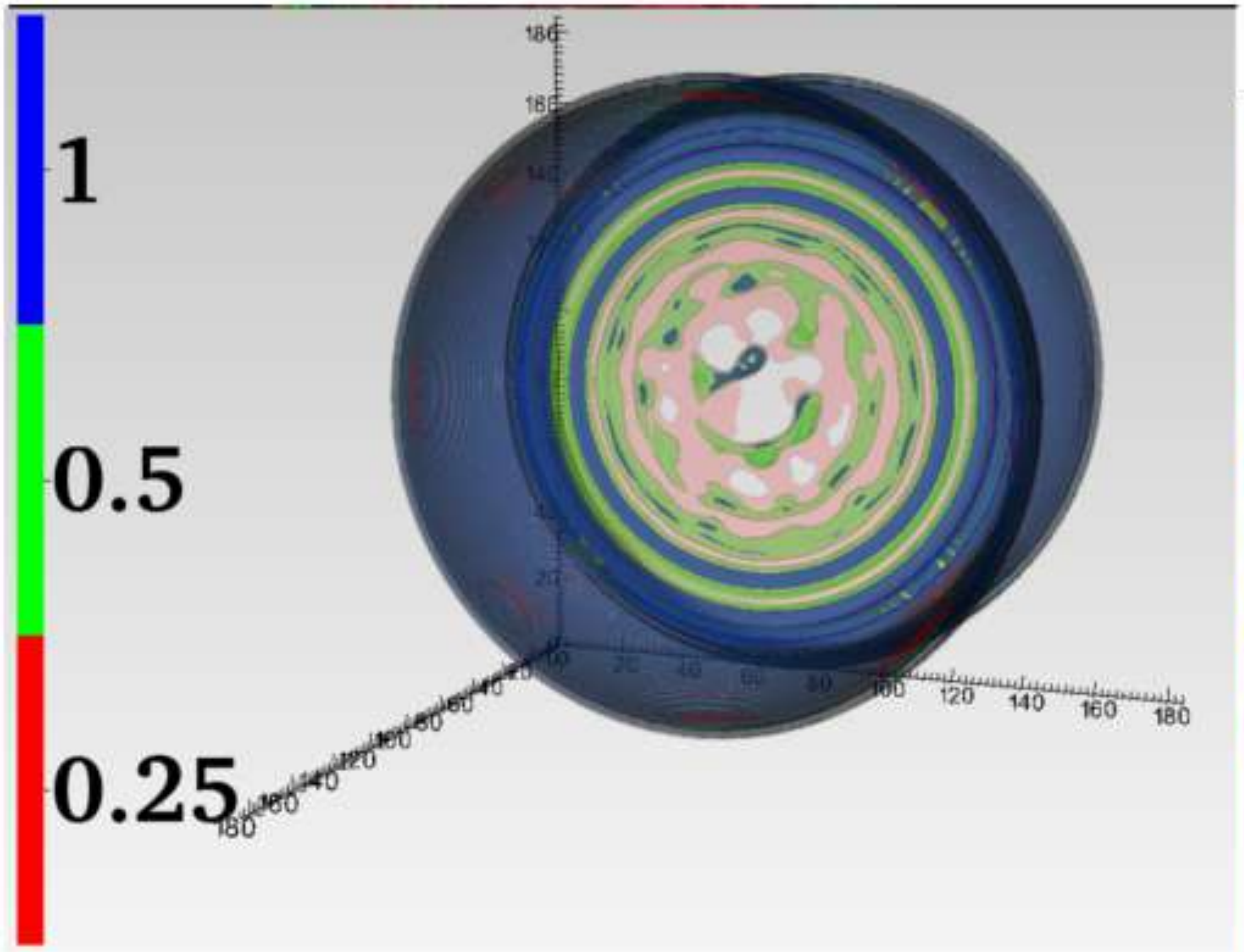


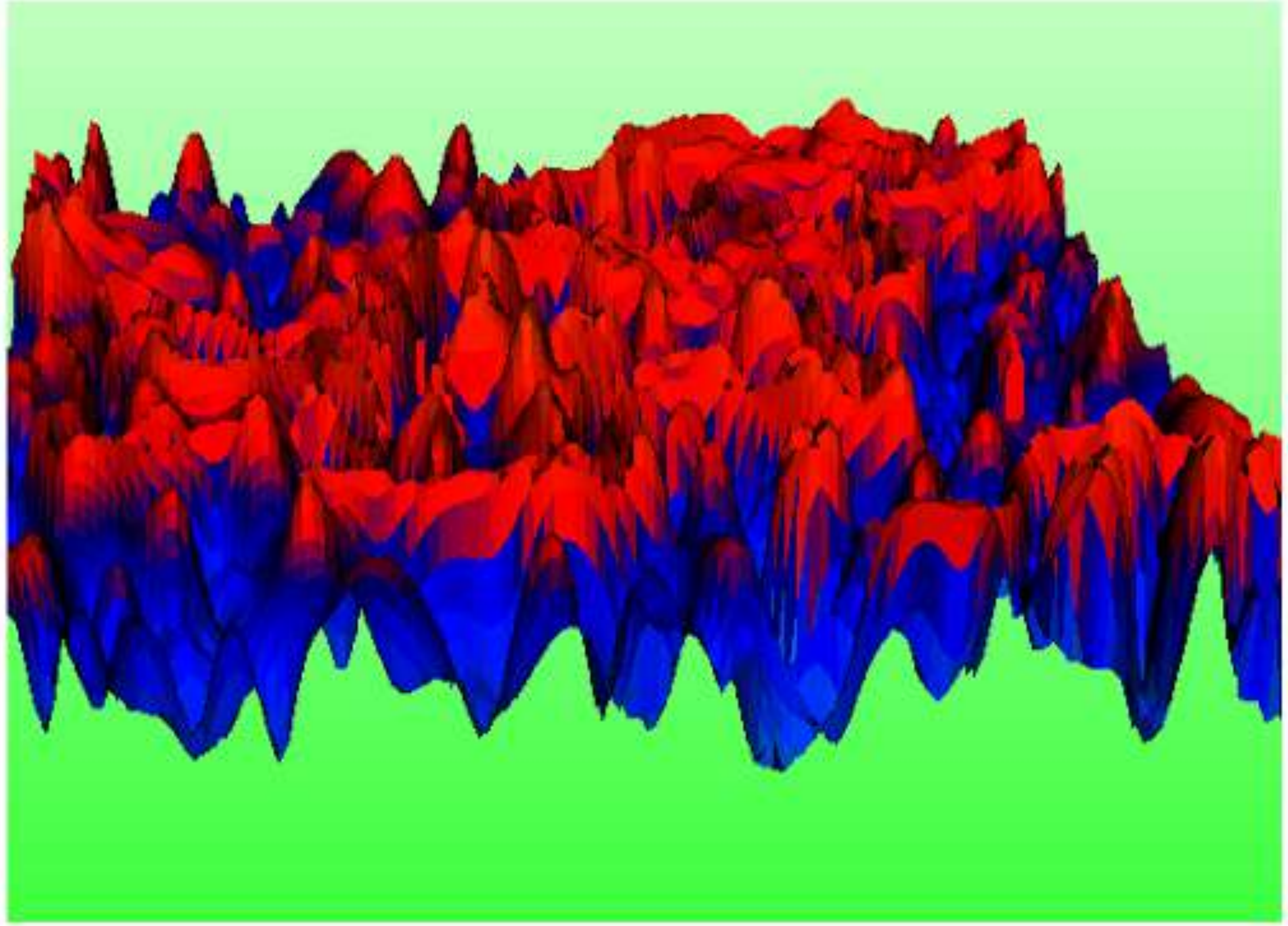


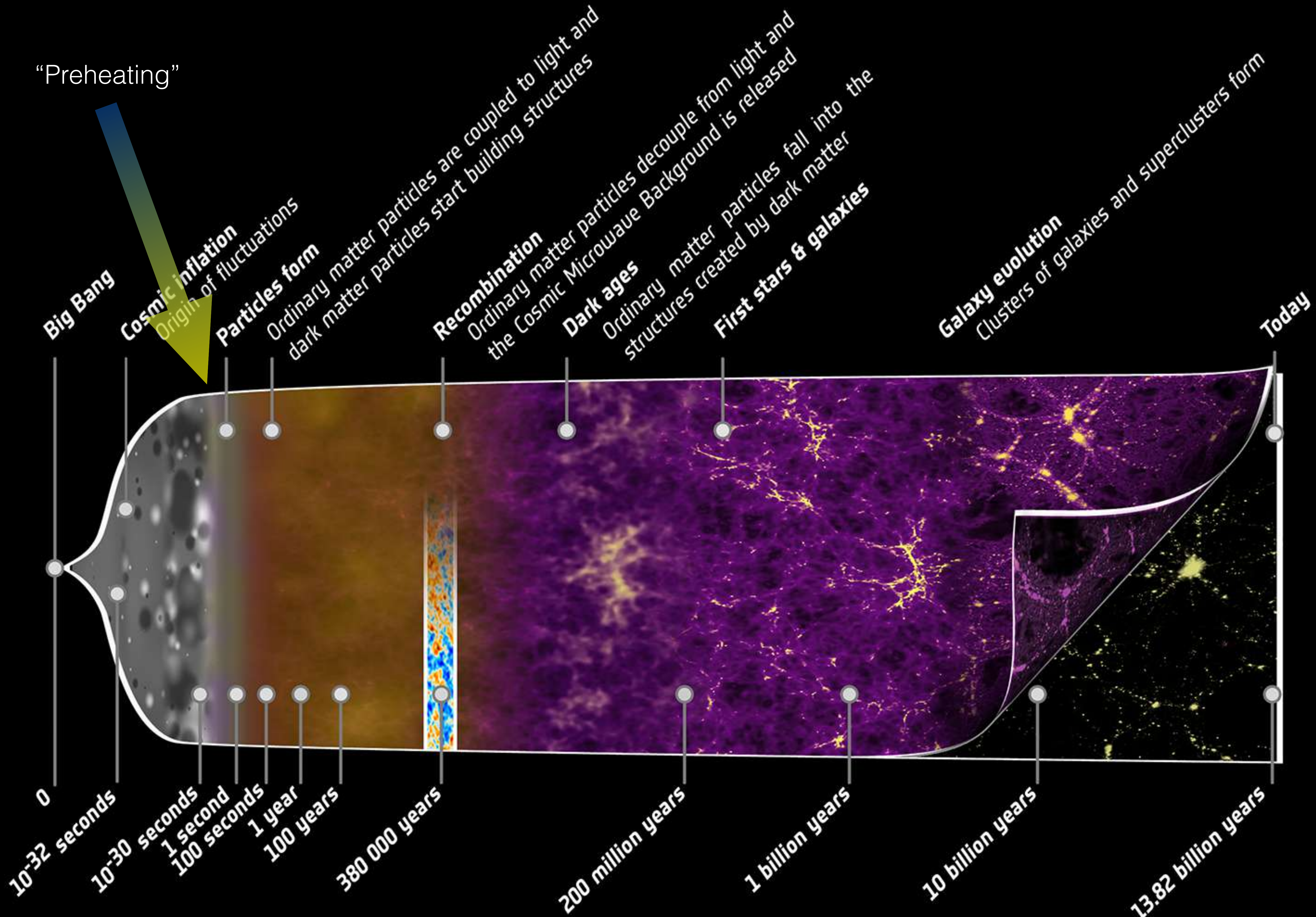


"Preheating"
"Phase Transitions?"
Big Bang
Cosmic inflation
Origin of fluctuations
Particles form
Ordinary matter particles are coupled to light and dark matter particles start building structures
Recombination
Ordinary matter particles decouple from light and the Cosmic Microwave Background is released
Dark ages
Ordinary matter particles fall into the structures created by dark matter
First stars & galaxies
Galaxy evolution
Clusters of galaxies and superclusters form
Today
0
10-32 seconds
10-30 seconds
1 second
100 seconds
1 year
100 years
380 000 years
200 million years
1 billion years
10 billion years
13.82 billion years
"False Vacuum Eternal Inflation"




1
0.5
0.25

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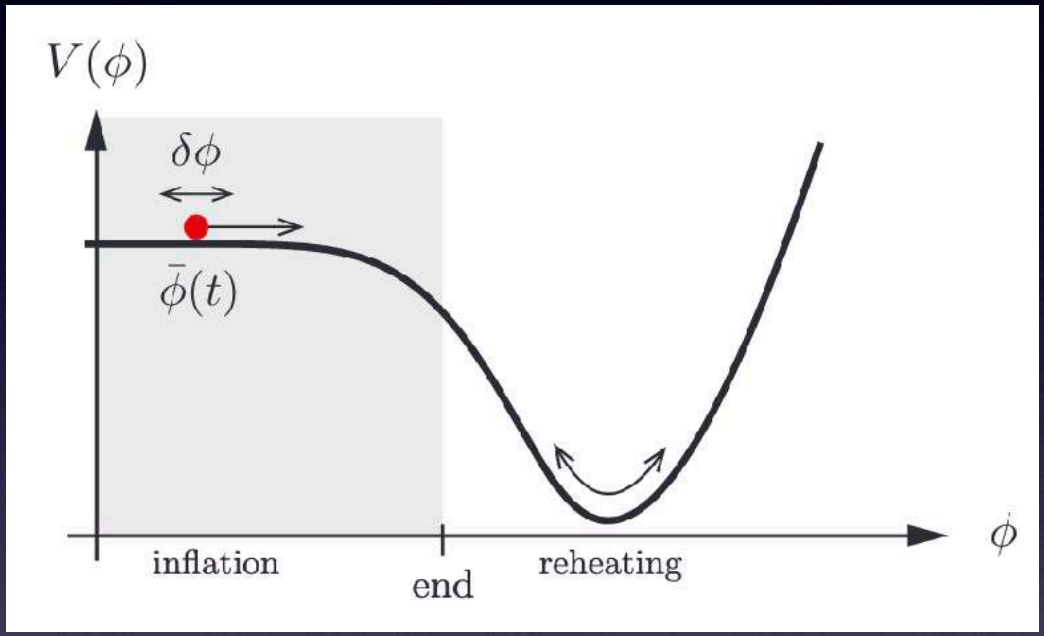
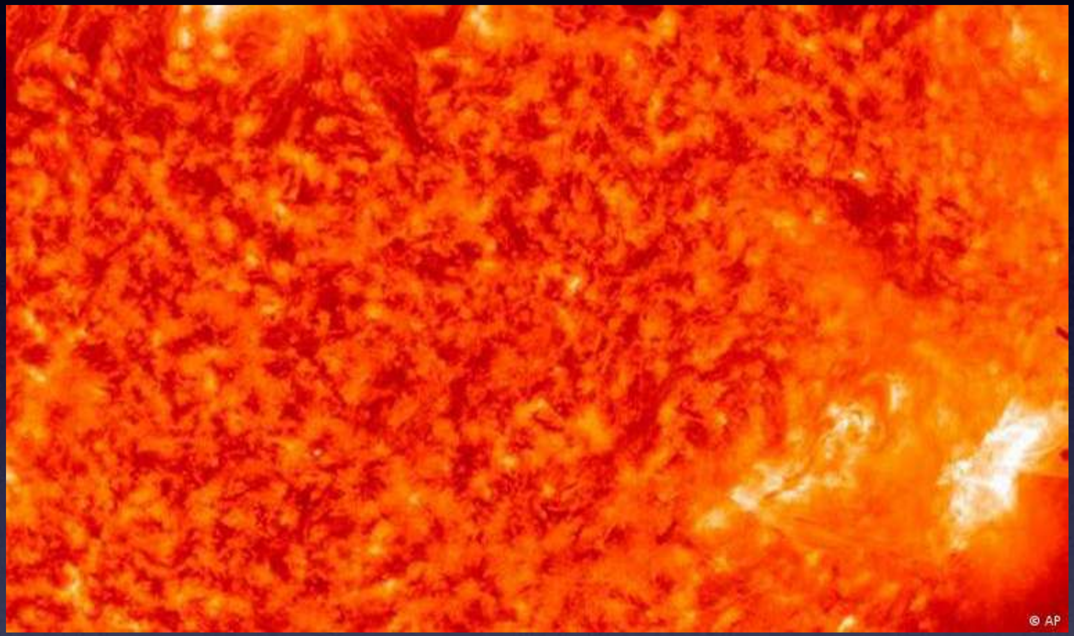
Starting the Big Bang





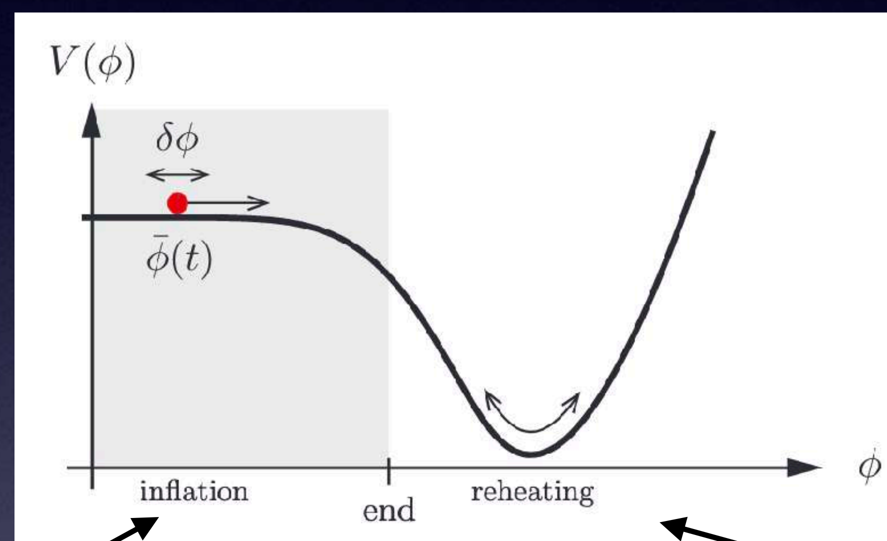
- Cold $T \approx 0$
- $\frac{S}{V} \approx 0$
- Few active d.o.f.

- Hot $(T \gtrsim T_{\mathrm{BBN}} \sim 10\mathrm{MeV})$
- $\frac{S}{V} \propto g_{\mathrm{eff}}(T)T^3$
- Many active d.o.f. (i.e. particles)

Huge production of entropy (information processing)

Theorist's View of the Early U

$$\mathscr{L}_\phi = -\frac{1}{2} G_{IJ} \partial_\mu \phi_I \partial^\mu \phi_J - V(\phi)$$



During Inflation

- Subhorizon homogeneity
- (Small) superhorizon perturbations
- $\phi(x,t) \to \bar{\phi}(t)$

End of Inflation

- $[\delta\hat{\phi}, \delta\hat{\dot{\phi}}] \implies \langle |\delta\phi_k|^2 \rangle, \langle |\delta\dot{\phi}_k|^2 \rangle > 0$
- $\phi(x,t) \to \bar{\phi}(t) + \delta\hat{\phi}(x,t)$
- Variety of instabilities

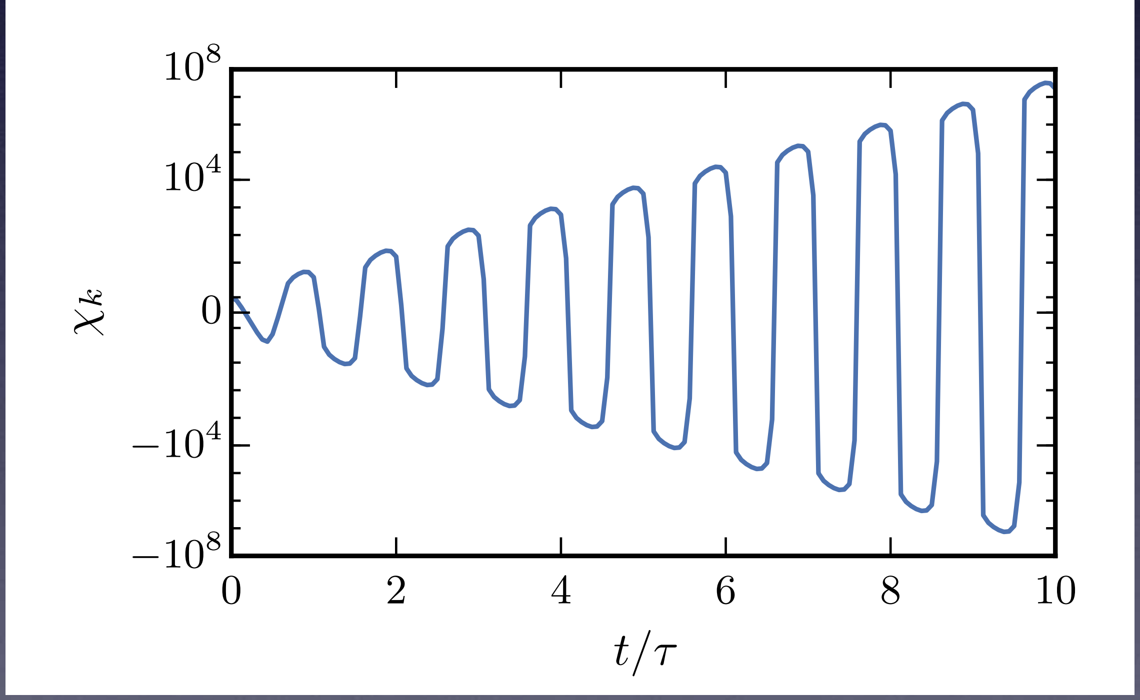
Preheating: Linear Theory

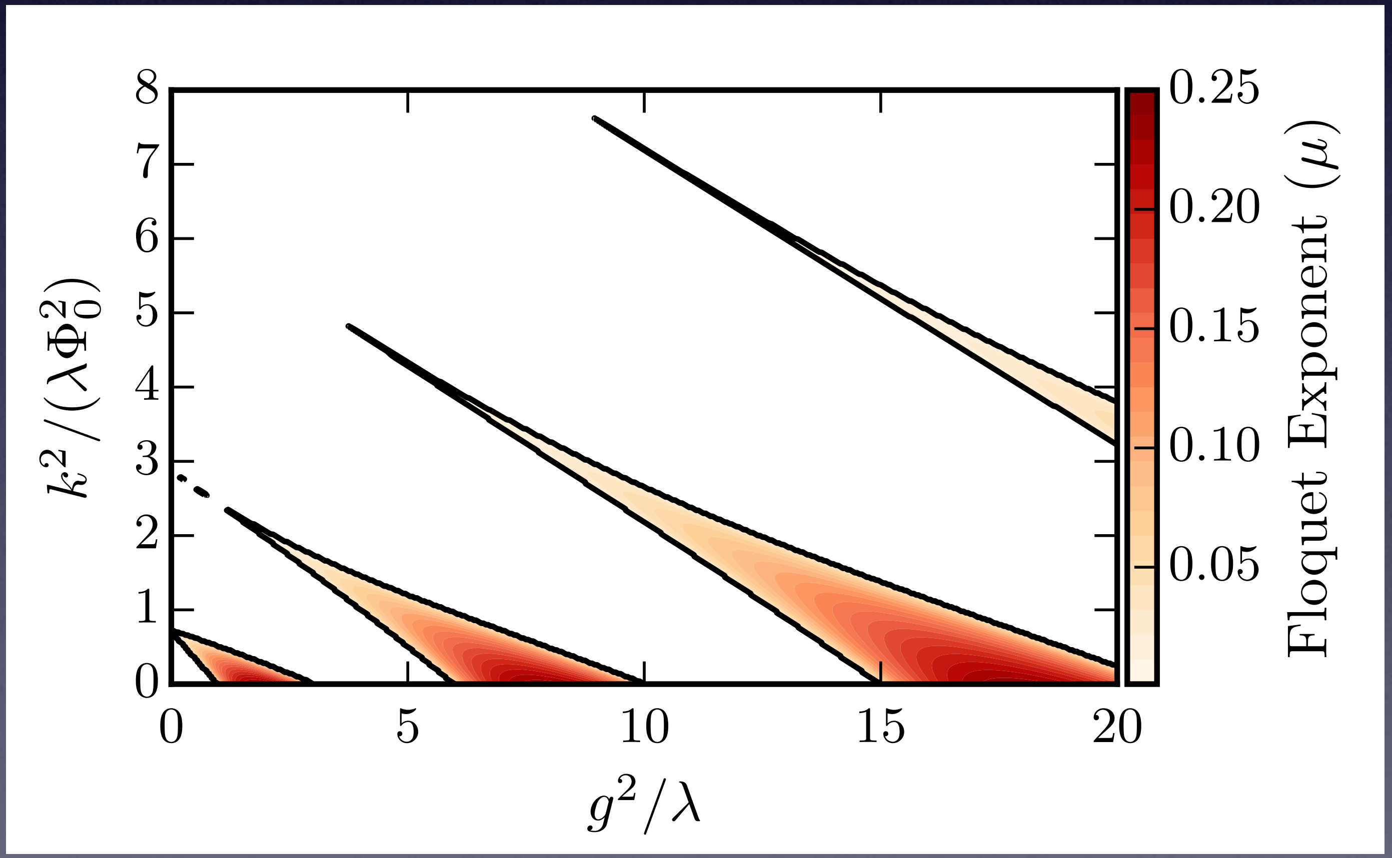
[e.g. Traschen, Bradendberger / Kofman, Linde, Starobinski]

$$\phi = \bar{\phi}(t) + \delta\phi$$

$$V(\phi) = \frac{\lambda}{4}\phi^4$$

$$\delta\ddot{\phi} + \left(k^2 + V''(\bar{\phi})\right)\delta\phi = 0$$





$\bar{\phi}$ acts as external driver

Full treatment includes backreaction and rescattering

Lattice Simulations

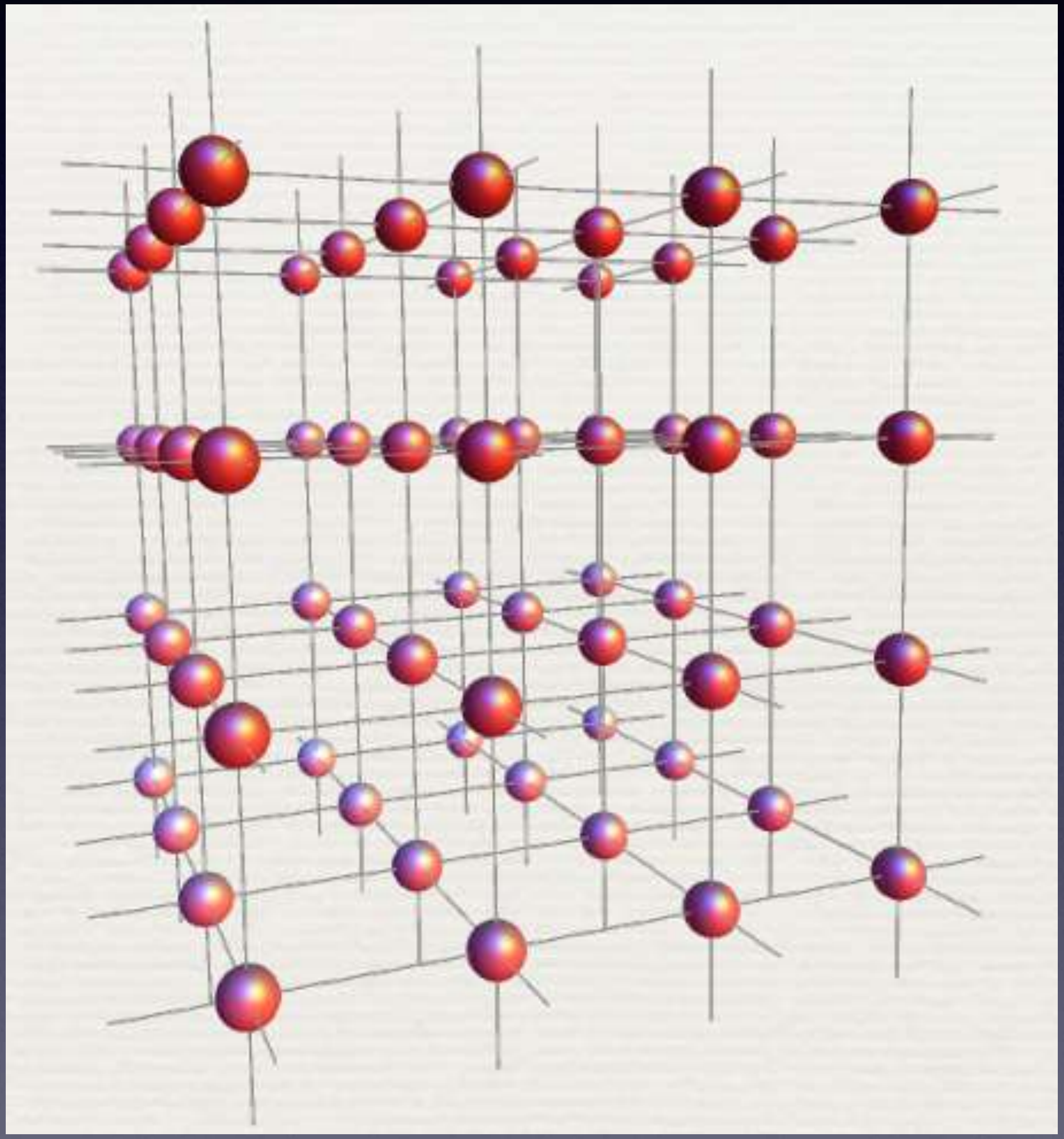
[Braden]

- Solve field equation

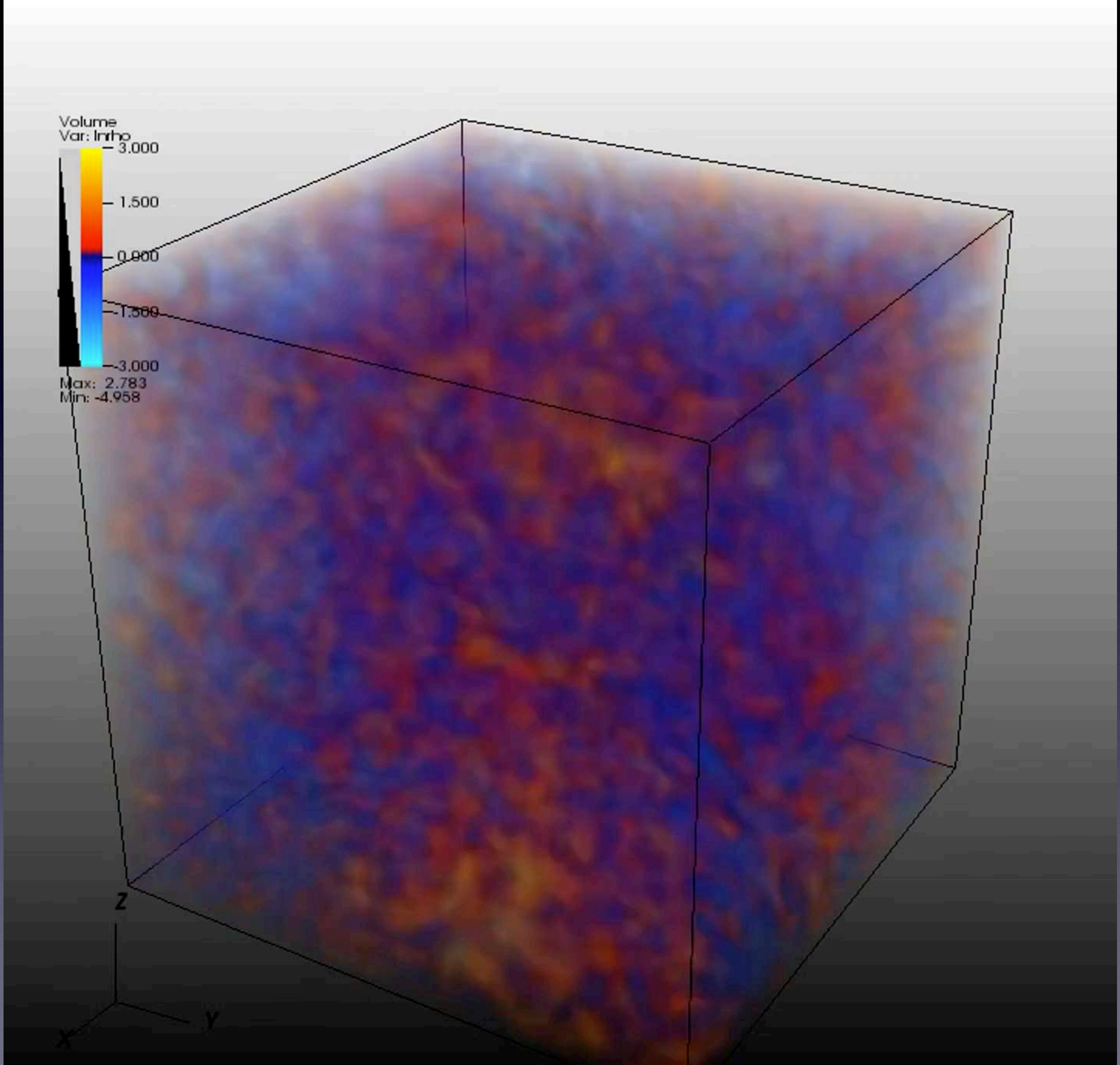
$$\ddot{\phi} + 3H\dot{\phi} + a^{-2}\nabla^2\phi + V'(\phi) = 0$$

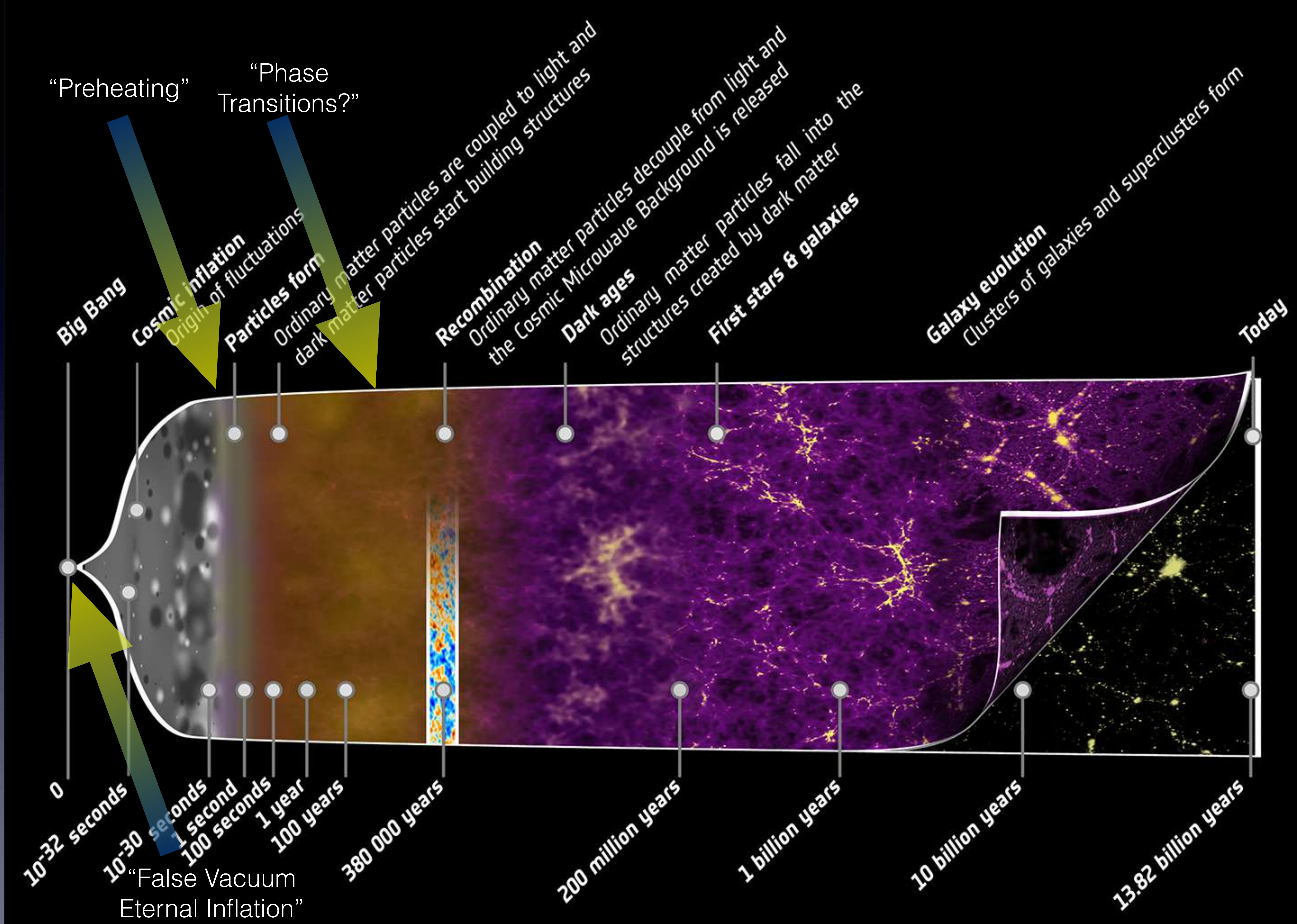
$$H^2 = \frac{\rho}{3M_P^2}$$

- Finite-difference or pseudospectral
- 10th order Gauss-Legendre (general) or 8th order Yoshida (nonlinear sigma model)
- Quantum fluctuations → random field realization



$\mathcal{O}(10^{-15})$ convergence


Volume
Var: lnrho
3.000
1.500
0.000
-1.500
-3.000
Max: 2.783
Min: -4.958
Z
Y
X



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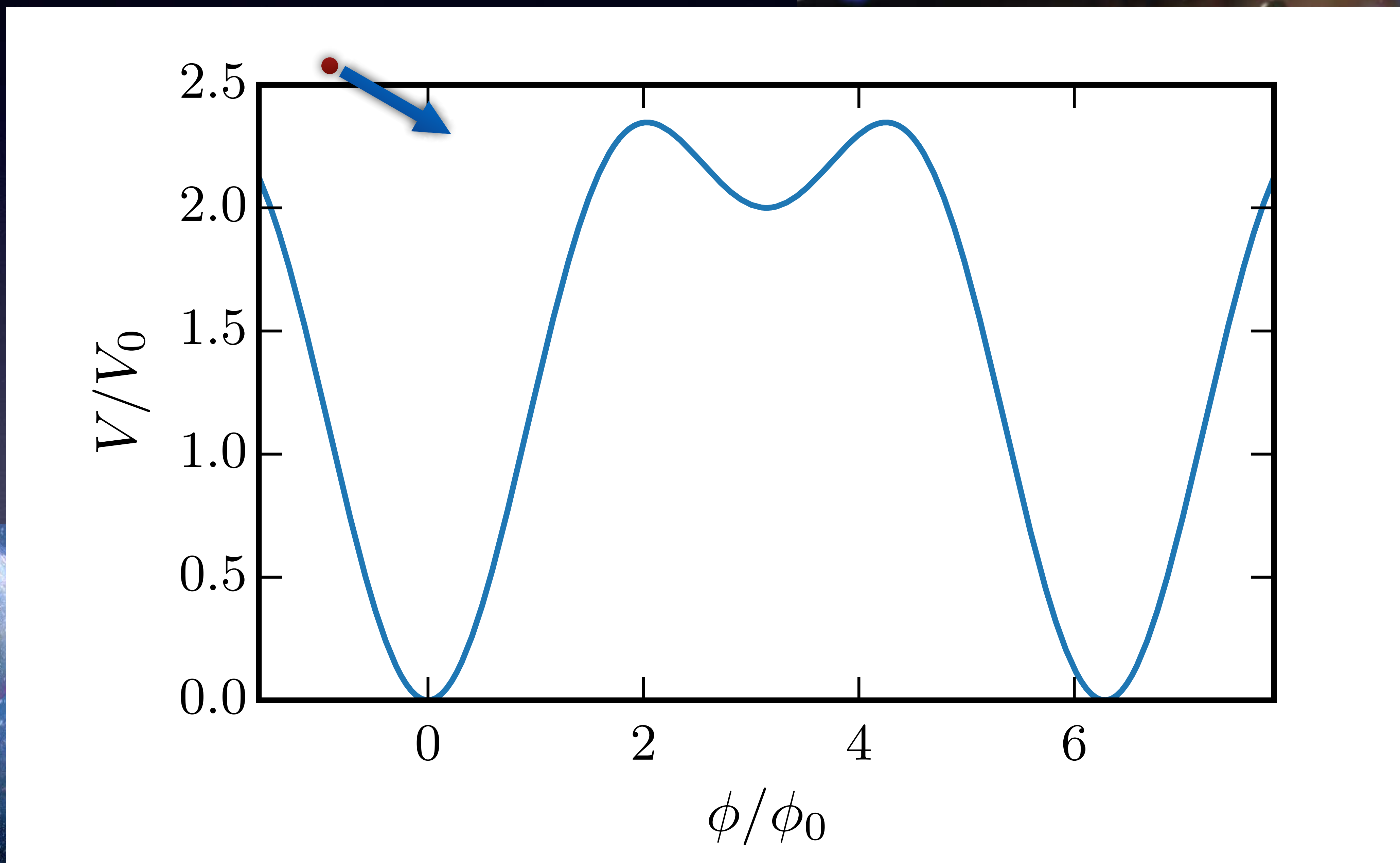
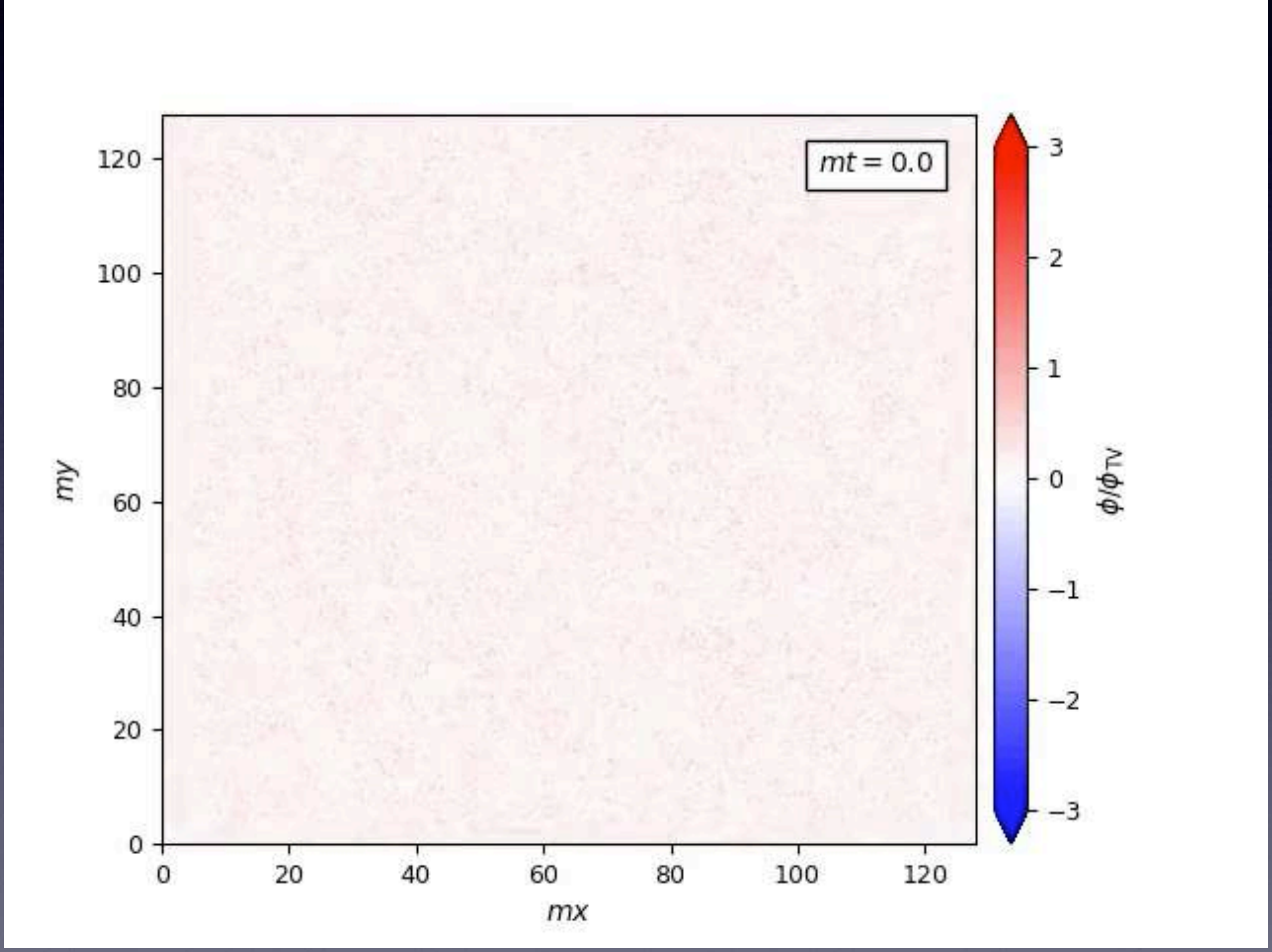
First Order Phase Transitions



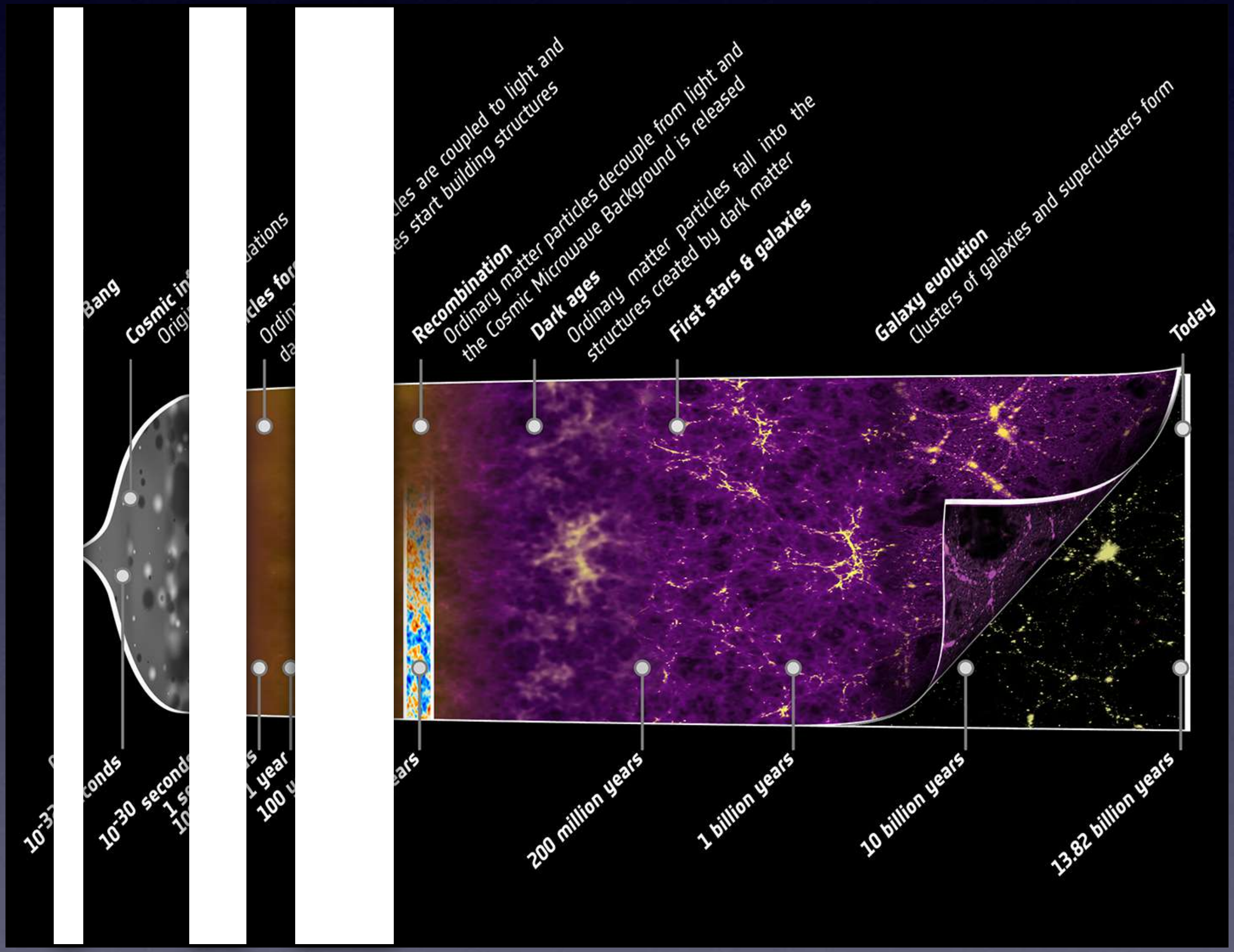
FVD in Action



Why Should I Care

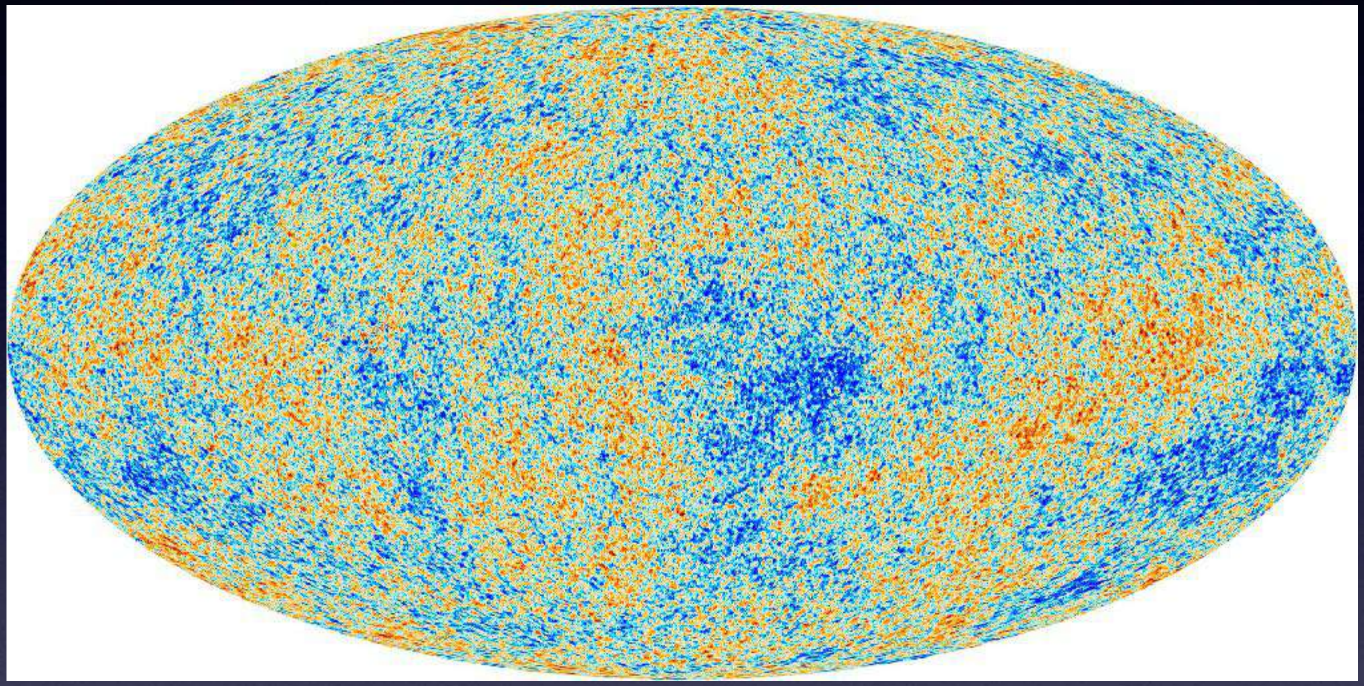
- Nonequilibrium physics is fundamental to nature
    - Intersection of nonlinear QFT and gravity

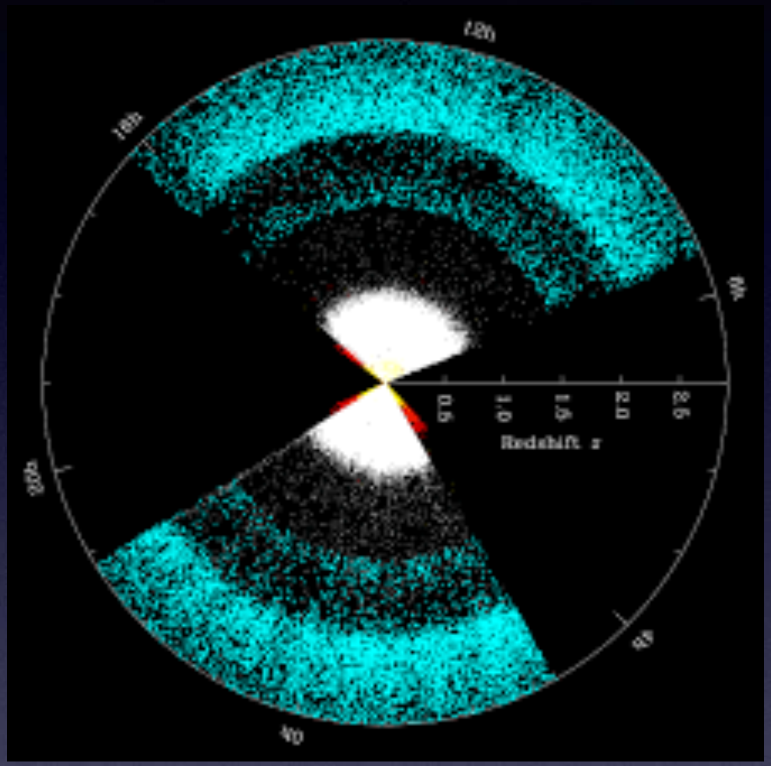
- Modern cosmology is incomplete otherwise



- Observational Considerations

Inflation and Cosmology





Standard Inflation: Models a few parameters

$$P_{\rm s}(k) = A_{\rm s} k^{n_{\rm s}-1}$$

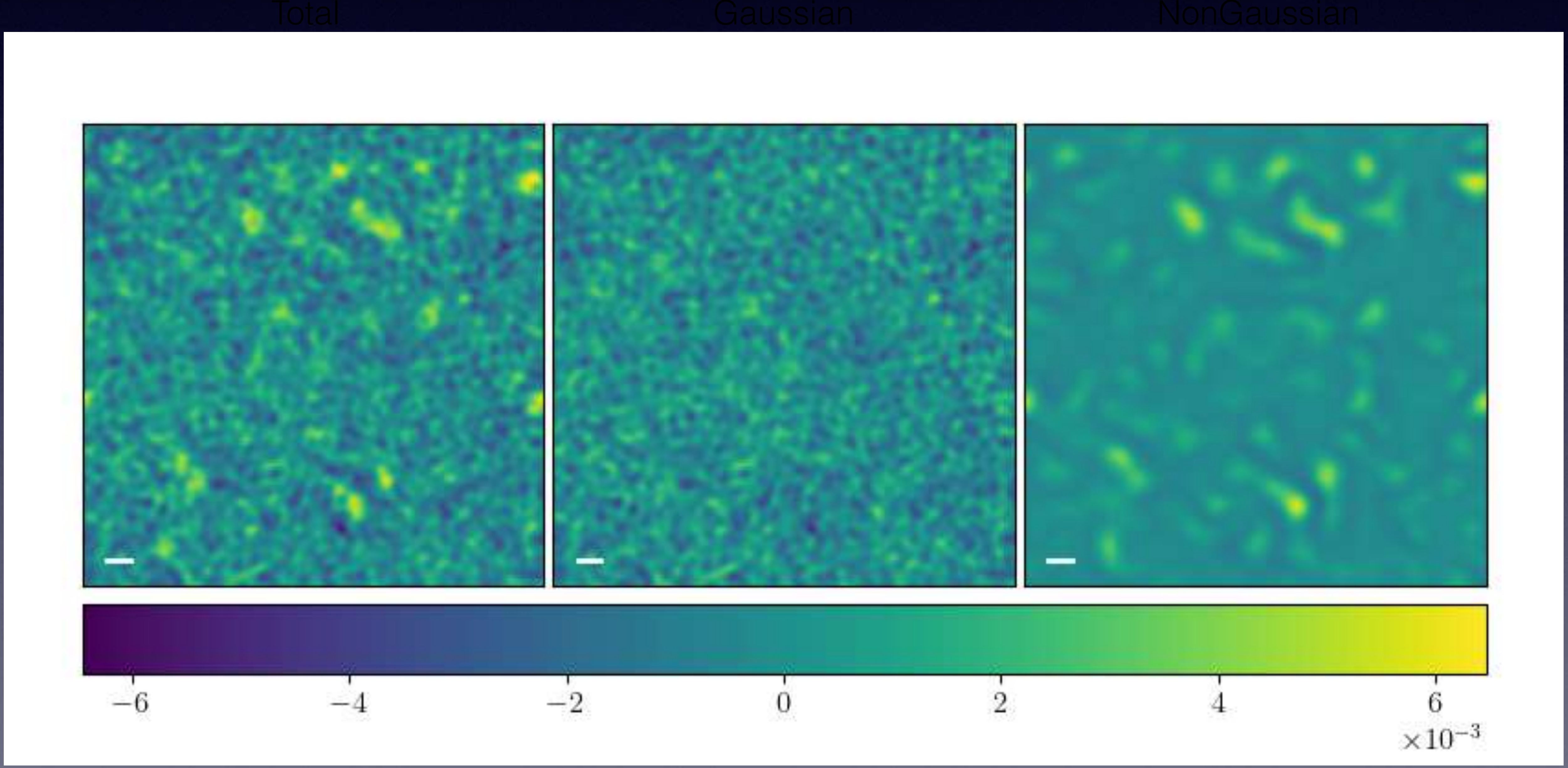
$r = 16\epsilon$

$f_{\rm NL}$ **Perturbative NonGaussianity**

What else can we use the data for?

Early U Nonlinearity Imprints Novel Density Fluctuations

$$\zeta = \zeta_{\mathrm{G}} + F_{\mathrm{NL}}(\chi)$$

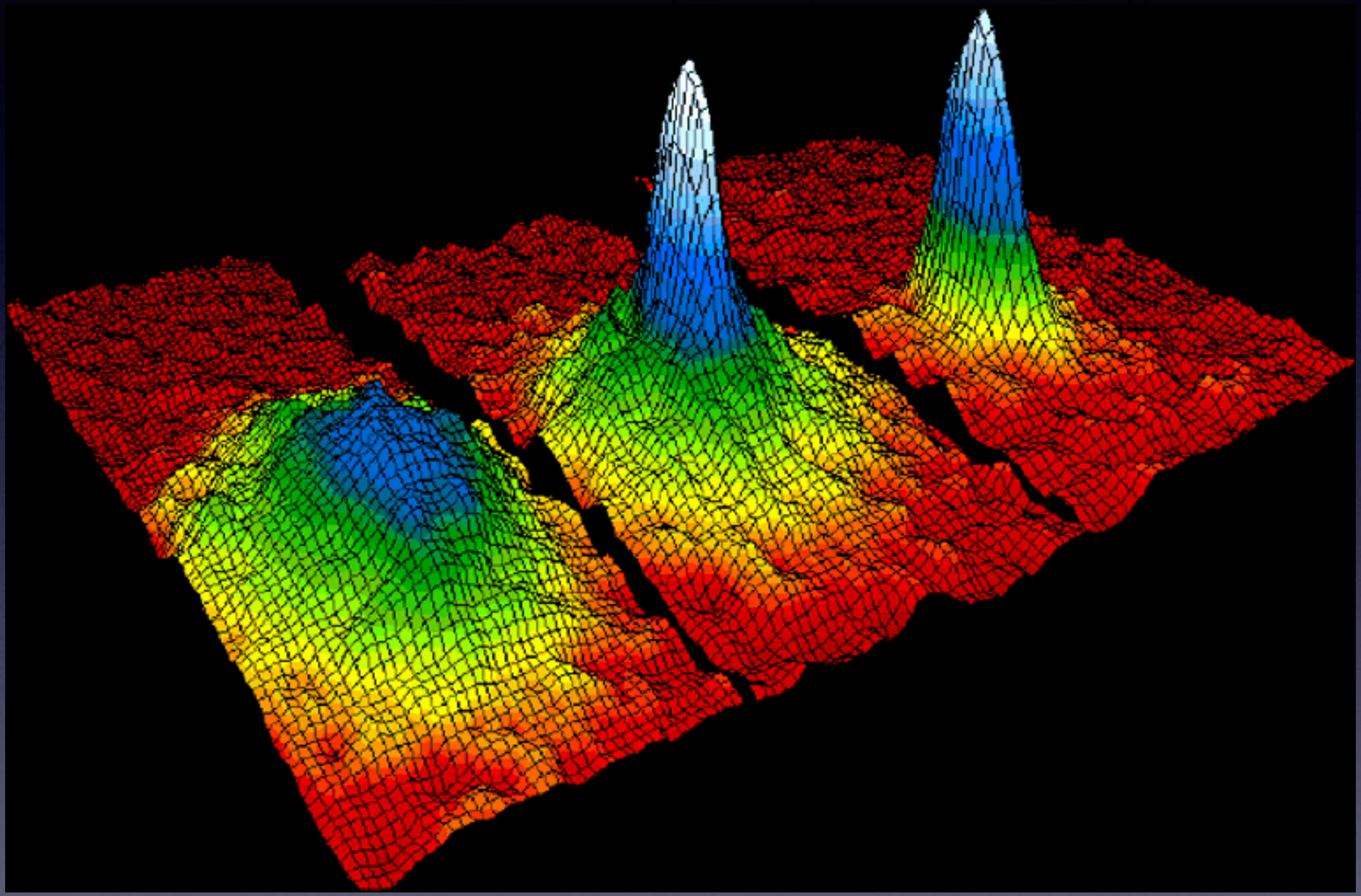


[Bond, Braden, Morrison]

Nonlinear, Nonperturbative, Nonequilibrium Quantum Field Theory (coupled to gravity)

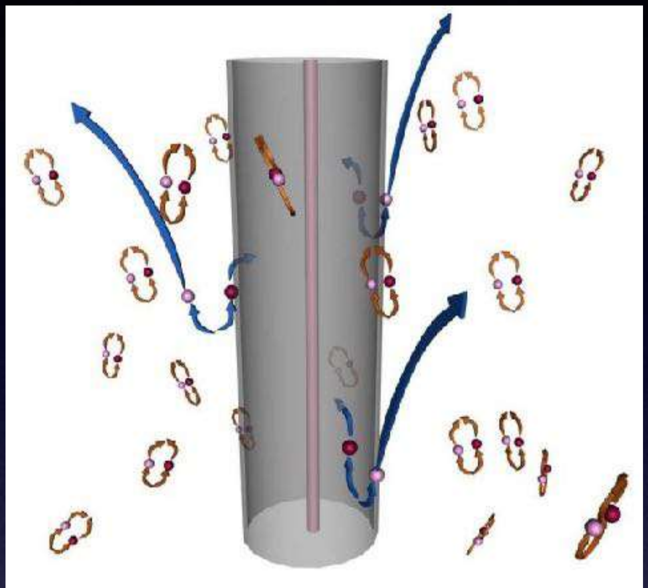
Our understanding of (p)reheating
and false vacuum decay rests
on reasonable but experimentally untested
approximations to non equilibrium QFT

Analogue Systems?

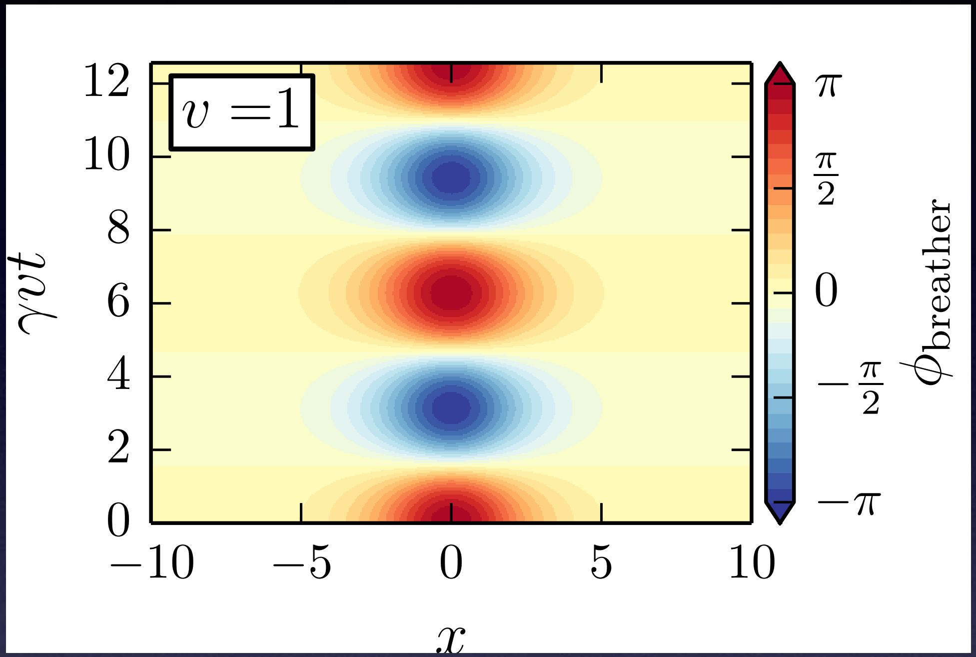


Bose-Einstein Condensates

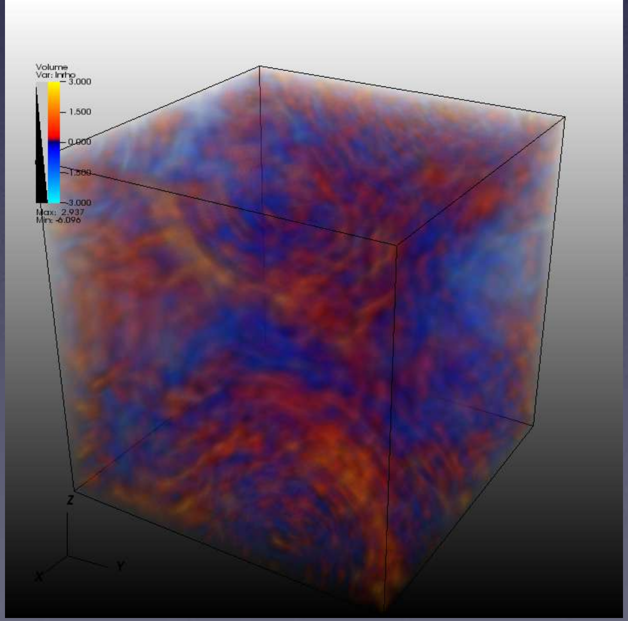
Analogue Systems



Hawking Radiation
(Linear, Quantum)

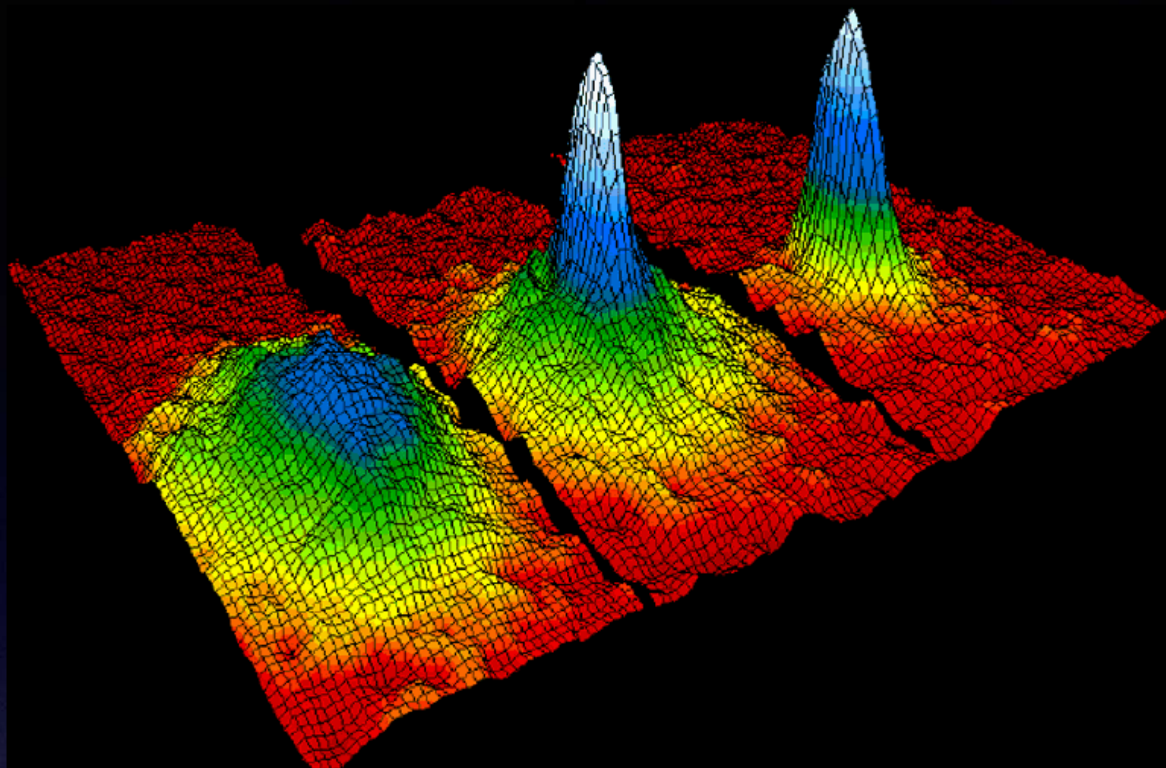


Solitons
(Nonlinear, Classical)

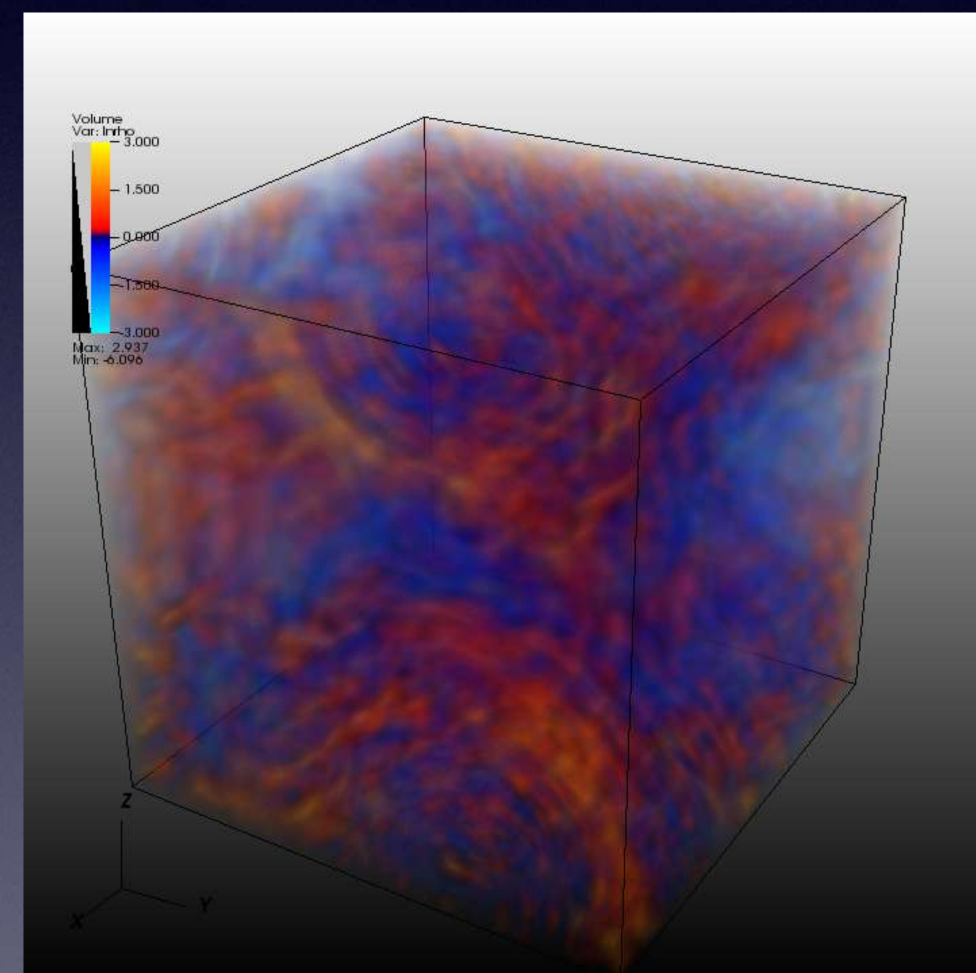


Preheating
(Linear, Quantum) ->
(Nonlinear, Classical)

Bubble Nucleation
(Nonlinear, Quantum)



$$\mathcal{L}_{\text{eff}} \sim G(\phi)\frac{\dot{\phi}^2}{2} - c_s^2\frac{(\nabla\phi)^2}{2} + \nu\Lambda\cos\phi + \ldots$$



Finite number effects

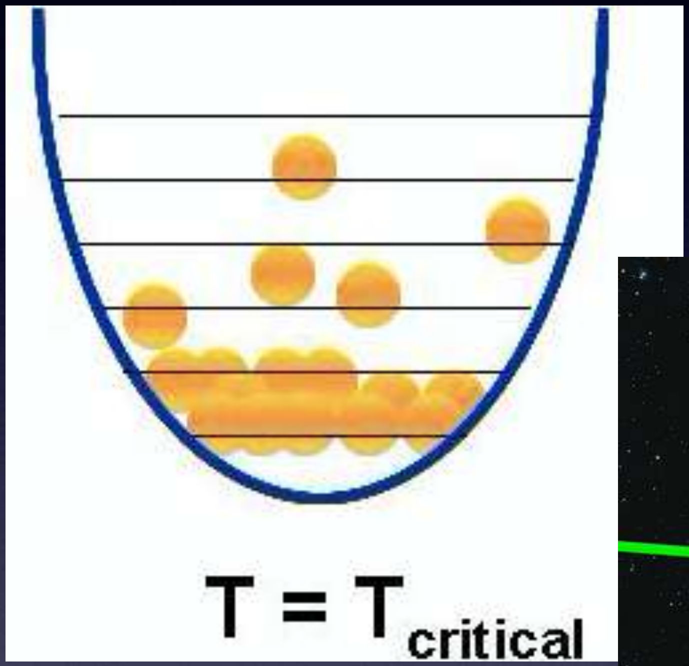
Trapping Potential

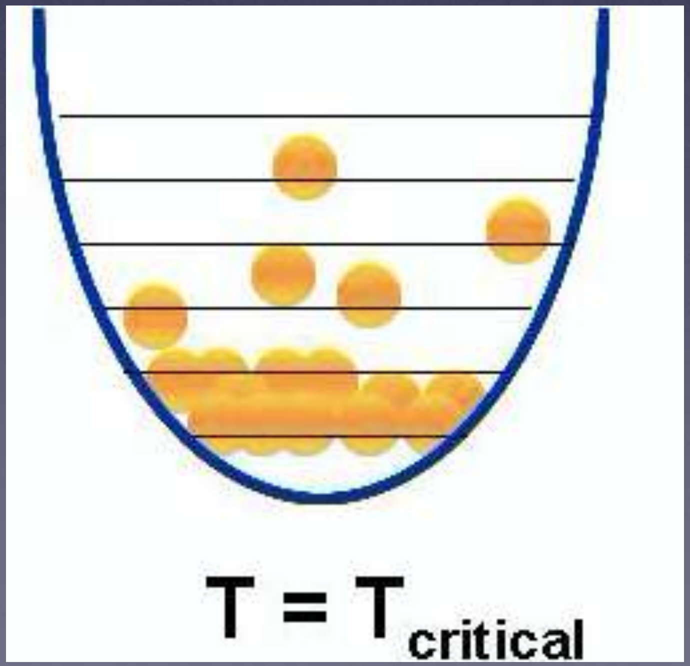
Finite Temperature

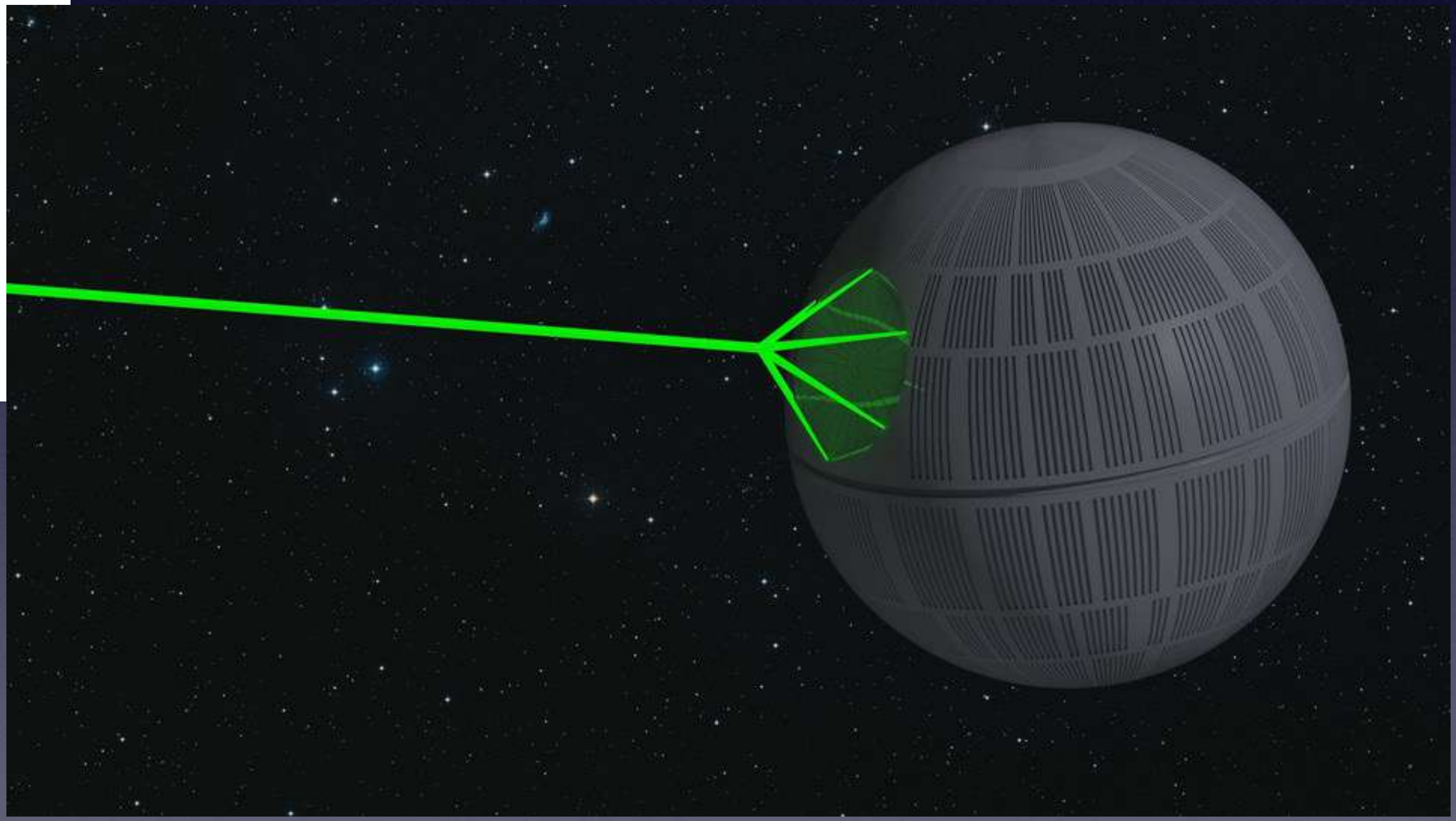
$$i\hbar\dot{\psi}_i = \left(-\delta_{ij}\frac{\hbar^2}{2m_i}\nabla^2 + V(\mathbf{x}) + g_{ij}|\psi_j|^2\right)\psi_i - \nu_{ij}\psi_j$$

Building an Analogue System

[Fialko, Opanchuk, Sidorov, Drummond, and Brand]
[**JB**, Johnson, Peiris, Weinfurtner]
[Billam, Gregory, Michel, and Moss]







Important Scale : Healing Length
Crossover between wave and particle dispersion relationship

BECs and Relativity

$$\psi_i = \sqrt{\rho_i} e^{-i\phi_i}$$

Can. Momentum
(Particle density)

Can. Position
(Complex phase)

Assumptions

$$\rho_i(x,t) = n_i + \delta\rho_i(x,t)$$

1) Homogeneous

2) Small

Useful limit $\tilde{\nu} \equiv \dfrac{\nu}{g\bar{n}} \ll 1$

First Hint: Relativistic Dispersion for $k < k_{\text{heal}}$

$$\hbar^2\omega^2 = m^2 + c^2k^2\left(1 + \frac{k^2}{k_{\text{heal}}^2}\right)$$

$$k_{\text{heal}}^2 = \frac{4mg\bar{n}}{\hbar^2} = 4\frac{m}{g\bar{n}}\frac{g^2\bar{n}^2}{\hbar^2}$$

Small Density Fluc ations

Preheating Dynamics

A convenient variable is $\varphi = \phi_2 - \phi$

Integrate out fluctuations in numb $Z_{\text{eff}} \propto \int d\phi e^{i\mathcal{L}_{\text{eff}}}$

Relative phas by sine-Gordon model

$$\mathscr{L}_{\text{eff}} \ldots \frac{(\nabla\varphi)^2}{2} + \nu\Lambda\cos\varphi + \ldots$$

$c_s^2 \approx \frac{g\ldots}{m}$ $\quad m_\varphi \approx \sqrt{\tilde{\nu}}\frac{2g\bar{n}}{\hbar}$ $\quad L_\varphi = \frac{c_s}{m_\varphi} \sim \frac{L_{\text{heal}}}{\sqrt{\tilde{\nu}}}$

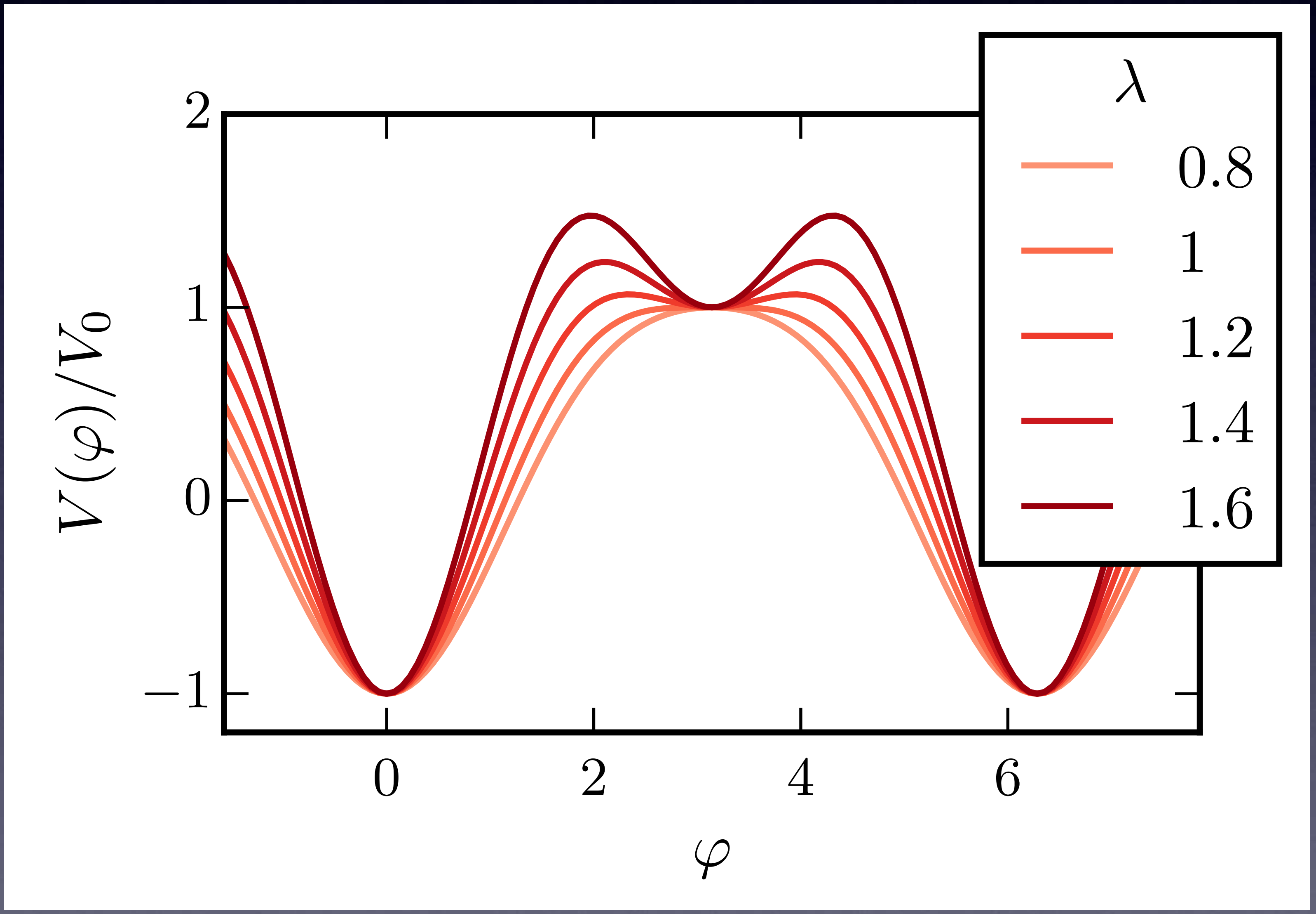
A Small Detour: False Vacuum Decay

Modulate Transition Rate

$$\nu = \nu_0 + \delta\hbar\omega\cos(\omega t)$$

Time Averaged Potential

$$\lambda = \delta \left(\frac{2g\bar{n}}{\nu_0} \right)^{1/2}$$



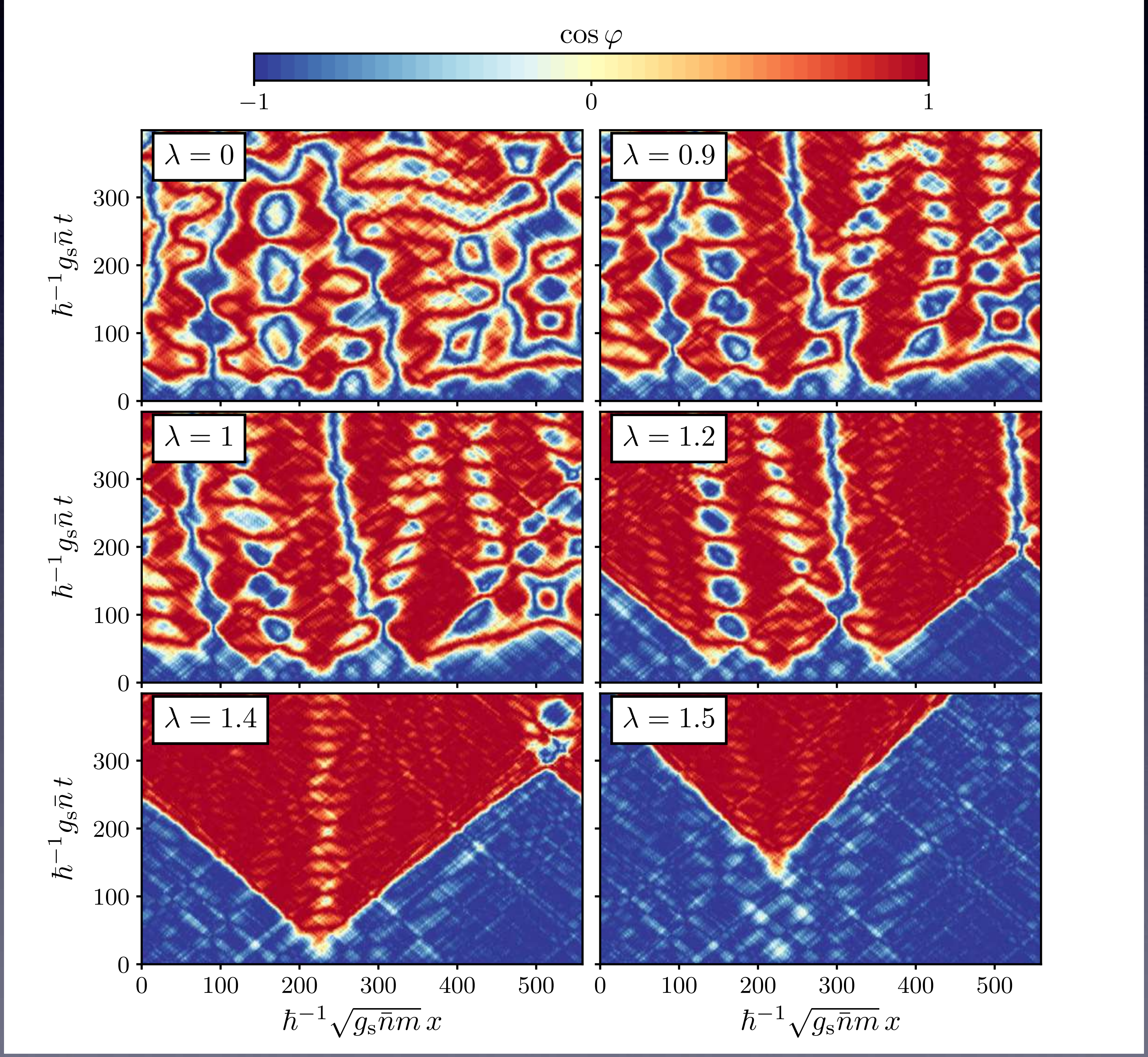
$$V(\phi) = V_0 \left(-\cos\left(\frac{\phi}{\phi_0}\right) + \frac{\lambda^2}{2}\sin^2\left(\frac{\phi}{\phi_0}\right) + 1 \right)$$

BEC Evolution

2nd-Order Phase Transition

Rapid 1st-Order Phase Transition

Slower 1st-Order Phase Transition



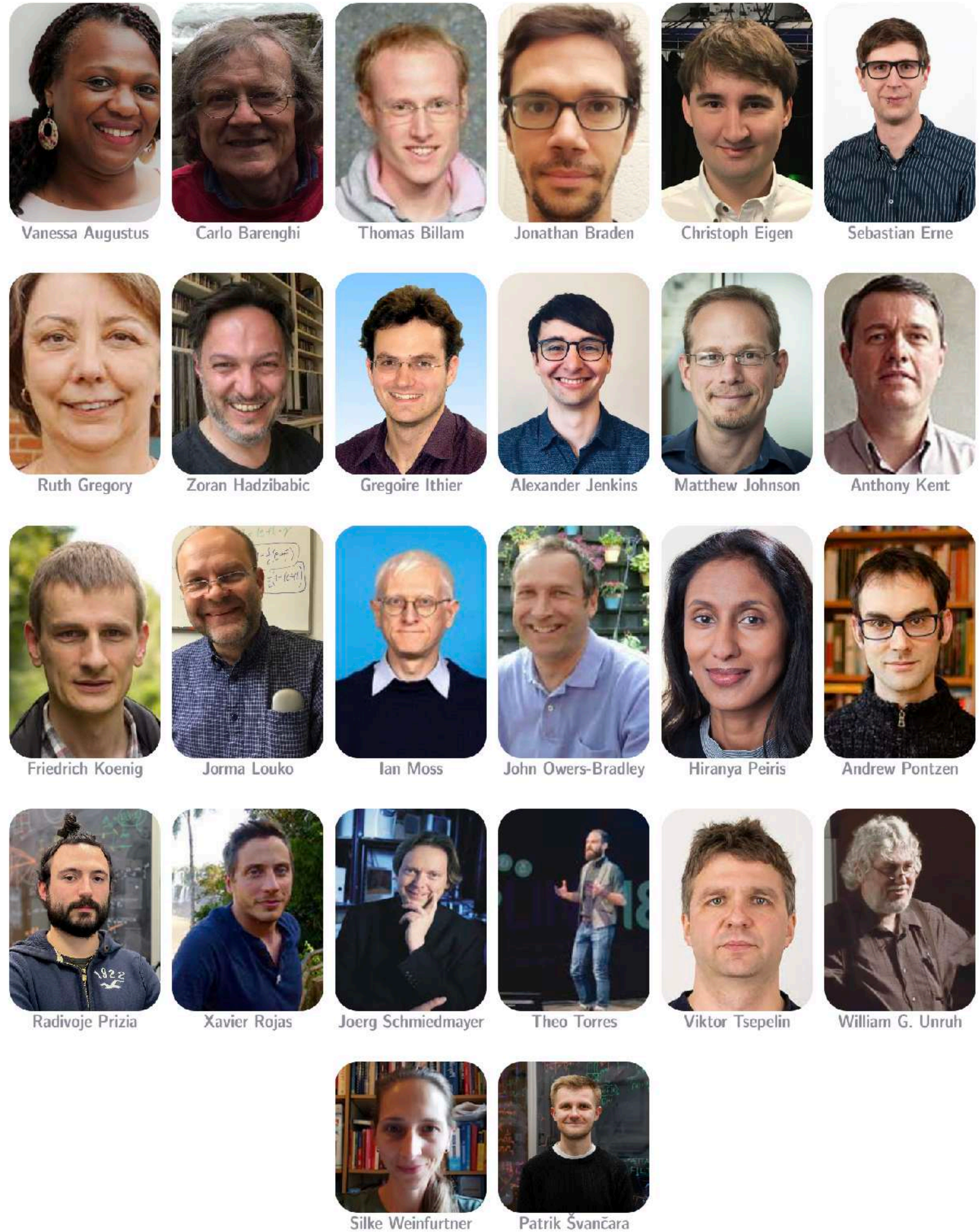
Increase Depth of Minimum

QSimFP

Quantum Simulators for Fundamental Physics

Building an analog FVD experiment @ Cambridge

More info at www.qsimfp.org



Back to our Regularly Scheduled Programming: Preheating

Small Density Fluctuations

A convenient variable is $\varphi = \phi_2 - \phi_1$

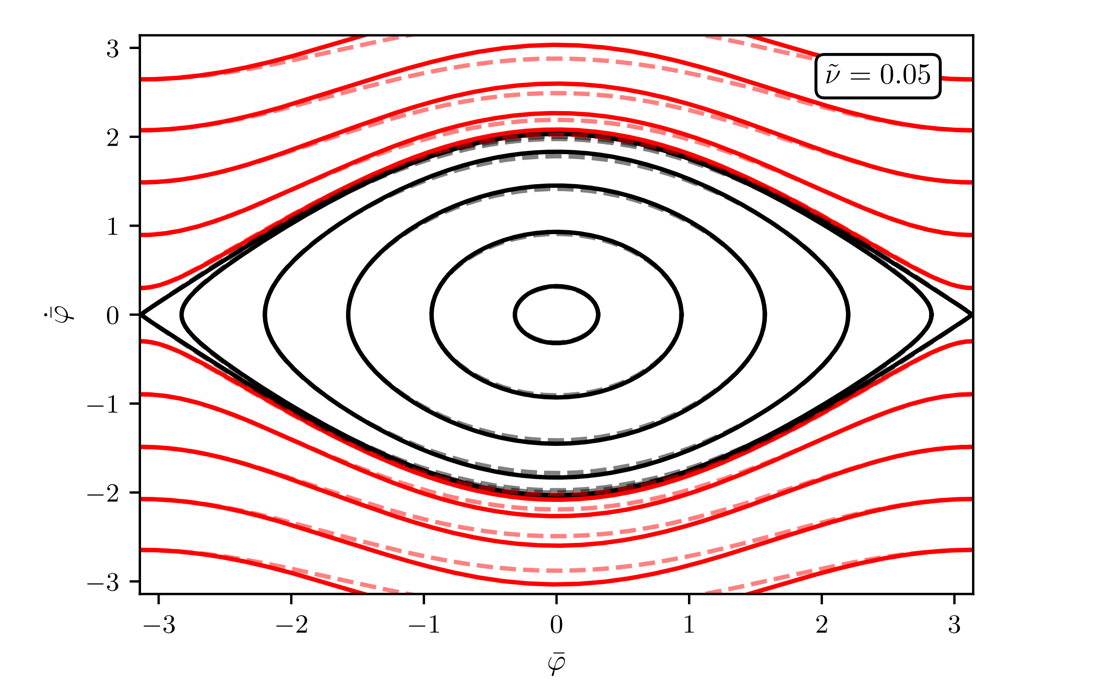
Integrate out fluctuations in number density $Z_{\text{eff}} \propto \int d\phi e^{i\mathcal{L}_{\text{eff}}}$

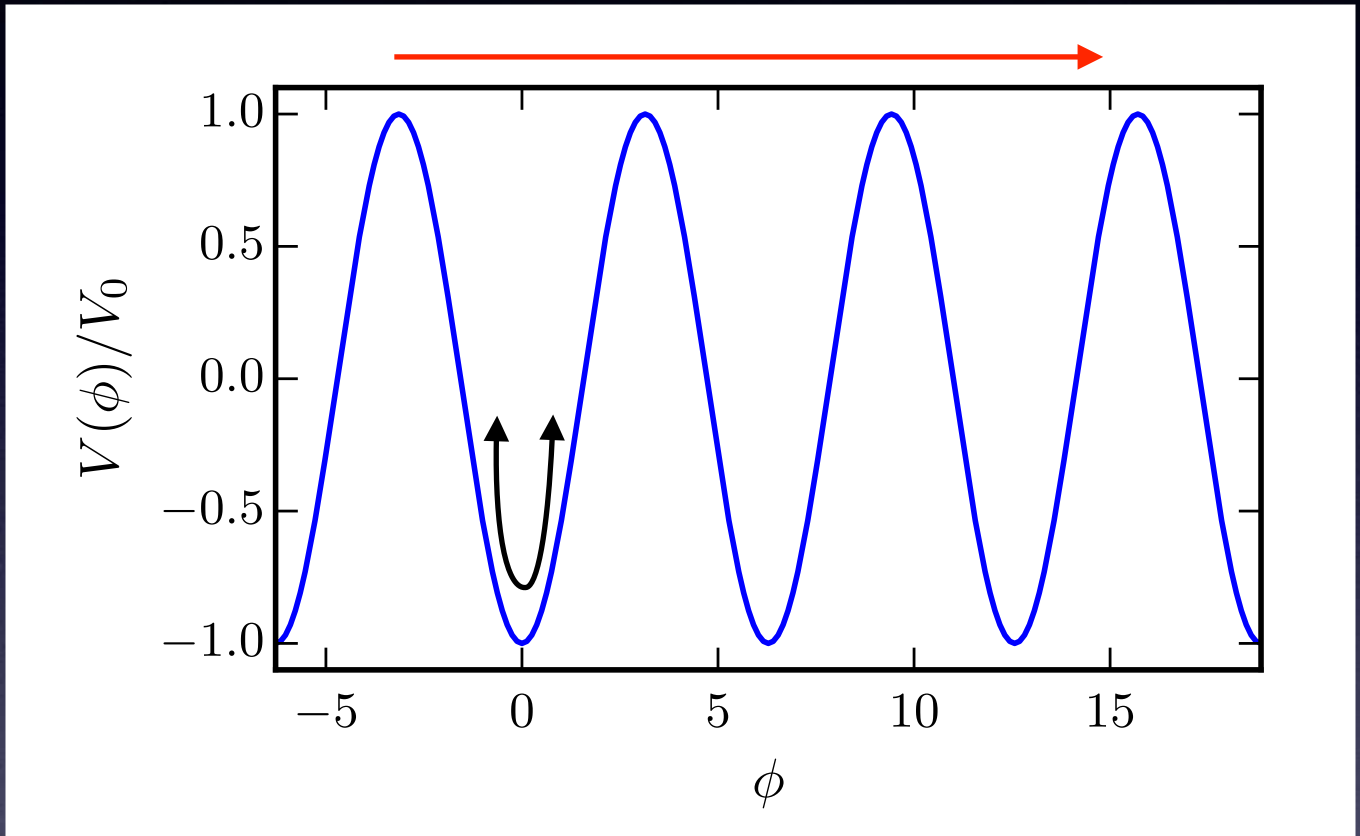
Relative phase governed by sine-Gordon model

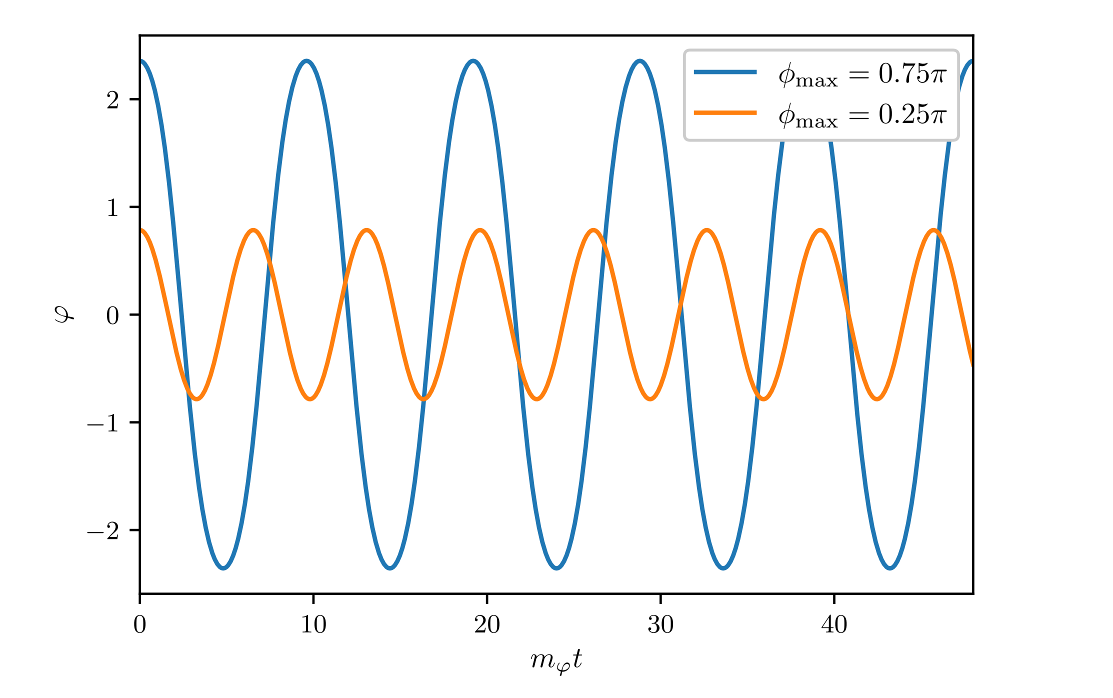
$$\mathscr{L}_{\text{eff}} \sim \frac{\dot{\varphi}^2}{2} - c_s^2 \frac{(\nabla \varphi)^2}{2} + \nu \Lambda \cos \varphi + \ldots$$

$$c_s^2 \approx \frac{g\bar{n}}{m} \qquad m_\varphi \approx \sqrt{\tilde{\nu}} \frac{2g\bar{n}}{\hbar} \qquad L_\varphi = \frac{c_s}{m_\varphi} \sim \frac{L_{\text{heal}}}{\sqrt{\tilde{\nu}}}$$

Homogeneous Background Evolution







Two periodic motion types

Black - single minimum

Red - scan minima

Perturbation Equations

$$\vartheta = \theta_1 + \theta_2 \qquad \varphi = \phi_1 - \phi_2 \qquad \Pi_\vartheta = \frac{\rho_1 + \rho_2}{2\bar{n}\sqrt{\tilde{\nu}}} \qquad \Pi_\varphi = \frac{\rho_2 - \rho_1}{2\bar{n}\sqrt{\tilde{\nu}}}$$

$$\psi_i(x, t) = \bar{\psi}_i(t) + \delta\psi_i(x, t)$$

$$\begin{aligned}
\frac{\mathrm{d}\delta\Pi_\vartheta}{\mathrm{d}\tilde{t}} =& \kappa^2 \left(\delta\vartheta + \sqrt{\tilde{\nu}}\bar{\Pi}\delta\varphi\right) \\
\frac{\mathrm{d}\delta\vartheta}{\mathrm{d}\tilde{t}} =& -\frac{\tilde{\nu}}{1-\tilde{\nu}\bar{\Pi}^2}\kappa^2\left(\delta\Pi_\vartheta - \sqrt{\tilde{\nu}}\bar{\Pi}\delta\Pi_\varphi\right) - \delta\Pi_\vartheta \\
& - \sqrt{\frac{\tilde{\nu}}{1-\tilde{\nu}\bar{\Pi}^2}}\sin\bar{\varphi}\delta\varphi + \left(\frac{\tilde{\nu}}{1-\tilde{\nu}\bar{\Pi}^2}\right)^{3/2}\bar{\Pi}\cos\bar{\varphi}\left(\delta\Pi_\varphi - \sqrt{\tilde{\nu}}\bar{\Pi}\delta\Pi_\vartheta\right) \\
\frac{\mathrm{d}\Pi_\varphi}{\mathrm{d}\tilde{t}} =& \kappa^2\left(\delta\varphi + \sqrt{\tilde{\nu}}\bar{\Pi}\delta\vartheta\right) \\
& + \cos\bar{\varphi}\sqrt{1-\tilde{\nu}\bar{\Pi}^2}\delta\varphi + \frac{\sqrt{\tilde{\nu}}\sin\bar{\varphi}}{\sqrt{1-\tilde{\nu}\bar{\Pi}^2}}\left(\delta\Pi_\vartheta - \sqrt{\tilde{\nu}}\bar{\Pi}\delta\Pi_\varphi\right) \\
\frac{\mathrm{d}\delta\varphi}{\mathrm{d}\tilde{t}} =& -\frac{\tilde{\nu}}{1-\tilde{\nu}\bar{\Pi}^2}\kappa^2\left(\delta\Pi_\varphi - \sqrt{\tilde{\nu}}\bar{\Pi}\delta\Pi_\vartheta\right) - \delta\Pi_\varphi \\
& + \frac{\tilde{\nu}\bar{\Pi}_\varphi\sin\bar{\varphi}}{\sqrt{1-\tilde{\nu}\bar{\Pi}^2}}\delta\varphi - \frac{\tilde{\nu}\cos\bar{\varphi}}{(1-\tilde{\nu}\bar{\Pi}^2)^{3/2}}\left(\delta\Pi_\varphi - \sqrt{\tilde{\nu}}\bar{\Pi}\delta\Pi_\vartheta\right) .
\end{aligned}$$

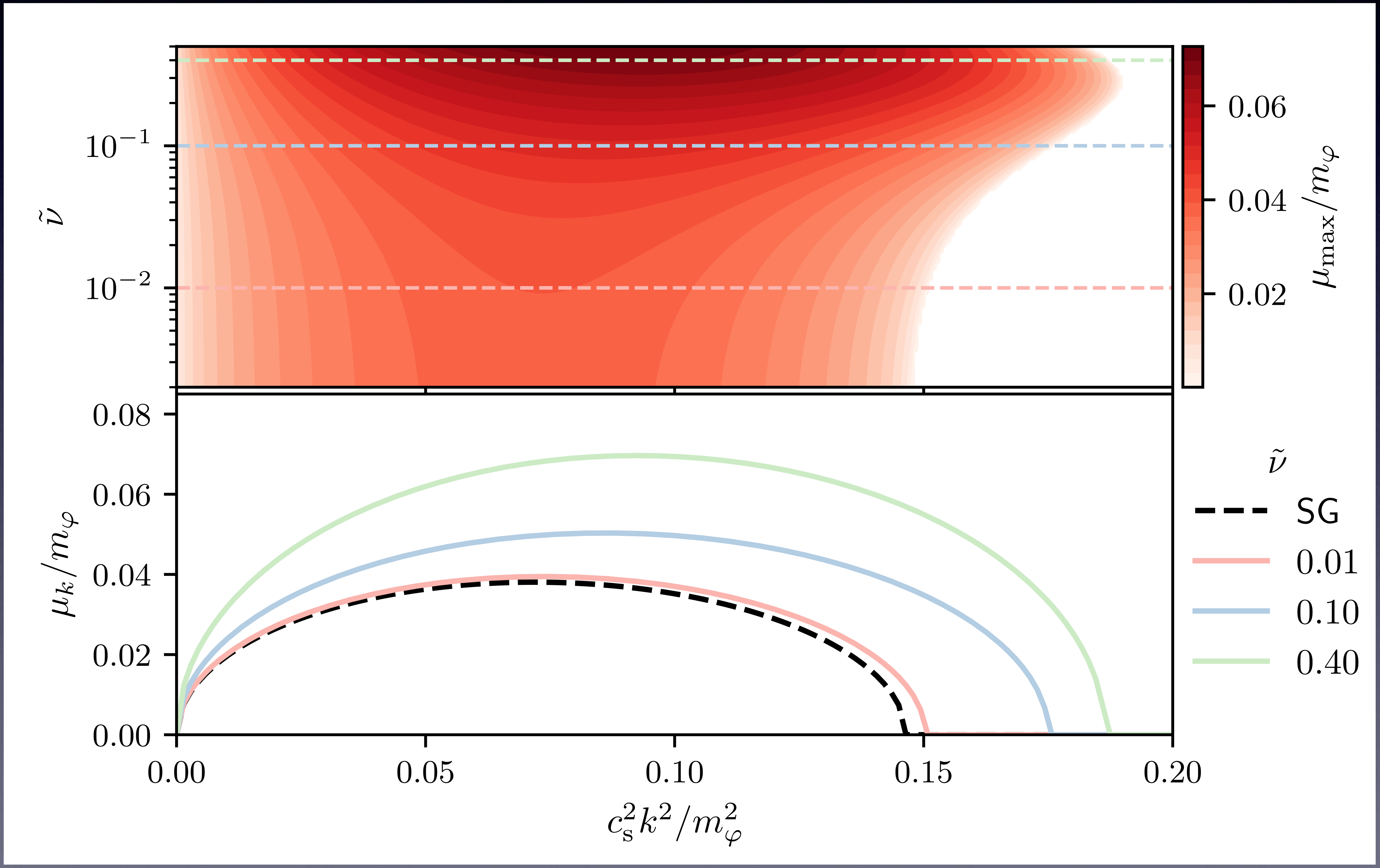
$$\frac{\mathrm{d}\mathbf{y}}{\mathrm{d}t} = L(t)\mathbf{y}$$

$$L(t+T) = L(t)$$

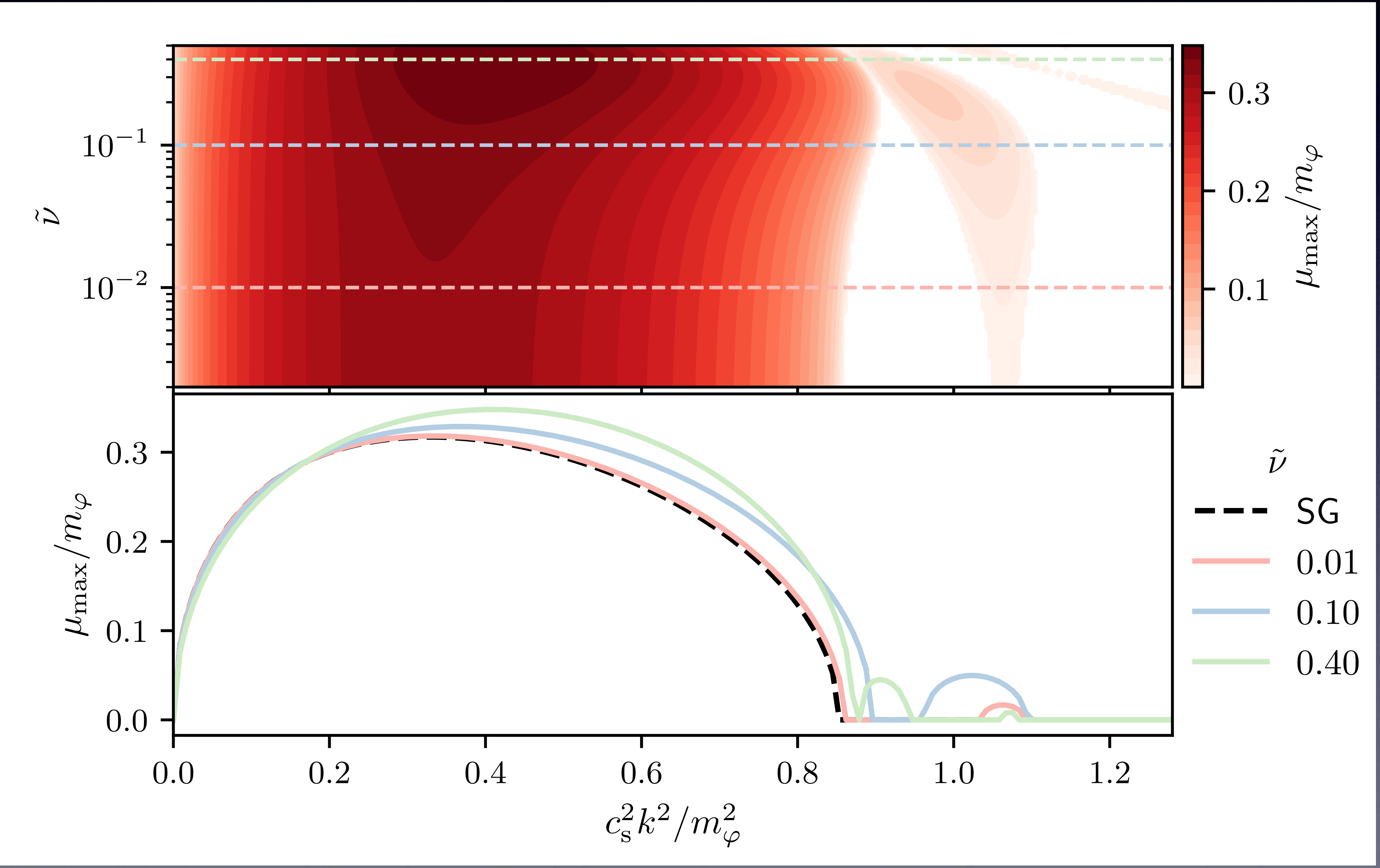
$\mathrm{Re}(\mu) > 0$: exponential growth

Periodic $\bar{\varphi}$ and $\bar{\Pi}_\varphi \Longrightarrow$ solutions $\delta\mathbf{y} = \mathbf{P}(t)e^{\mu t}$

Linear Instability
$(\phi_{\max} = 0.25\pi)$

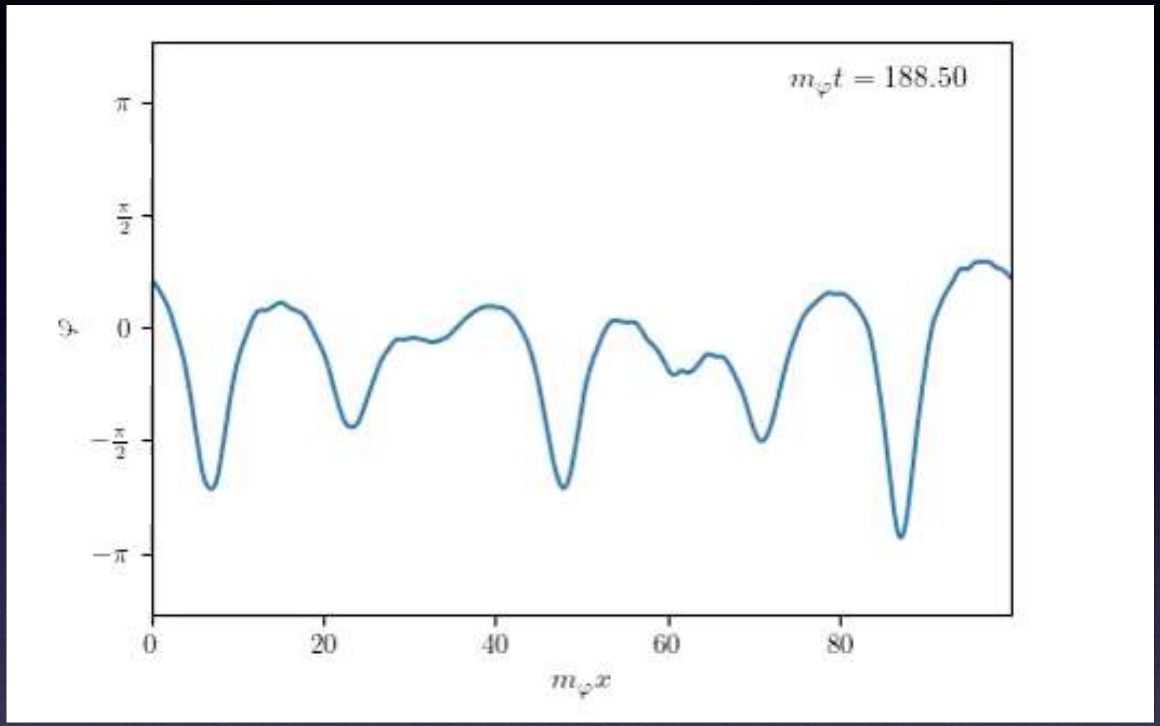


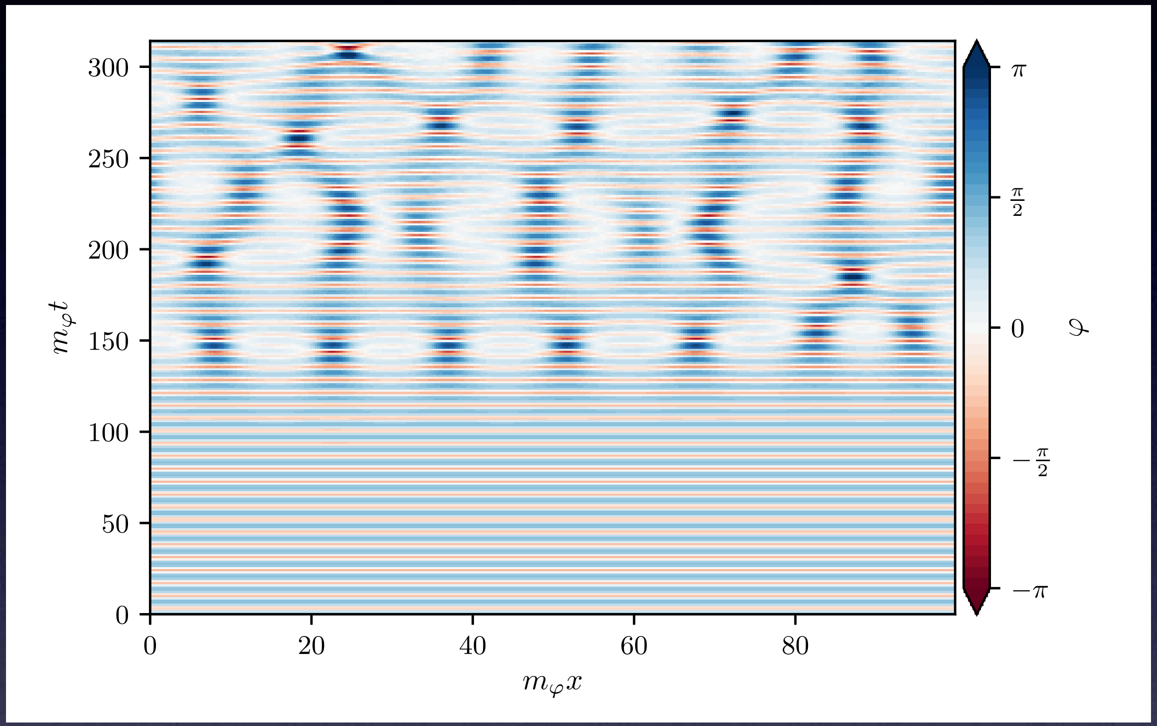
Linear Instability
$(\phi_{\max} = 0.75\pi)$

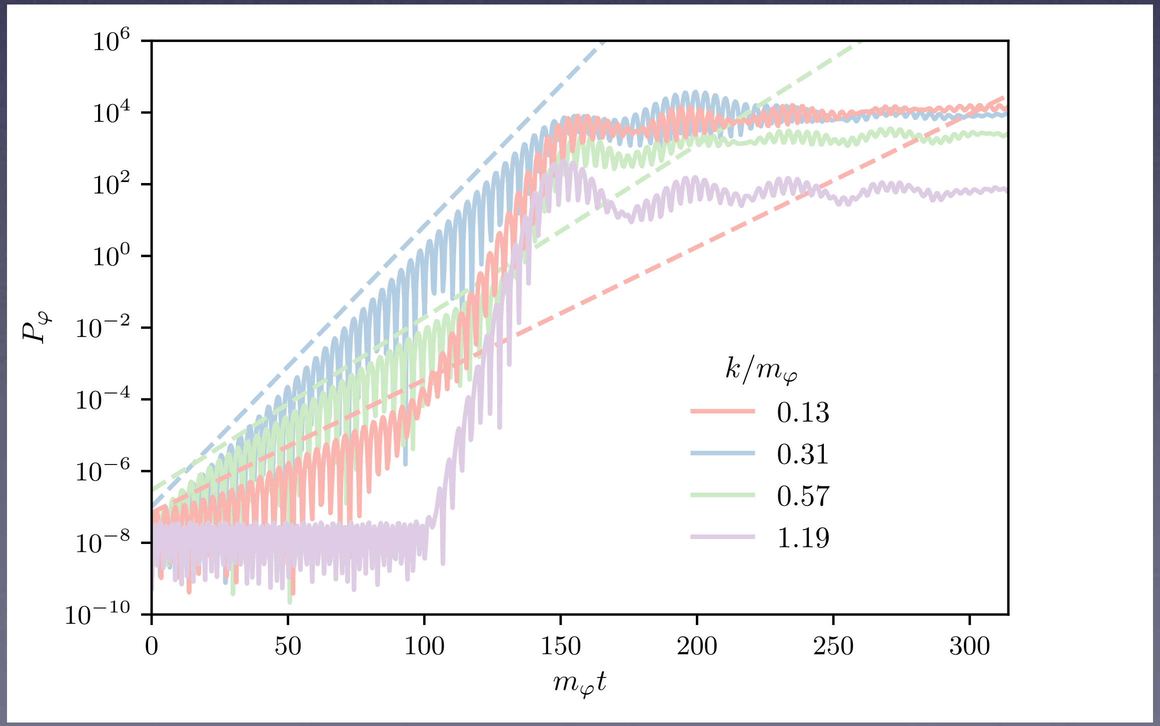


What About Nonlinearity

1D Analog Preheating

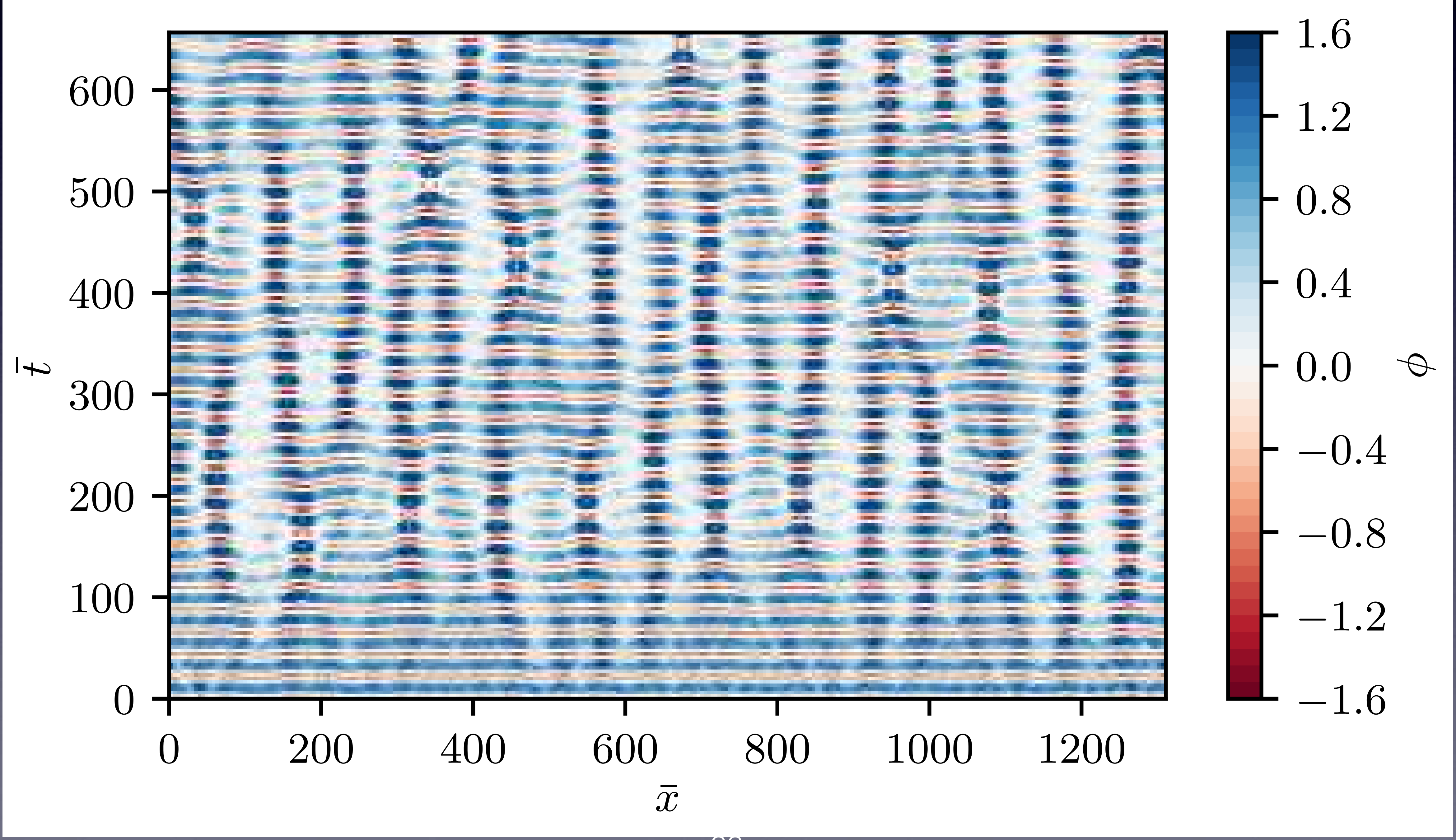




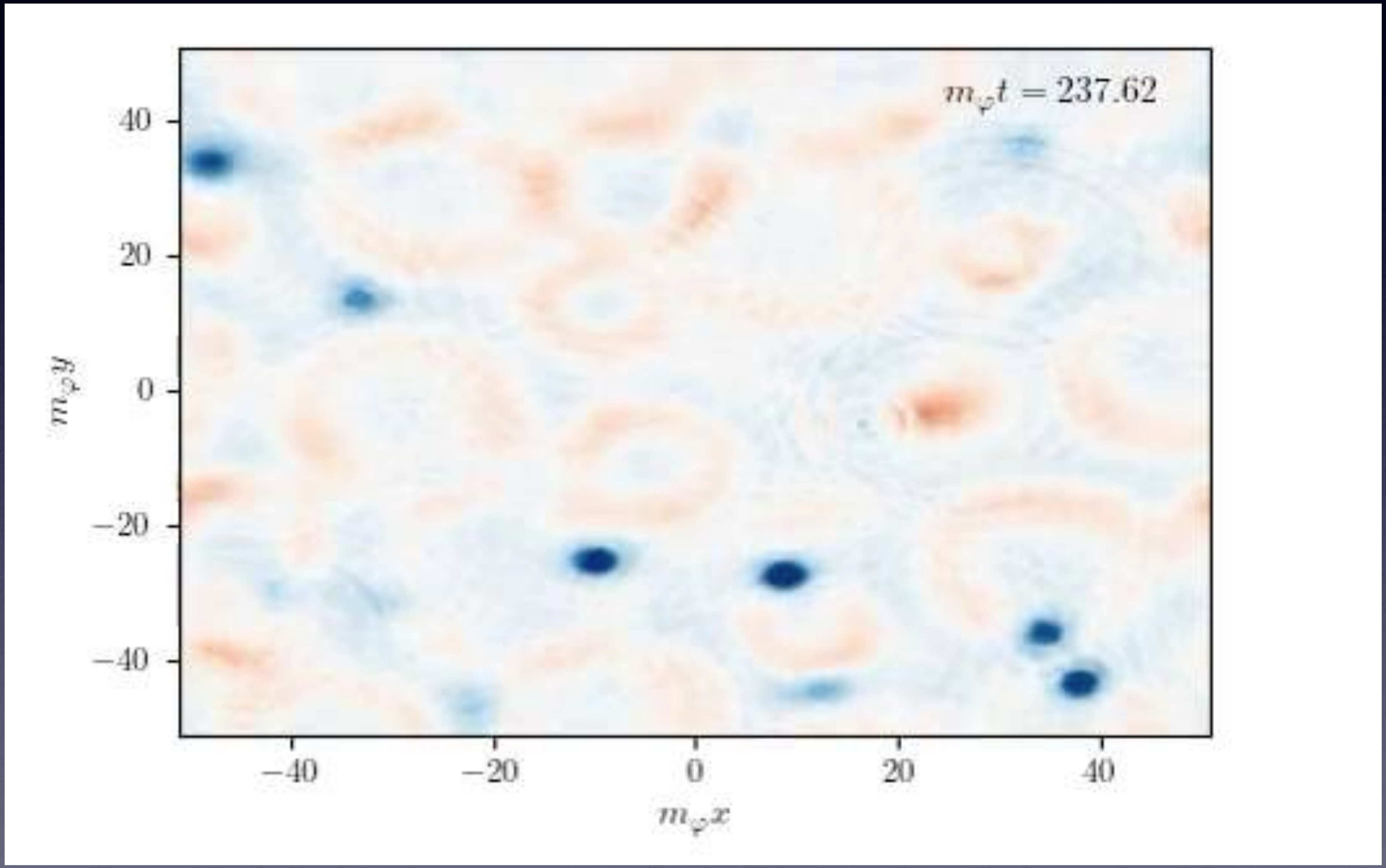


Early : Linear (Floquet) theory
Intermediate : Rescattering
Late : Solitary waves

Larger initial fluctuations (i.e. fewer particles)



2D Analog Preheating



Field Momentum $\dot{\varphi}$ with mean removed

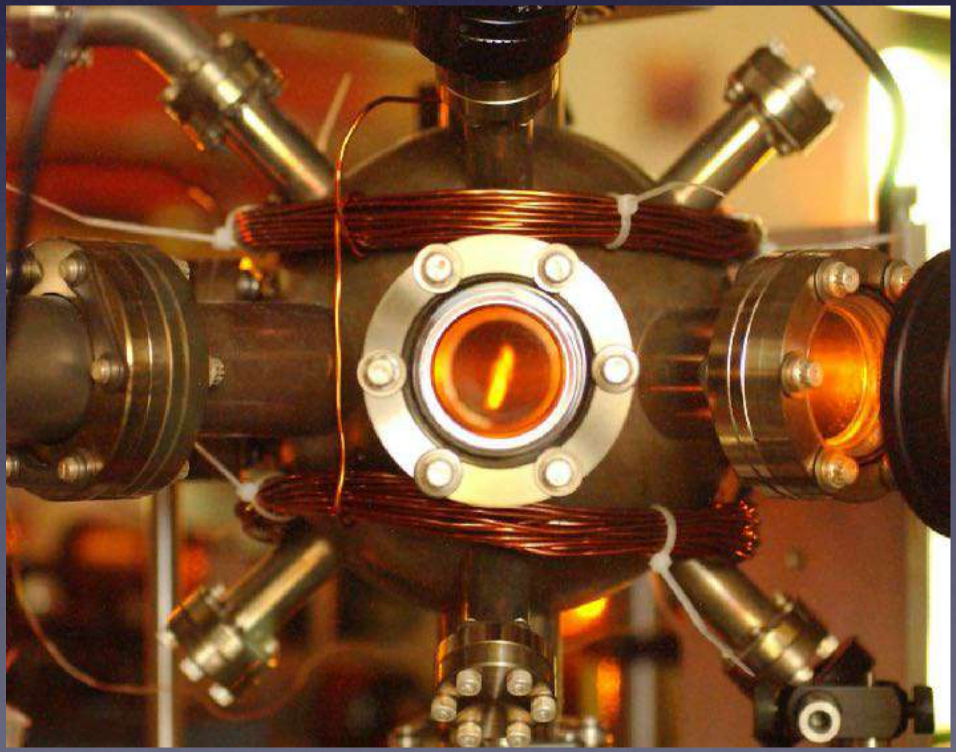
BUT …

[c.f. JB, Bond, Mersini-Houghton, for relativistic field example]

… Real BECs are Trapped

$$i\hbar\dot{\psi}_i = \left(-\delta_{ij}\frac{\hbar^2}{2m_i}\nabla^2 + V(x) + g_{ij}|\psi_j|^2\right)\psi_i - \nu_{ij}\psi_j$$

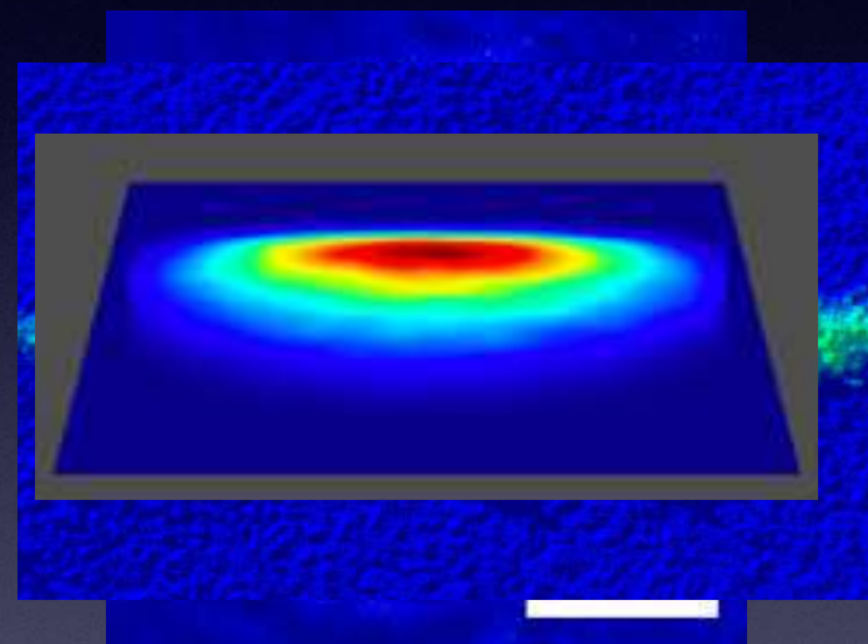
$V(x)$=0 homogeneous: $\bar{\psi}$ $\rightarrow$ $V(x) \neq 0$ inhomogeneous $\bar{\psi}(x)$



Does parametric resonance still work?

Dimensional Reduction

Idea : Integrate out trapped directions



Pancake Trap

Cigar Trap

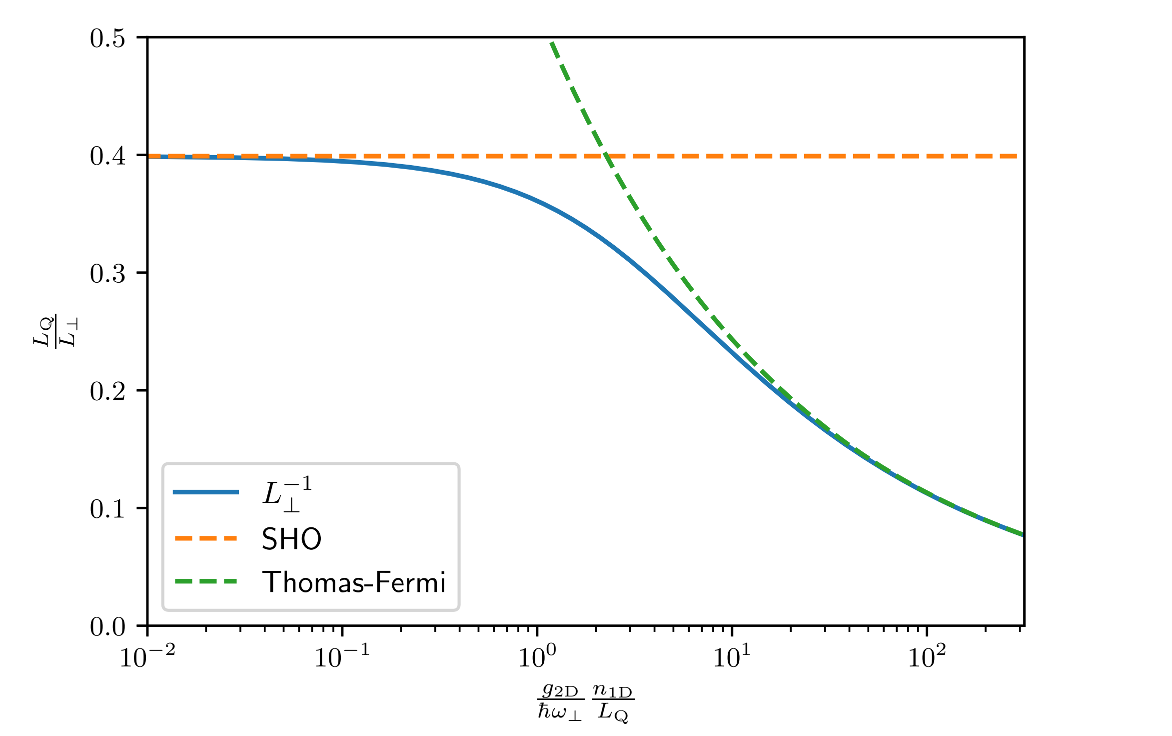
Doughnut Trap

$$i\hbar\dot{\psi}_i = \left(-\delta_{ij}\frac{\hbar^2}{2m_i}\nabla^2 + V(\mathbf{x}) + g_{ij}|\psi_j|^2\right)\psi_i - \nu_{ij}\psi_j$$

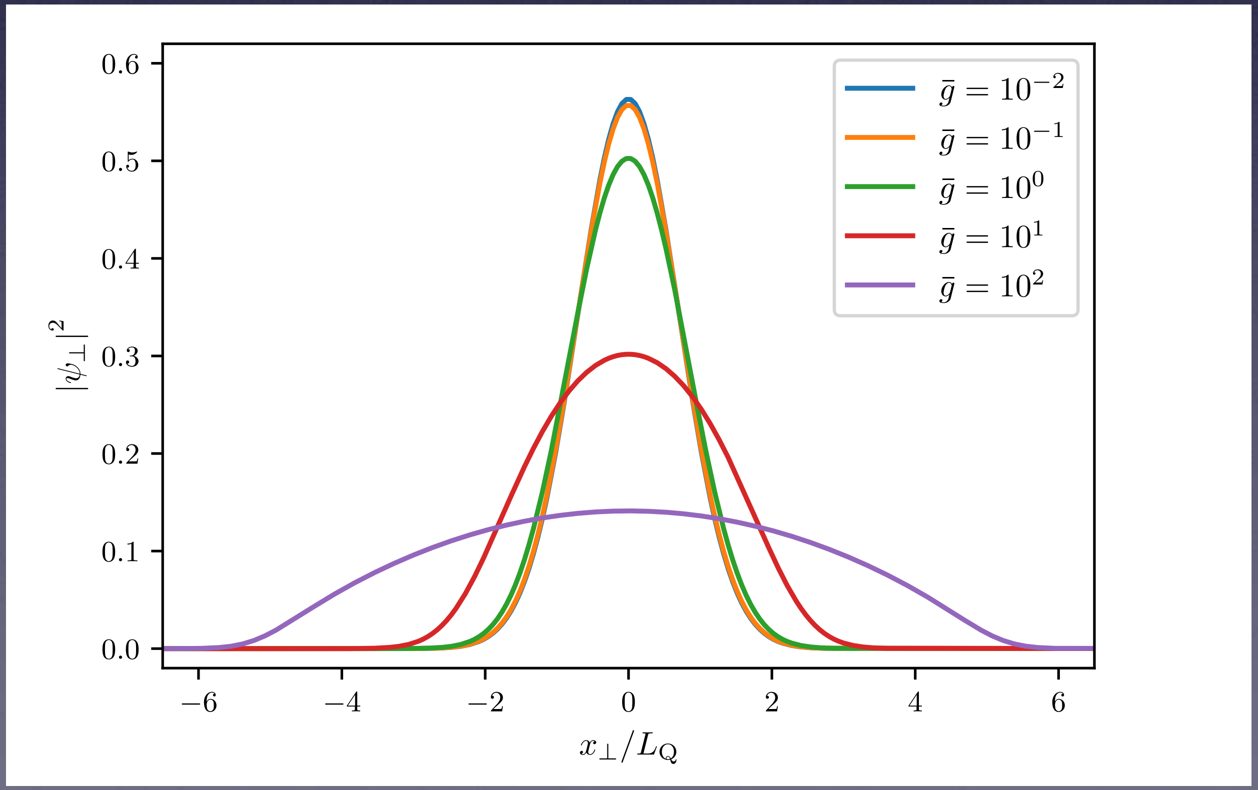
$$\frac{g^{D-1}}{g^D} = \frac{1}{L_\perp} = \frac{\int dx_\perp n^2}{\int dx_\perp n} = \frac{\int dx_\perp |\psi_\perp|^4}{\int dx_\perp |\psi_\perp|^2}$$

Harmonic Trap

$$V(x_\parallel, x_\perp) = \frac{1}{2} m\omega_\perp^2 x_\perp^2 \qquad L_\mathrm{Q}^2 = \frac{\hbar}{m\omega_\perp}$$

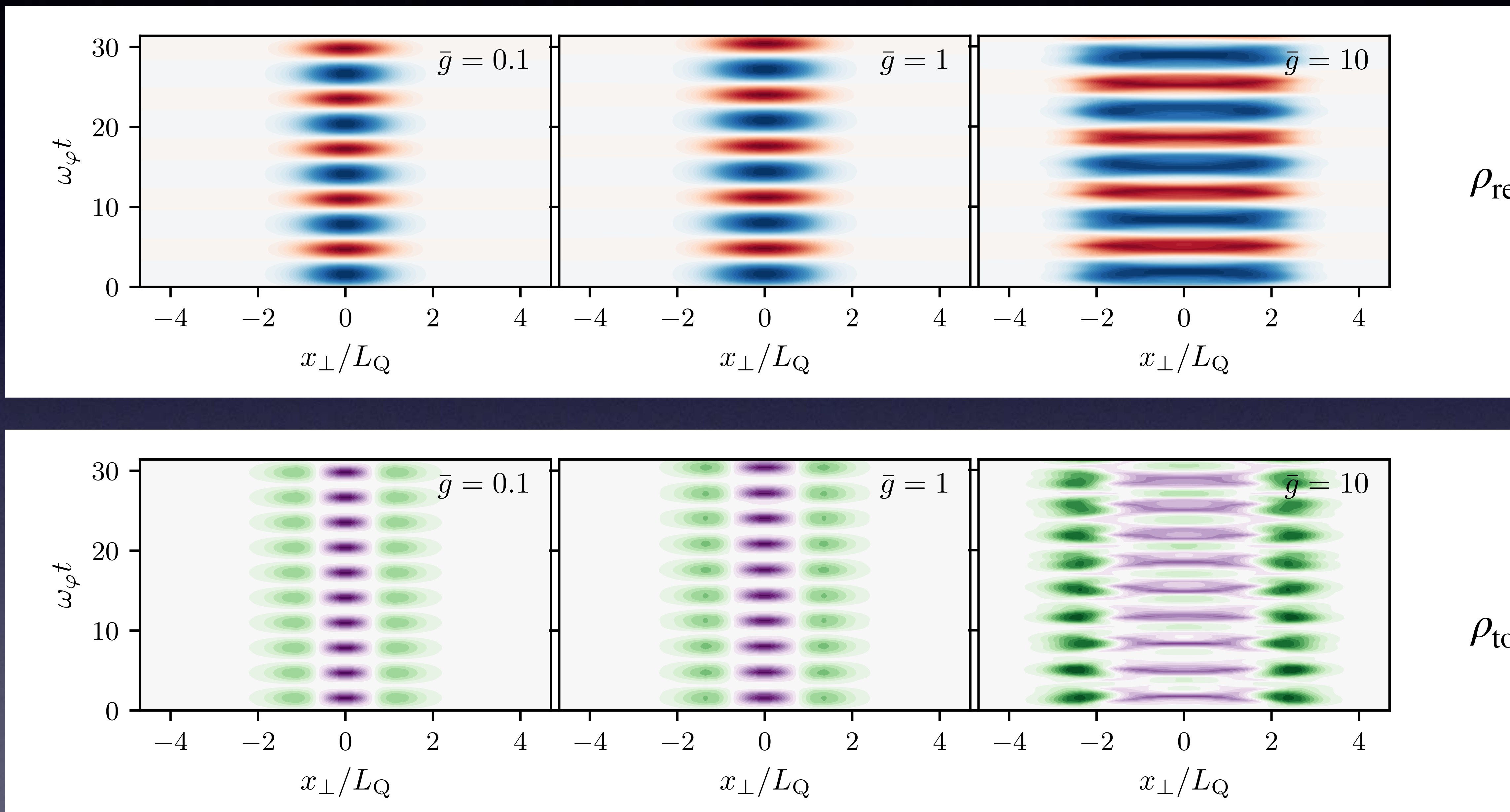


$$\mu\psi_\perp = \left[-\frac{\hbar^2}{2m}\nabla_\perp^2 + V_\mathrm{trap}(x_\perp) + g^D \left|\psi_\perp\right|^2 \right] \psi_\perp$$



$$\bar{g} \sim \frac{L_\mathrm{Q}^2}{L_\mathrm{heal}^2} \sim \frac{1}{\tilde{\nu}} \frac{m_\varphi^2 L_\mathrm{Q}^2}{c_s^2}$$

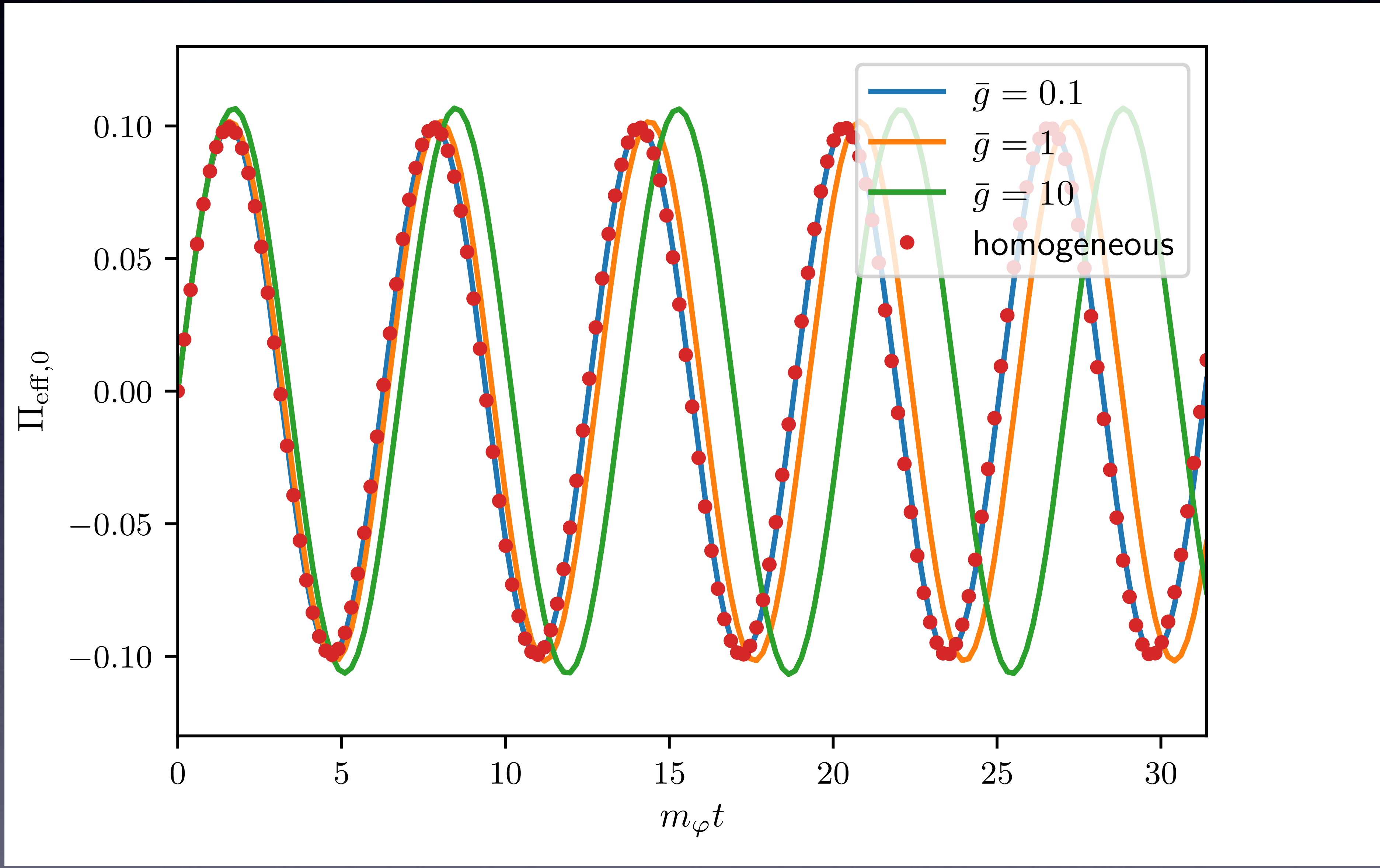
Background Evolution



SHO limit → **Thomas-Fermi Limit**

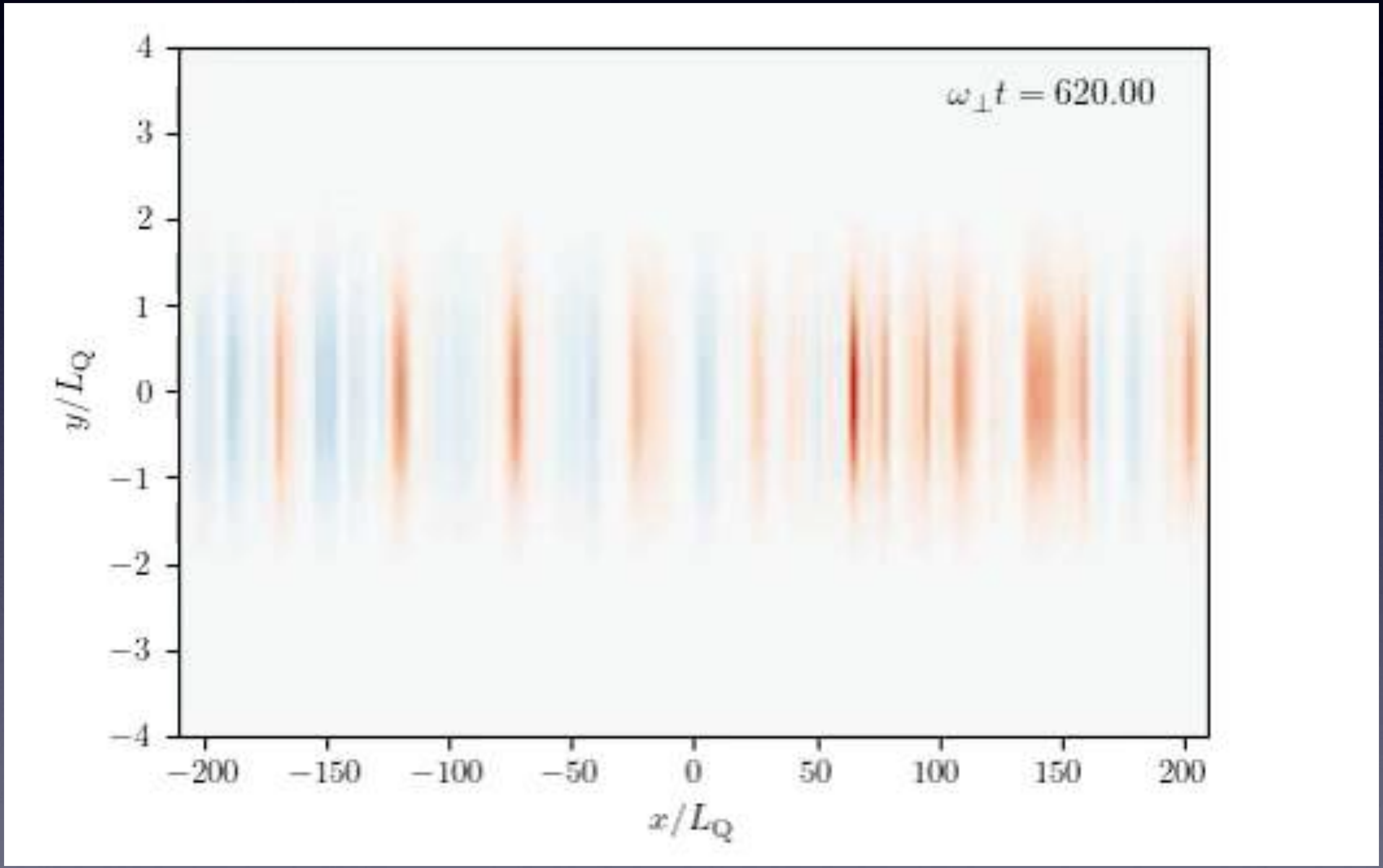
Decrease strength of transverse trap

Evolution Matches Dimensionally Reduced Simulations



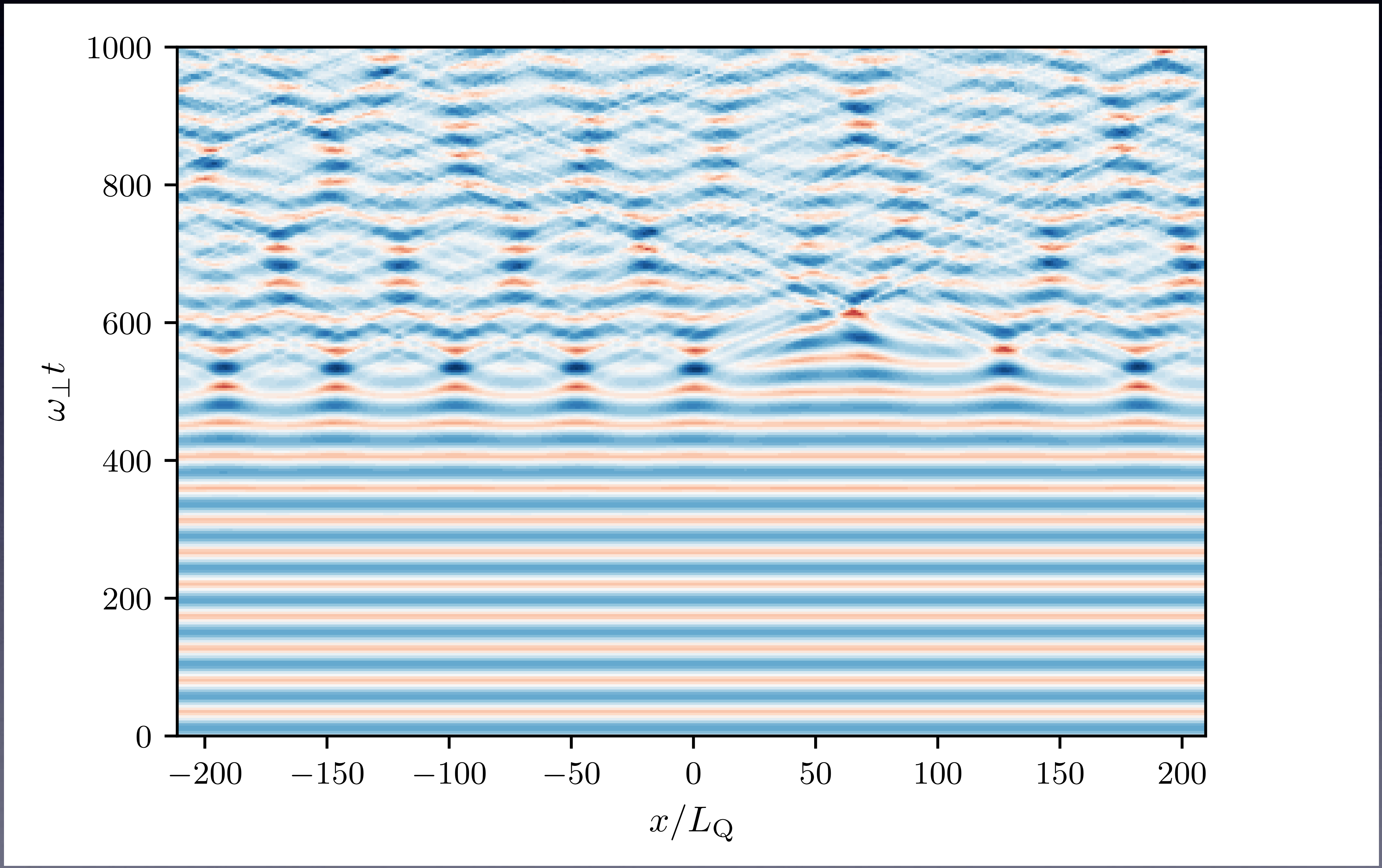
Is trapping maintained with parametric resonance?

Preheating in a Trap



Trapping Direction

Effective 1D Field Evolution



Summary

- Coupled dilute gas Bose-Einstein condensates can behave as relativistic fields
- Setups exist to mimic end-of-inflation (self-induced parametric resonance -> nonlinearity)
- Linear theory : BEC and cosmological calculation match, deviations controlled by tunable parameter
- Solitary waves in nonlinear regime
- Persists in more realistic case of trapped BEC
- Similar setup can also be used for vacuum decay