

Minkowski Functionals as a tool for Non-Gaussianities and anisotropy: extensions to CMB Polarization

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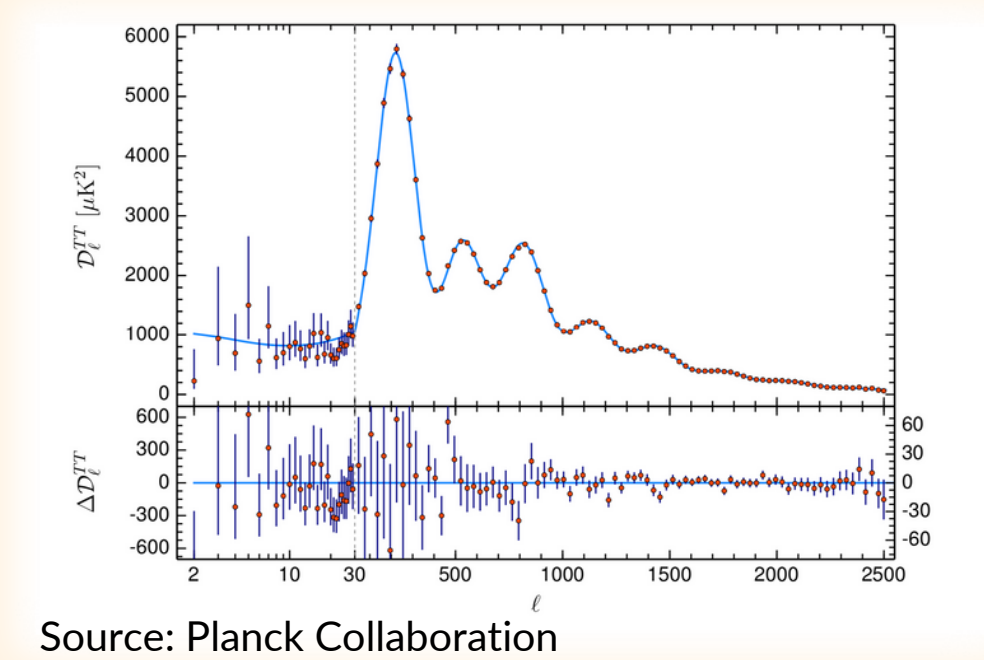
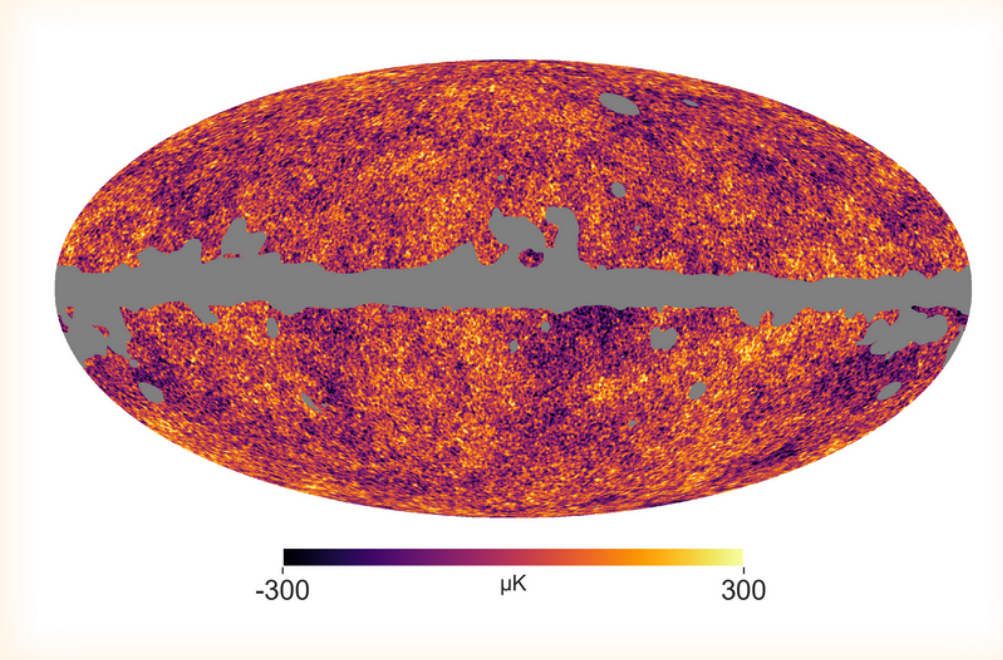
Cosmology from Home 2023

Outline

- Introduction
- Minkowski Functionals on CMB polarization
- Applications of Minkowski Functionals
- Software
- Conclusions

Gaussian fields are easy to describe

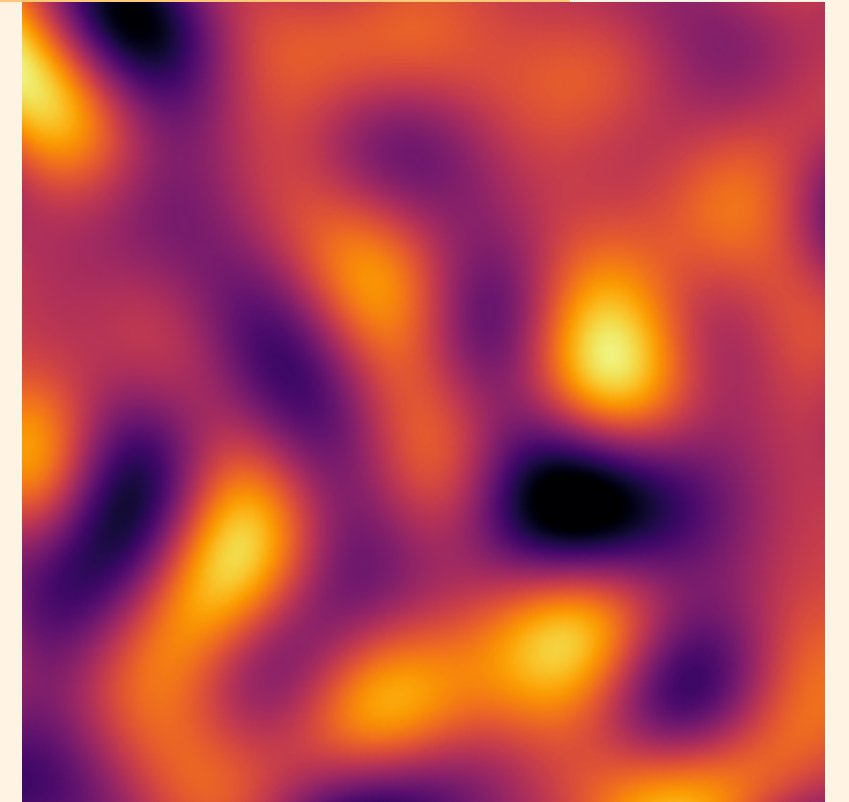
- Gaussian → Physical process fully described by 2pt correlation function



- Blind to non–Gaussianity and anisotropy
- Other tools: 3/4pt correlation functions, extrema statistics, Betti numbers, Minkowski Functionals
 - f_{NL} is important, but not the only way

Minkowski Functionals are higher order statistics

- We consider a field (e.g., T or δ)
- Let u be a threshold (e.g., 2σ)
- We define the **excursion set** $A(u)$ as the regions of the field above u
- Minkowski Functionals (MFs) are:
 - V_0 : area of $A(u)$
 - V_1 : boundary length of $A(u)$
 - V_2 : Euler–Poincaré characteristic of $A(u)$ (#regions – #holes)



MFs are accurately predicted

- For isotropic Gaussian fields, the expectation is known and variance is small
- The three factors decouple:

$$\mathbb{E} [V_j(A_u)] \approx \rho_j(u) V_0(\mathbb{S}^2) \mu^{j/2}$$

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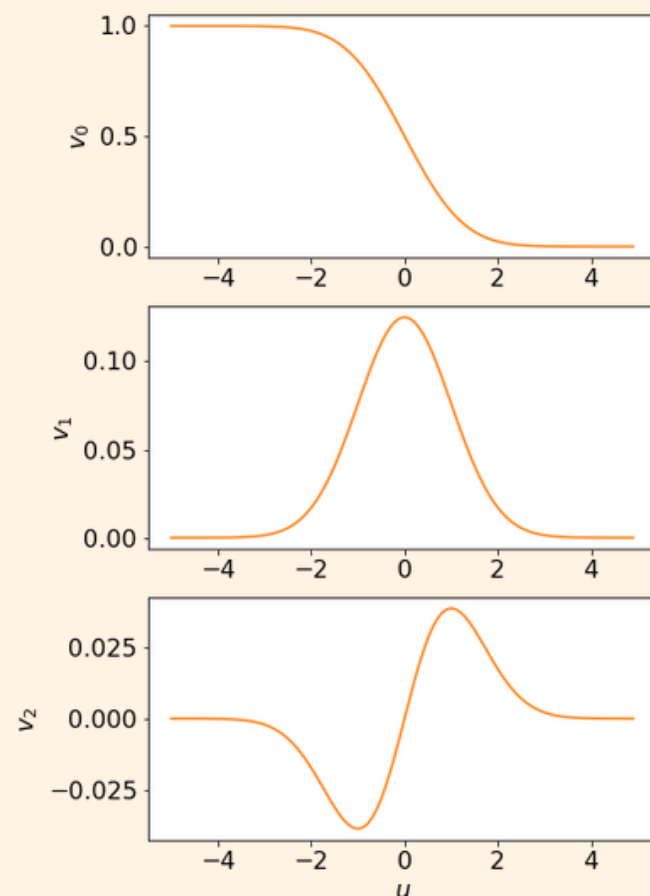
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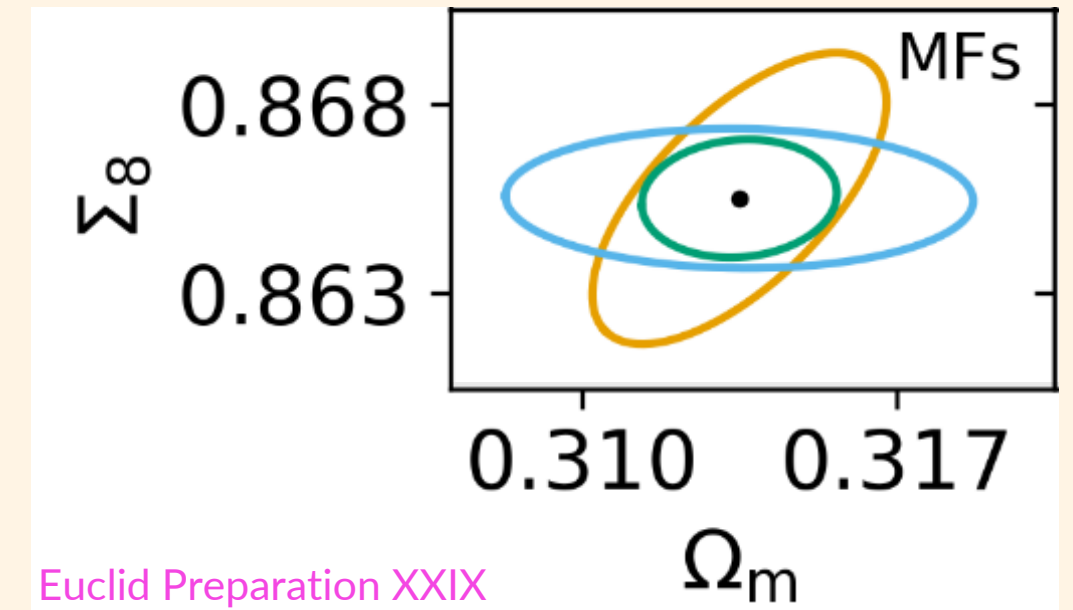
$$\begin{aligned} \frac{\mathbb{E} [V_0(A_u)]}{4\pi} &= 1 - \Phi(u) \\ \frac{\mathbb{E} [V_1(A_u)]}{4\pi} &= \frac{1}{8} \exp\left(-\frac{u^2}{2}\right) \mu^{1/2} \\ \frac{\mathbb{E} [V_2(A_u)]}{4\pi} &= \frac{\mu}{\sqrt{(2\pi)^3}} \exp\left(-\frac{u^2}{2}\right) u \end{aligned}$$

MFs have plenty of applications

- Any deviation is due to non–Gaussianity and/or anisotropy
- Early Universe (e.g., T): test for primordial non–Gaussianity
 - Planck 2018 VII (isotropy & statistics)

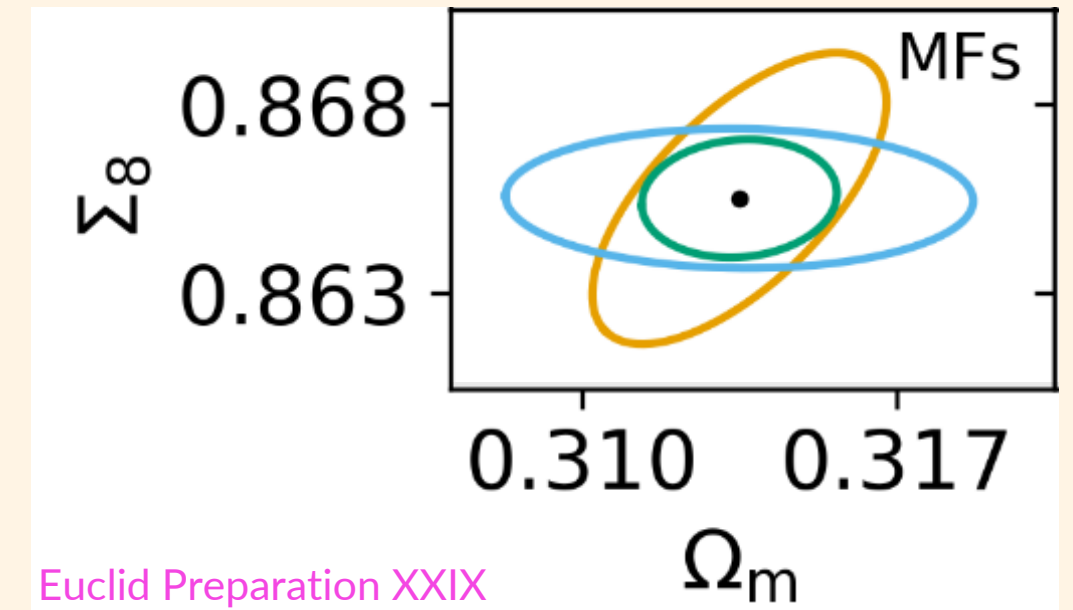
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 - Euclid Preparation XXIX (2023), Grewal+ (2022),
Zürcher+ (2022), ...



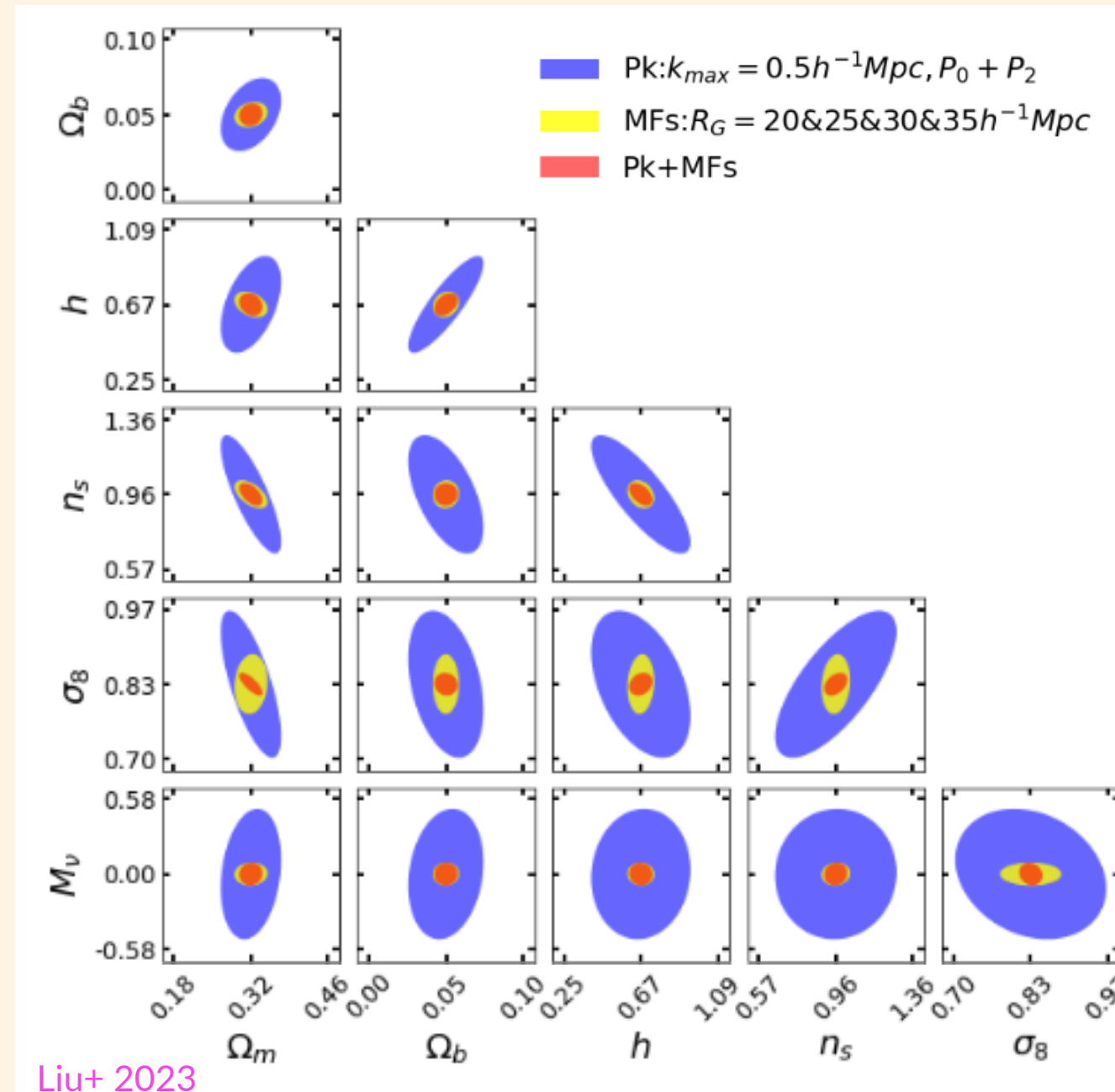
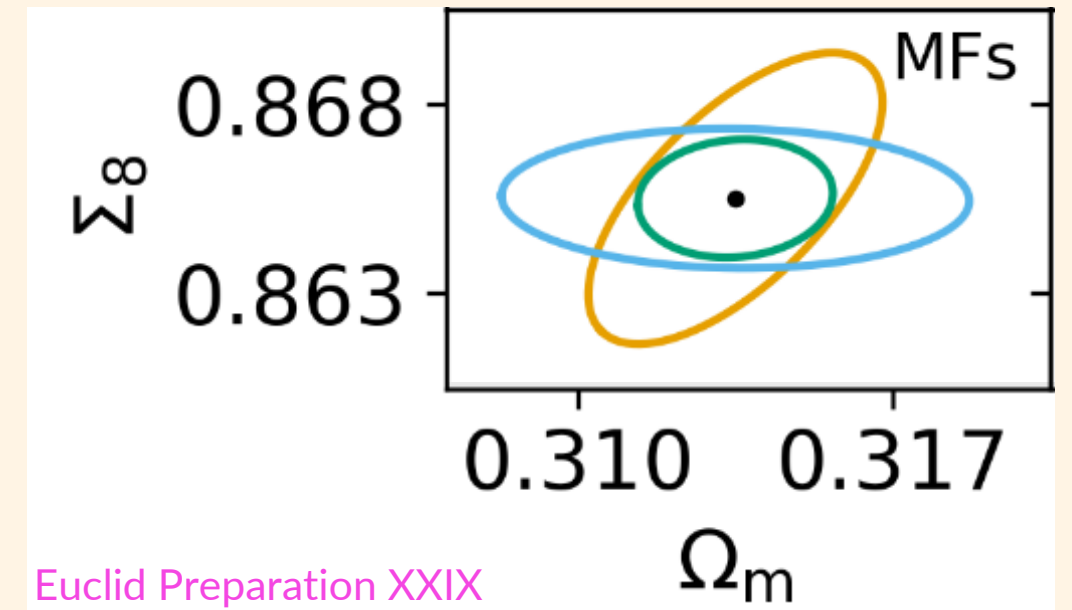
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- Foregrounds (e.g., Galactic):
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- Large Scale Structure (e.g., galaxy distribution):
 - Liu+ (2023), Appleby+ (2022), Spina (2021), ...



We extend MFs to modulus of polarization P^2

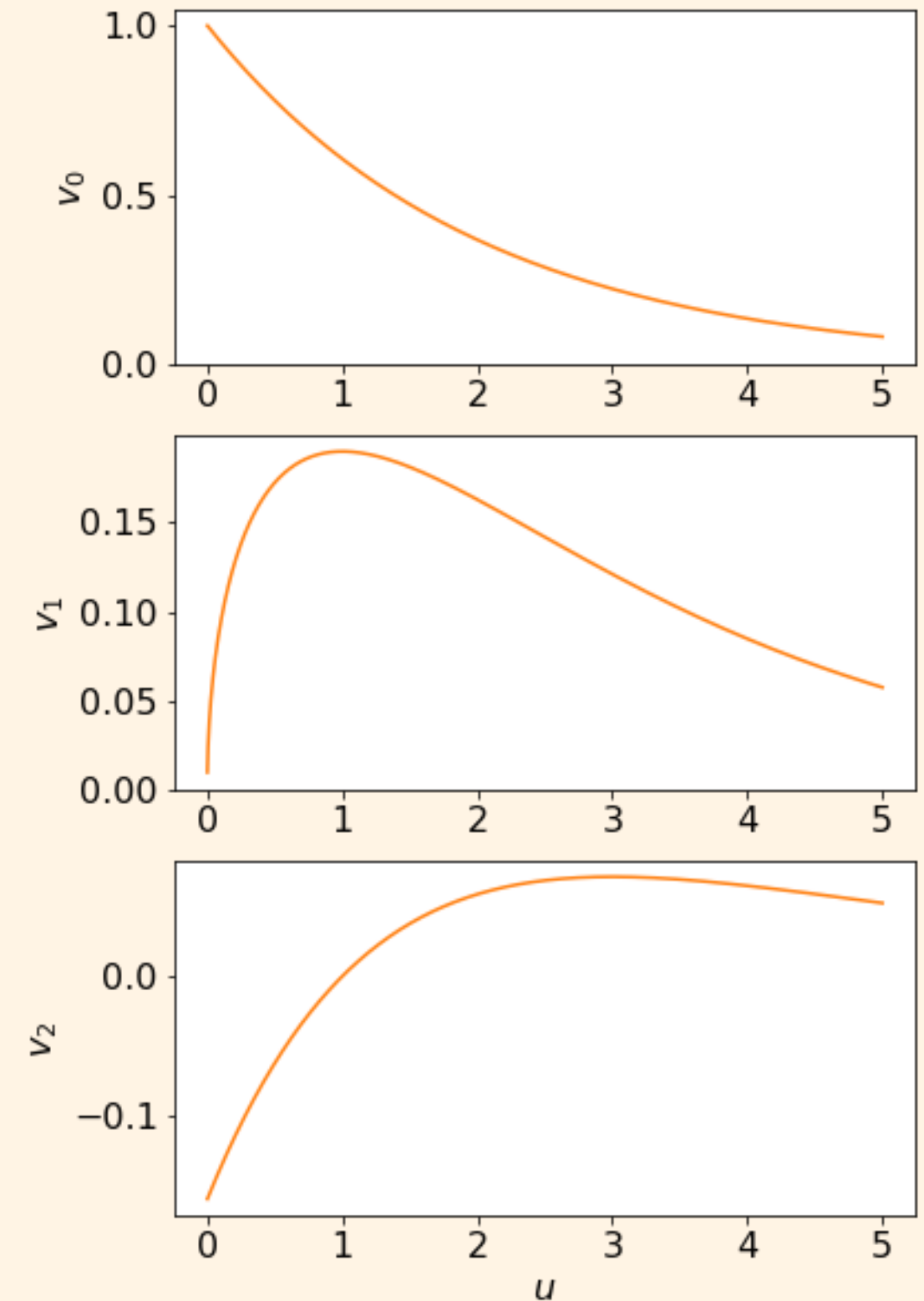
**Minkowski Functionals of CMB polarisation intensity with Pynkowski:
theory and application to Planck data**

Alessandro Carones,^{1,2*} Javier Carrón Duque,^{1,2} Domenico Marinucci,³ Marina Migliaccio,^{1,2} Nicola Vittorio^{1,2}

arXiv: 2211.07562

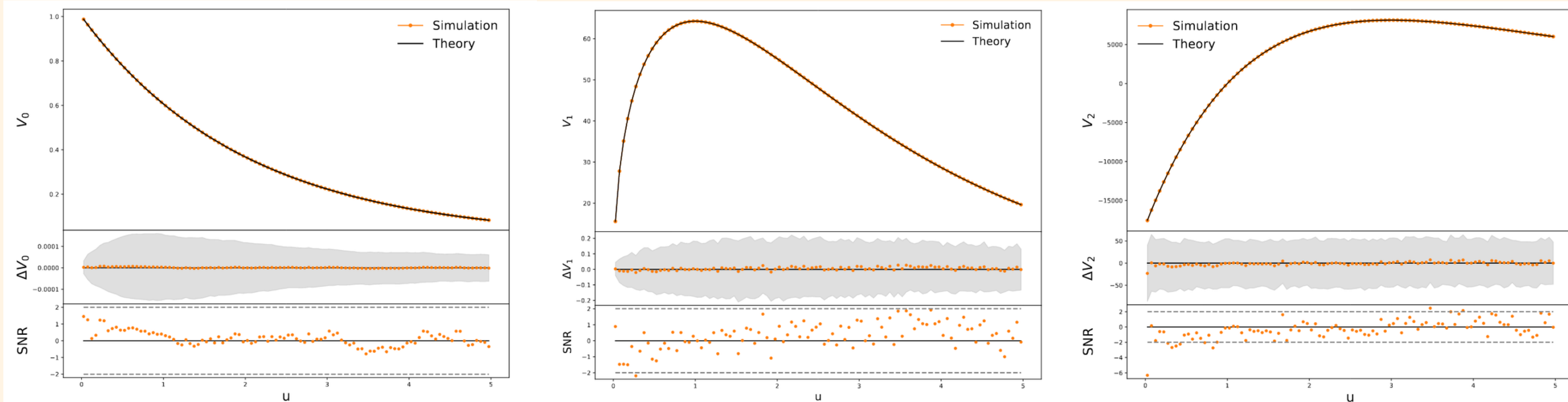
- We generalize the theoretical formula for $P^2 = Q^2 + U^2$

$$\frac{\mathbb{E} [V_0(A_u)]}{4\pi} = \exp(-u/2)$$
$$\frac{\mathbb{E} [V_1(A_u)]}{4\pi} = \frac{\sqrt{2\pi}}{8} \sqrt{\mu u} \exp(-\frac{u}{2})$$
$$\frac{\mathbb{E} [V_2(A_u)]}{4\pi} = \mu \frac{(u - 1) \exp(-u/2)}{2\pi}$$



Simulations are compatible with the theory (P^2)

- Excellent compatibility between theory and isotropic Gaussian simulations

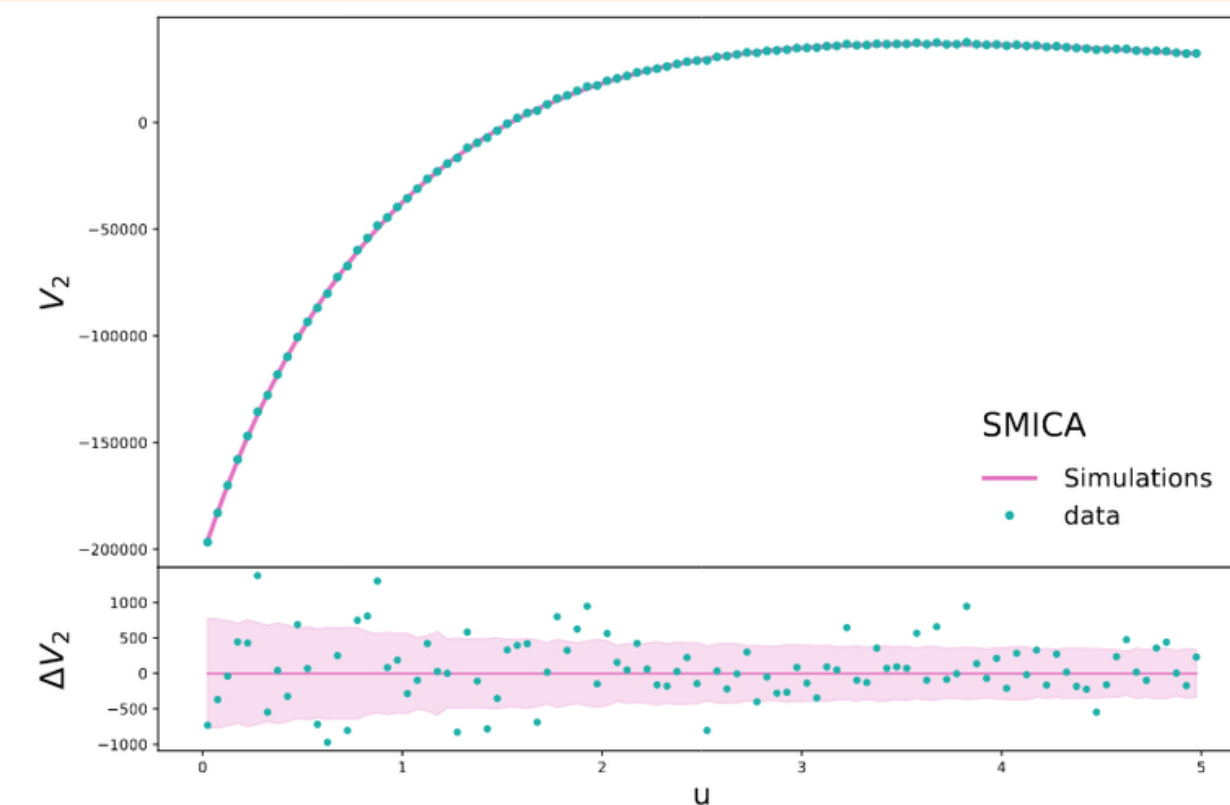
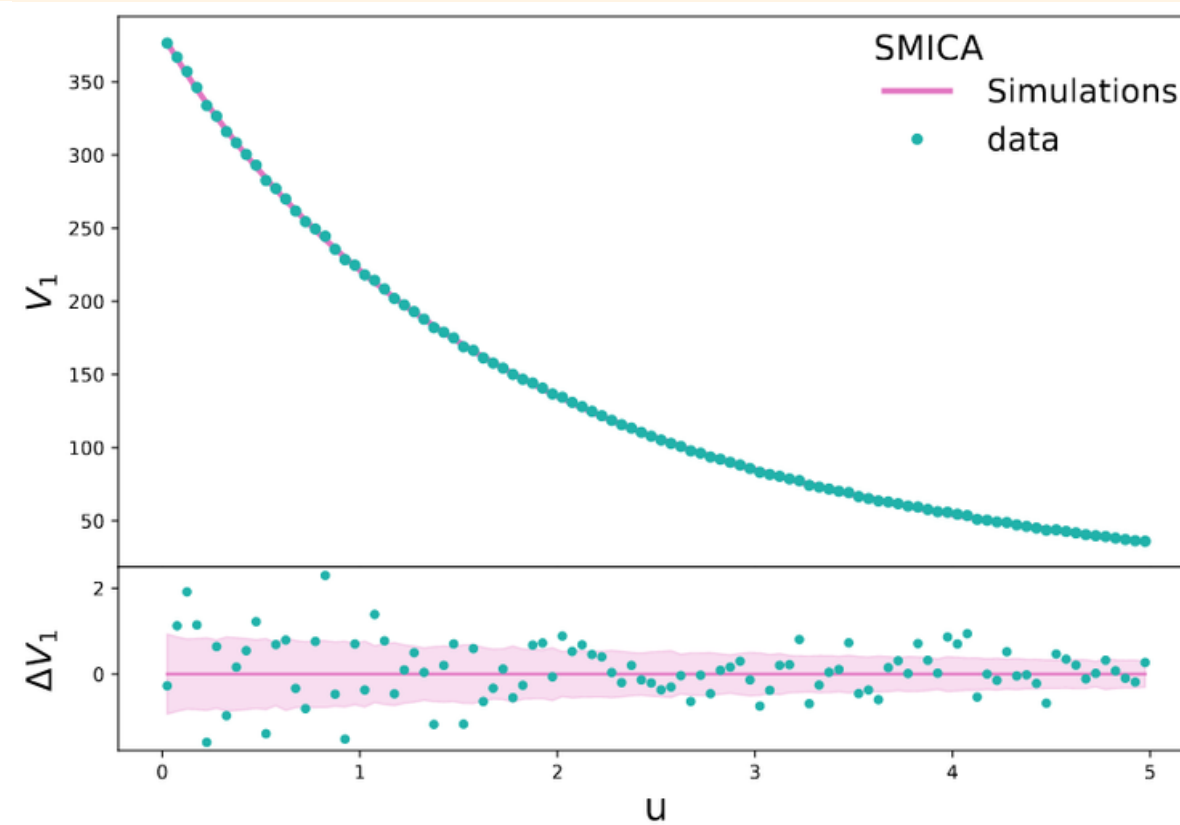
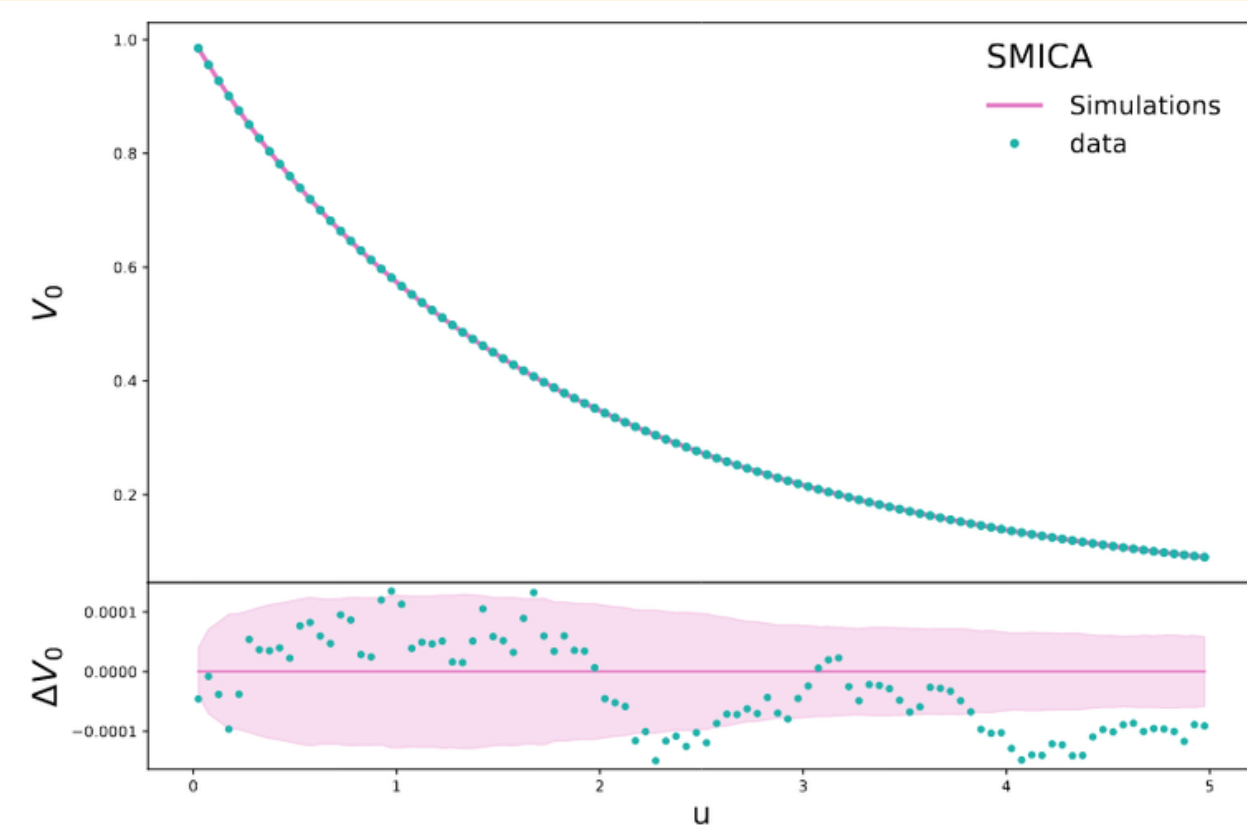


- Shown:
 - MF
 - Residual in simulations (1σ)
 - mean / σ (mean)

Planck is compatible with realistic simulations (P^2)

- Realistic simulations with anisotropic noise (observational strategy)
- No significant deviation (SMICA & SEVEM)

		χ^2	p_{exc} (%)
V_0	SMICA	1.012	44.7
	SEVEM	0.993	47.0
V_1	SMICA	1.010	47.7
	SEVEM	1.144	17.0
V_2	SMICA	0.812	86.7
	SEVEM	1.084	30.7



There is **more information** in the polarization field

- Polarization is a spin-2 complex field
- Information is lost in any scalar projection (P, E, B, Q, U, \dots)

There is **more information** in the polarization field

- Polarization is a spin−2 complex field
- Information is lost in any scalar projection (P, E, B, Q, U, \dots)
- We analyse the full polarization information using

$$f(\phi, \theta, \psi) = Q(\phi, \theta) \cos(2\psi) - U(\phi, \theta) \sin(2\psi)$$

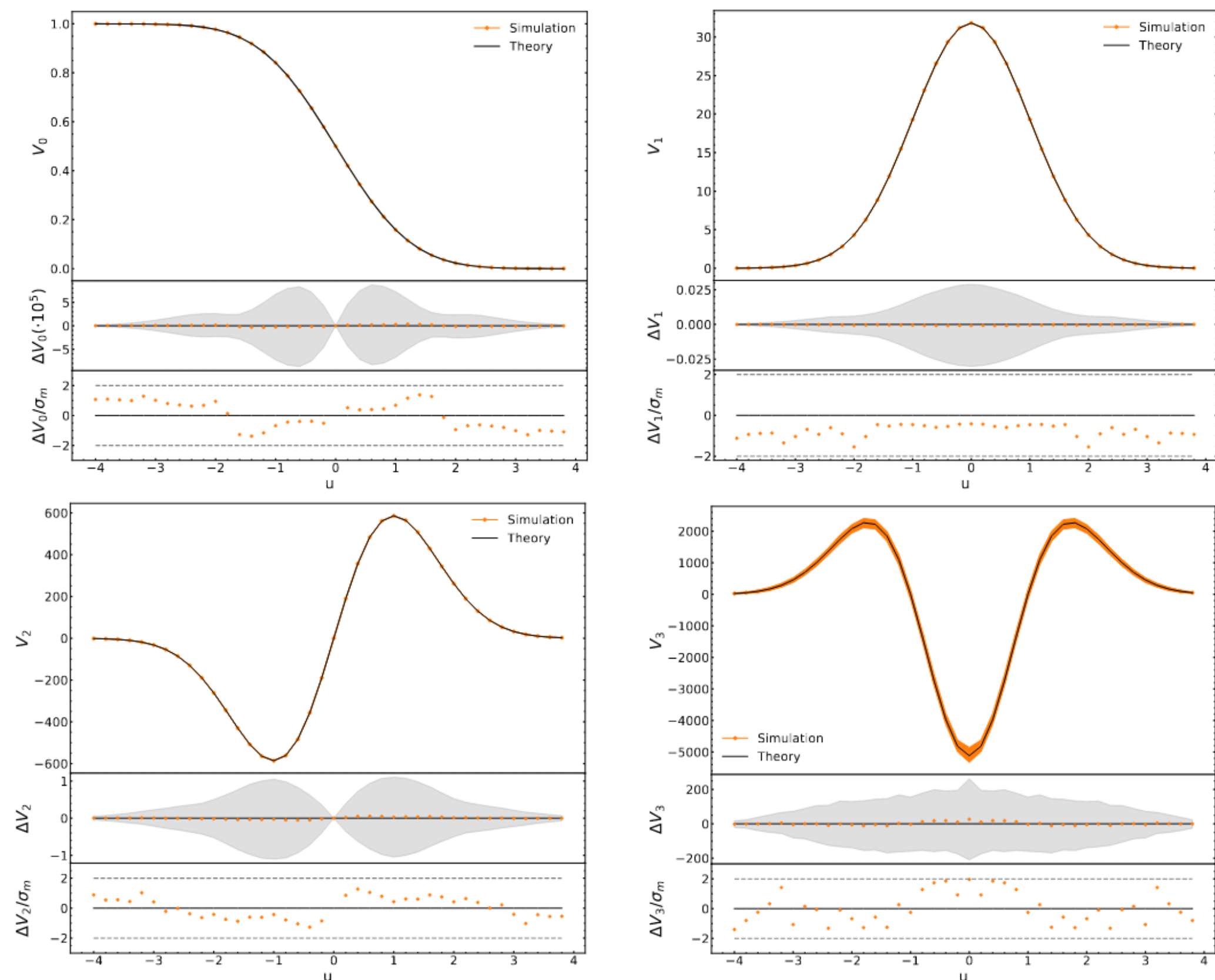
- This is defined in $SO(3)$, a 3D manifold

Minkowski Functionals in $SO(3)$ for
the spin−2 CMB polarisation field

J. Carrón Duque,^{a,b,1} A. Carones,^{a,b} D. Marinucci,^c M.
Migliaccio,^{a,b} and N. Vittorio^{a,b}

[arXiv: 2301.13191](https://arxiv.org/abs/2301.13191)

Simulations are compatible with the theory (f)



- 3D space \rightarrow 4 MFs
- Shown:
 - MF
 - Residual in simulations (1σ)
 - mean / $\sigma(\text{mean})$
- No significant deviation is found

We analyse Galactic dust models

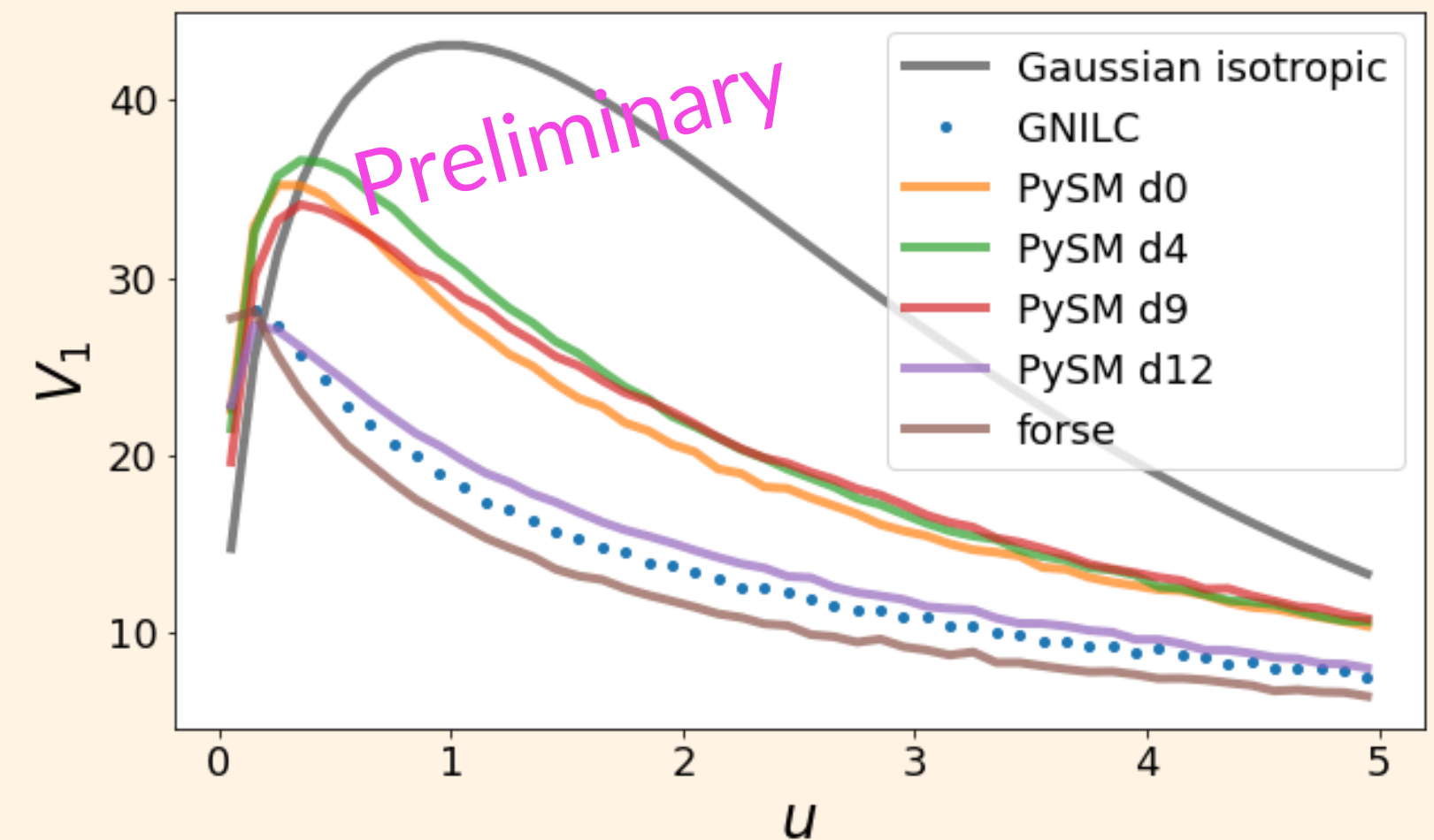
(w/ Giuseppe Puglisi)

- Non–Gaussian foregrounds are important to test component separation methods
- Simulating non–Gaussian foregrounds is not trivial:

- Modulation
- Very different approaches from PySM
- Generative Neural Networks

(forse: Krachmalnicoff&Puglisi, 2020)

- We are assessing different models

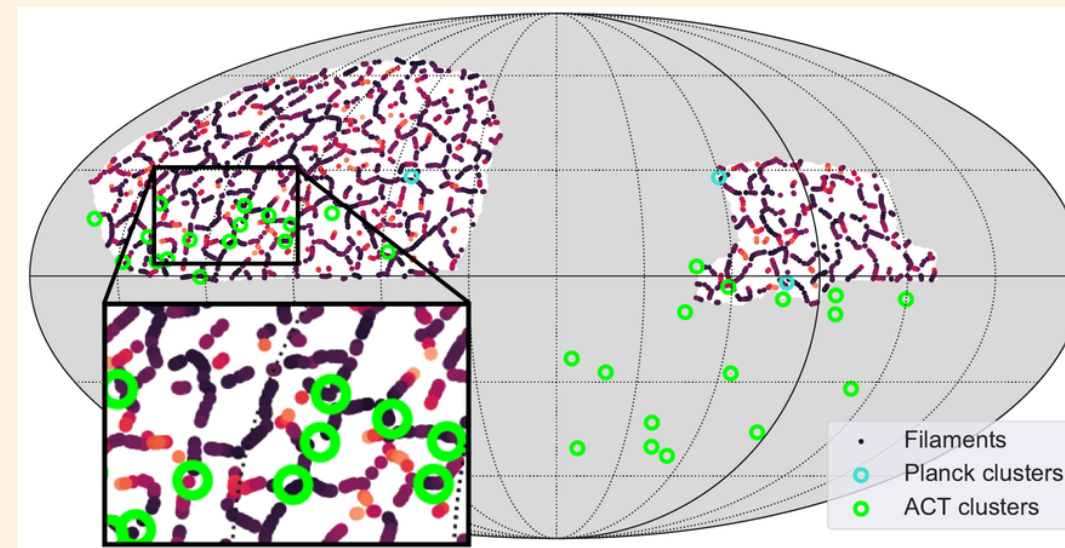
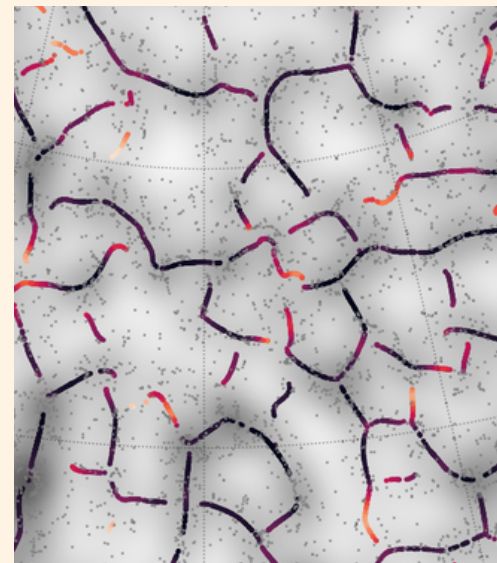


We produced a **Cosmic Filaments** catalogue

- Publicly available: www.javiercarron.com/catalogue
- $0.05 < z < 2.2$
- Promising results in different areas

A novel cosmic filament catalogue from SDSS data[★]

Javier Carrón Duque^{1,2}, Marina Migliaccio^{1,2}, Domenico Marinucci³, and Nicola Vittorio^{1,2}



MFs can be applied to the 3D density field

- The LSS is NOT Gaussian: lots of information in its non–Gaussianities

Primordial non–Gaussianities

- Consequence of Inflation
- MFs are well suited for some models
- Blind or model dependent

Late Universe non–Gaussianities

- Consequence of Gravity and Baryonic effects
- Dominant, especially at small scales

- Can MFs distinguish both origins?
 - Can we include the effect of Gravity?
- Can MFs constrain cosmological parameters effectively?
 - Yes, at least with forward modelling
- How do they compare to other statistics?
 - Theoretical models, degeneracies, systematics, ...

MFs can have many other applications

- We are exploring, among others:
 - Galactic dust polarised emission
 - Morphology of LSS
 - Forecast for future missions
 - CMB power asymmetry
 - **+ new ideas?**

We develop **Pynkowski** as a Python package

- Pynkowski is fully documented and modular
- Theory module: theoretical prediction of different kinds of fields (Gaussian, χ^2 , f , ...)
- Data module: different kinds of data structures (np arrays, healpix maps, ...)
- Stats module: different higher-order statistics (MFs, maxima/minima distribution, ...)
- All modules are easy to expand

Now available!



<https://github.com/javicarron/pynkowski>

```
$ pip install pynkowski
```

Pynkowski is **easy** to use



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```

```
import numpy as np
import healpy as hp

import pynkowski as mf    # For Minkowski Functionals

# Define the thresholds for the excursion sets
us = np.linspace(-5., 5., 100)

# Load the CMB map and angular power spectrum
my_map = ...
my_cls = hp.anafast(my_map)    # or load from file

# Compute the Minkowski Functionals on my map
data_map = mf.Healpix(my_map, normalise=True, mask=None)    # Default parameters

v0_data = mf.V0(data_map, us)
v1_data = mf.V1(data_map, us)
v2_data = mf.V2(data_map, us)

# Compute the Minkowski Functionals on a Gaussian random field with the same power spectrum
gaussian_field = mf.SphericalGaussian(my_cls, normalise=True, fsky=1.)    # Default parameters

v0_theory = mf.V0(gaussian_field, us)
v1_theory = mf.V1(gaussian_field, us)
v2_theory = mf.V2(gaussian_field, us)
```

Python

Takeaway points

- Minkowski Functionals are useful tools to study **non–Gaussianities** and isotropy, with many applications in both the Early and Late Universe
- We have expanded the formalism to **CMB polarization** in two ways: the polarization intensity P^2 , and the full information in the spin map
- We have created **Pynkowski** to ease the application of MFs to the cosmological community

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Thank you!