As Good As It Gets

Towards the best understood H_0 from strong gravitational lensing

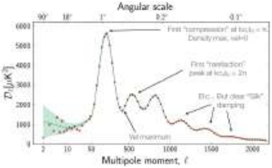
Jenny Wagner

Probes of H_0 are probes of cosmic distances

Comoving distance:

$$D_{\mathrm{com}}(z) = \frac{c}{H_0} \int_0^z \frac{\mathrm{d}x}{E(x)}$$

CMB



- all-sky observation
- foreground subtraction
- sound wave scale from power spectrum

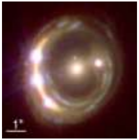
→ cosmology-depend. H_0

Cepheid+SN

- local neighbourhood
- different classes
- pulse-luminosity relation

→ "local" H_0

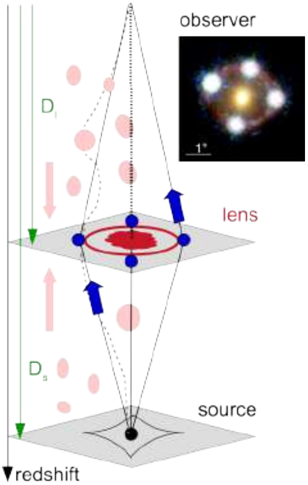
Strong grav. lensing



- wide redshift range
- various probes
- only based on weak light deflection

→ **optimum H_0?!**

Image credits: ESA/Planck, O. Doré; H. Bond (STScI and Penn State University); ESA/Hubble, NASA, S. Suyu et al.



$$t_A - t_B = \frac{(1+z_l)}{c} \frac{D_l D_s}{D_{ls}} [\phi_A - \phi_B] \propto \frac{1}{H_0}$$

$$\phi_A - \phi_B = \frac{1}{2} (\boldsymbol{x}_A - \boldsymbol{x}_B)^\top (\boldsymbol{\alpha}_A + \boldsymbol{\alpha}_B) - (\psi_A - \psi_B)$$

Measuring H_0 requires:

- an arrival time difference $t_A - t_B$ (observed)
- a distance between images $\boldsymbol{x}_A - \boldsymbol{x}_B$ (observed)
- a distance-redshift relation $D(z)$ (cosmology)
- *local* lens props at image positions (lensing)

break degeneracies in lensing and cosmology

Image credits: ESA/Hubble, NASA, S. Suyu et al.

Derivation of all exact lensing degeneracies (arXiv:1809.03505)

Formalism is degenerate in local lens properties
$\rightarrow$ degeneracies for each multiple image system

$$t_A - t_B = \frac{1+z_\mathrm{d}}{c} \frac{D_\mathrm{l} D_\mathrm{s}}{D_\mathrm{ls}} \left(\phi_A - \phi_B \right)$$

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Mathematical divergence theorems imply

- $\boldsymbol{\alpha}(\boldsymbol{x}) = \nabla \psi(\boldsymbol{x})$, $\boldsymbol{y} = \boldsymbol{x} - \boldsymbol{\alpha}(\boldsymbol{x})$
  → $\phi_A - \phi_B$ invariant, as long as

  $$\delta\psi_A - \delta\psi_B = -\left(\boldsymbol{x}_A - \boldsymbol{x}_B\right)^\top \delta\boldsymbol{y}$$

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- $\Delta\psi(\boldsymbol{x}) = 2\kappa(\boldsymbol{x})$ with $\forall\, \boldsymbol{x} \in \partial\mathcal{A} :\ \psi(\boldsymbol{x}) = 0$
  → $\psi(\boldsymbol{x}) = \frac{1}{\pi} \int_\mathcal{A} \mathrm{d}^2 \tilde{\boldsymbol{x}}\, \kappa(\tilde{\boldsymbol{x}}) \ln|\boldsymbol{x} - \tilde{\boldsymbol{x}}|$
  → $\phi_A - \phi_B$ invariant, as long as

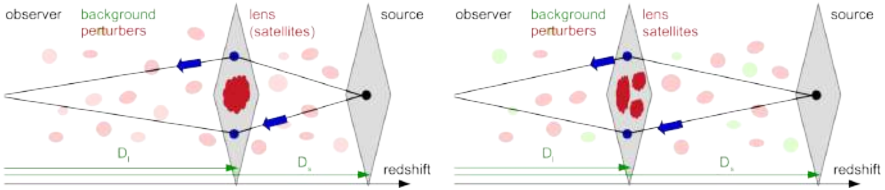
  $\delta\kappa(\boldsymbol{x})$ is a null-set of the integral measure

Extension from the lens plane into cosmology (arXiv:1904.07239)

Measure integrated quantities along the line of sight

$$t_A - t_B = \frac{1 + z_\mathrm{d}}{c} \frac{D_\mathrm{l} D_\mathrm{s}}{D_\mathrm{ls}} \left(\phi_A - \phi_B \right)$$

scenario 1 scenario 2



1) underdetermination of splitting (freedom of gauge choice):

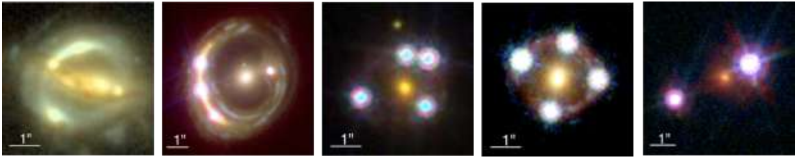
metric + perturbers + lensing

2) Invisible dark matter can be redistributed

Wagner MNRAS (2019); Ellis & Bruni Phys. Rev. D (1989)

Breaking degeneracies by mass density models

Critical curve reveals similar lens morphology for many lenses



- galaxy-scale lenses (elliptical) power-laws:

$$\rho(\boldsymbol{r}) \propto \left(1 + \frac{|\boldsymbol{r}|}{r_\mathrm{s}}\right)^{-\alpha} \qquad \text{(SIS, NIS, SIE, ...)}$$

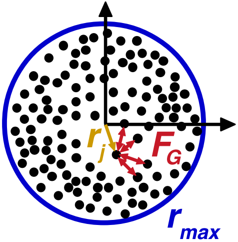
- cluster-scale lenses combined power-laws:

$$\rho(\boldsymbol{r}) \propto \left(\frac{|\boldsymbol{r}|}{r_\mathrm{s}}\right)^{-\alpha} \left(1 + \left(\frac{|\boldsymbol{r}|}{r_\mathrm{s}}\right)^{\beta}\right)^{-\gamma} \qquad \text{(NFW, PIEMD, ...)}$$

Why combinations of power-law profiles?

Derivation of power-law density profiles (arXiv:2002.00960)

Self-gravitating ensemble of n_p i.i.d. particles form continuous $\rho(r)$



- "microscopic" probability for a particle at r_j:

$$p(r_j) = N\left(1 + \frac{r_j}{r_\sigma}\right)^{-\alpha}$$

- "macroscopic" normalisation given by finite volume:

$$\int_{V_\mathrm{max}} \mathrm{d}V\, p(r_j) \stackrel{!}{=} 1 \rightarrow N = N(\alpha, r_\sigma, r_\mathrm{max})$$

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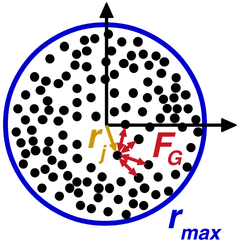

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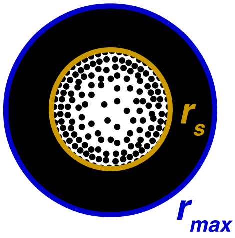


$\rightarrow$ most likely joint spatial distribution determines α

$$\frac{\partial_\alpha N(\alpha, r_\sigma, r_\mathrm{max})}{N(\alpha, r_\sigma, r_\mathrm{max})} - \frac{1}{n_\mathrm{p}} \sum_{j=1}^{n_\mathrm{p}} \ln\left(1 + \frac{r_j}{r_\sigma}\right) \stackrel{!}{=} 0$$

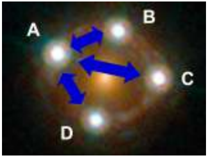
$\rightarrow$ treat particles as (inhom.) sampling of $\rho(r)$

$\rightarrow$ most common $\rho(r)$ are special cases!

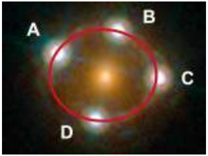
H_0 from galaxy-scale time-delay cosmography (TDCOSMO)

$$t_A - t_B = \frac{1+z_\mathrm{d}}{c} \frac{D_\mathrm{l} D_\mathrm{s}}{D_\mathrm{ls}} \left(\phi_A - \phi_B\right)$$

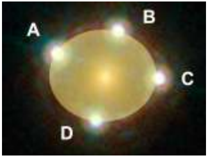
time delays



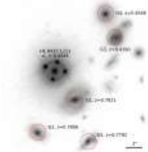
lens model



vel. dispersion



surroundings



$$P(\mathcal{V}|\mathcal{O}) = P(\mathcal{O}|\mathcal{V})\, P(\mathcal{V}|\mathcal{A})\, P(\mathcal{A})$$

obs.: $\mathcal{O} = \{t_A - t_B, \sigma, ...\}$, param.: $\mathcal{V} = \{H_0, \Omega_m, \Omega_\Lambda, p_\mathrm{lm}\}$, assumptions: $\mathcal{A}$

$\mathrm{H_0}$ from galaxy-scale time-delay cosmography (TDCOSMO)

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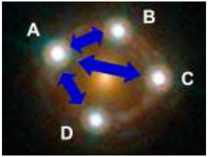
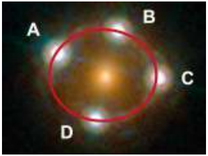


lens model



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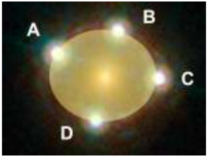

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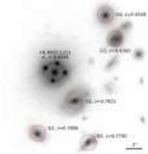


$$P(\mathcal{V}|\mathcal{O}) = P(\mathcal{O}|\mathcal{V})\, P(\mathcal{V}|\mathcal{A})\, P(\mathcal{A})$$

obs.: $\mathcal{O} = \{t_A - t_B, \sigma, ...\}$, param.: $\mathcal{V} = \{H_0, \Omega_m, \Omega_\Lambda, p_\mathrm{lm}\}$, assumptions: $\mathcal{A}$

Still need to sample $E(z)$ to get H_0!

Observation-based $E(z)$ from supernovae (arXiv:1812.04002)

Reconstruct $E(z)$ from SNe Ia without any assumption on H_0

- Pantheon sample: 1048 SNe Ia out to $z = 2.3$
- given $d = \mu + M$ and the covariance matrix Σ_μ
- $D_\mathrm{L} = 10^{\frac{\mu}{5}+1} = 10^{\frac{d-M}{5}+1}$, $\Sigma_\mathrm{obs} = 10^{-\frac{2M}{5}} \tilde{\Sigma}_\mathrm{obs}(\Sigma_\mu)$

assume scaling with M equivalent to scaling with H_0

$$D_\mathrm{L} \equiv 10^{-\frac{M}{5}} \tilde{D}_\mathrm{obs} = \frac{c}{H_0} \tilde{D}_\mathrm{mod} = \frac{c}{H_0}(1+z) \int_0^z \frac{\mathrm{d}x}{E(x)}$$

- Einstein-de-Sitter basis $\{\Phi_\alpha(z)\}_\alpha$: $\tilde{D}_\mathrm{obs}(z,c) = \sum_\alpha c_\alpha \Phi_\alpha(z)$
- Starobinski-approach: $\tilde{D}_\mathrm{obs} \stackrel{!}{=} \tilde{D}_\mathrm{mod} \;\Rightarrow \tilde{E}(z)$
- normalisation $E(z=0) \stackrel{!}{=} 1 \;\Rightarrow E(z) = \frac{\tilde{E}(z)}{\tilde{E}(z=0)}$

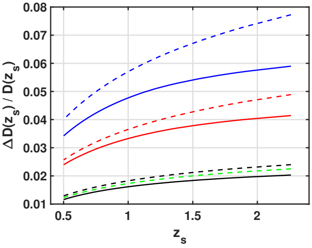
$E(z)$ without any assumption about H_0

Wagner & Meyer MNRAS (2019); Scolnic et al. ApJ (2018); Mignone & Bartelmann A&A (2008); Starobinski JETP Lett. (1998)

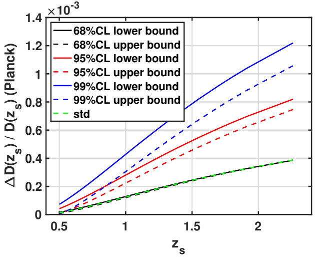
Distance ratio from supernovae (arXiv:1812.04002)

- **convert to** $\tilde{D}_\mathrm{A}(z) = \frac{1}{(1+z)} \int_0^z \frac{\mathrm{d}x}{E(x)}$ **for** $\tilde{D}_\mathrm{s}$, $\tilde{D}_\mathrm{l}$, $\tilde{D}_\mathrm{ls}$
- **calculate** $D \equiv \frac{\tilde{D}_\mathrm{l}\tilde{D}_\mathrm{s}}{\tilde{D}_\mathrm{ls}}$

relative imprecision of D (Pantheon)



relative imprecision of D (Planck)



observation-based $E(z)$ gains independence of cosmological parameters but costs precision

Summary: the best understood H_0-value

- H_0 **from time delay differences**

  - well-accessible observables:
    few TDD, luminous part of the lens
    $\longrightarrow$ **understanding of relevant astrophysical effects**

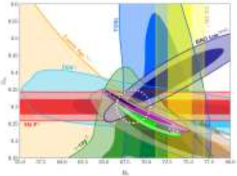
  - simple lensing formalism:
    gravity is described by a linear potential theory
    $\longrightarrow$ **understanding of its degeneracies**

  - simple lens description:
    morphology constrained by fundamental interactions
    $\longrightarrow$ **understanding of occurring lens shapes**

- **BUT...**

  - percent-precision seems feasible?
    $\longrightarrow$ **is percent-precision reasonable?**

  - **can this approach really solve the H_0-tension?**



Thank you for your attention

I gratefully acknowledge

- inspiring discussions with my colleagues, collaborators, and friends
- the invitation to present my work here!

My idea to solve the H_0-tension:

arXiv:2203.11219

Further information:
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