Freezing-In Gravitational Waves

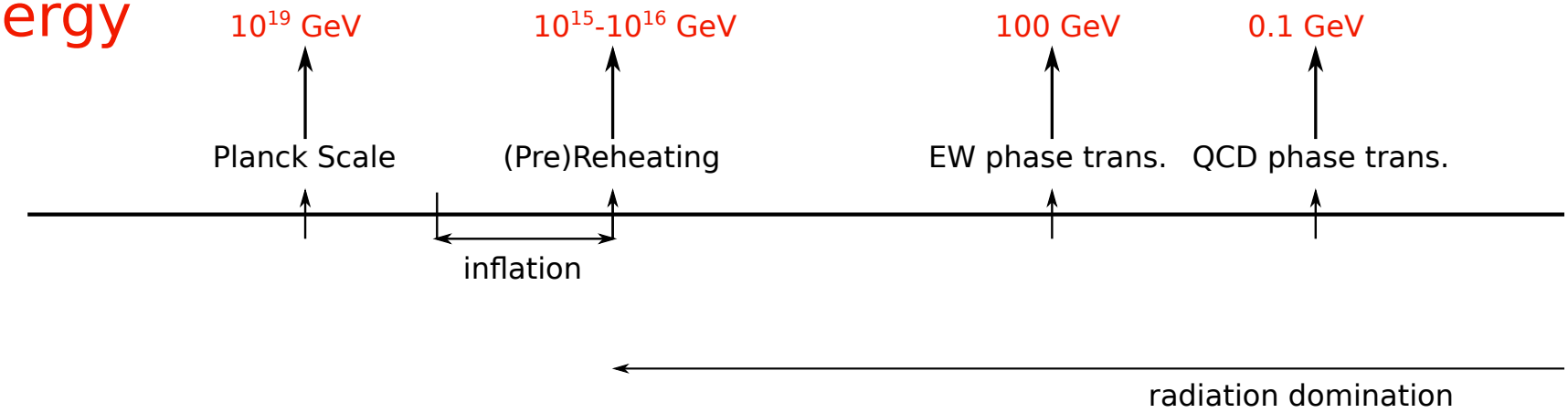
Jan Schütte-Engel

Cosmology from Home 2023

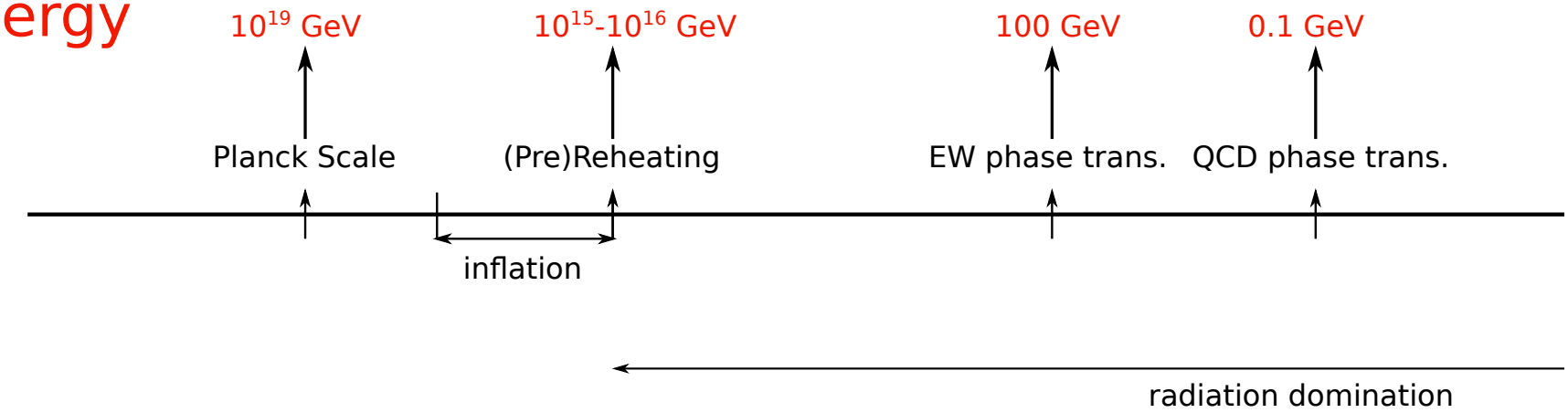
based on

[Ringwald, **JSE**, Tamarit 20], [Ghiglieri, **JSE**, Speranza 22]

Energy


10^{19} GeV
10^{15}-10^{16} GeV
100 GeV
0.1 GeV
Planck Scale
(Pre)Reheating
EW phase trans.
QCD phase trans.
inflation
radiation domination

Energy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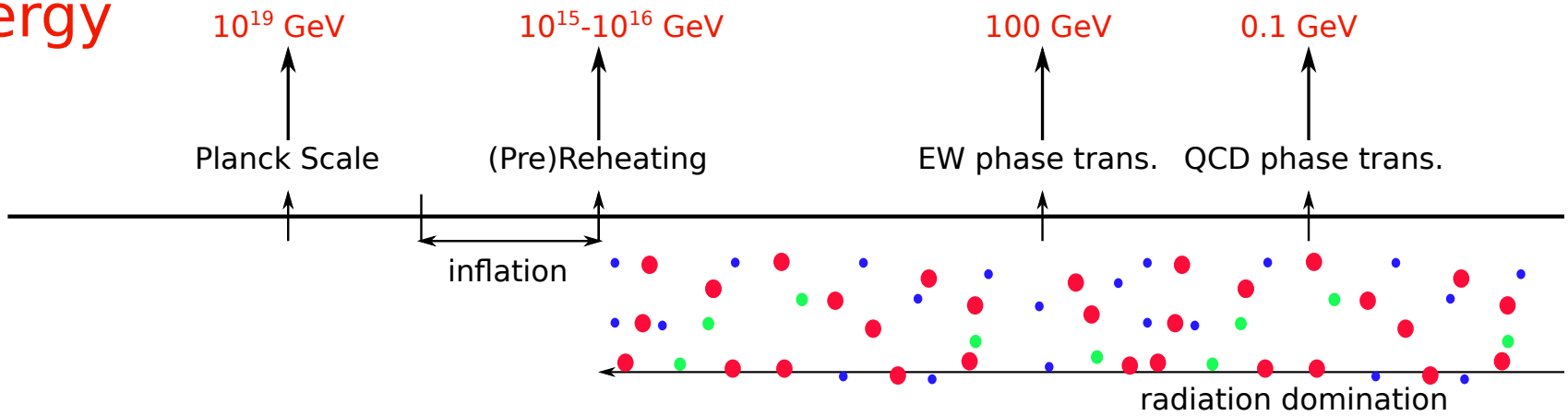
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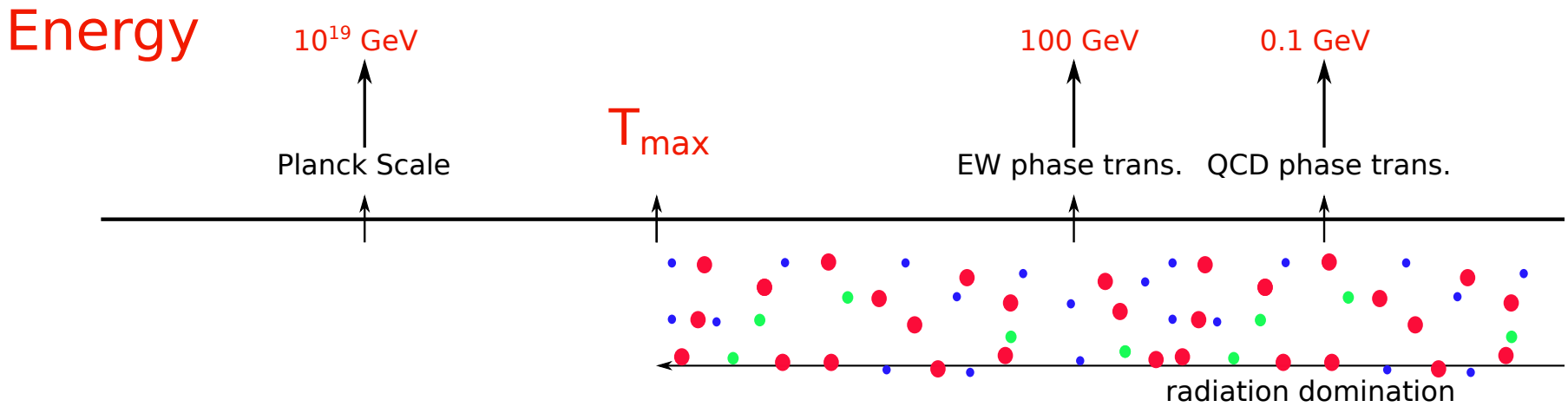
Energy
10^{19} GeV
Planck Scale
T_{max}
100 GeV
EW phase trans.
0.1 GeV
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radiation domination

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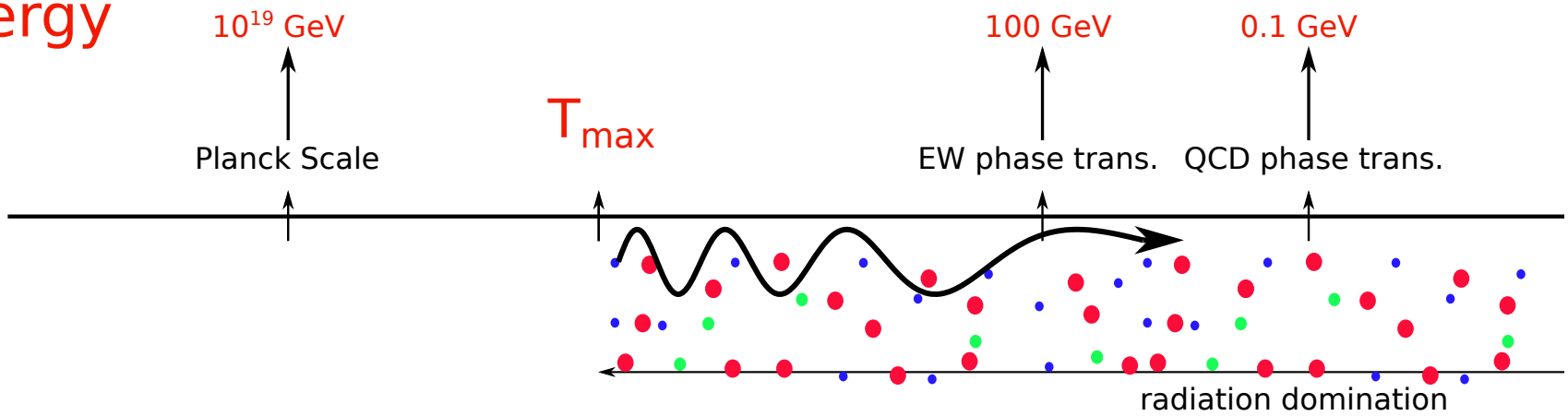
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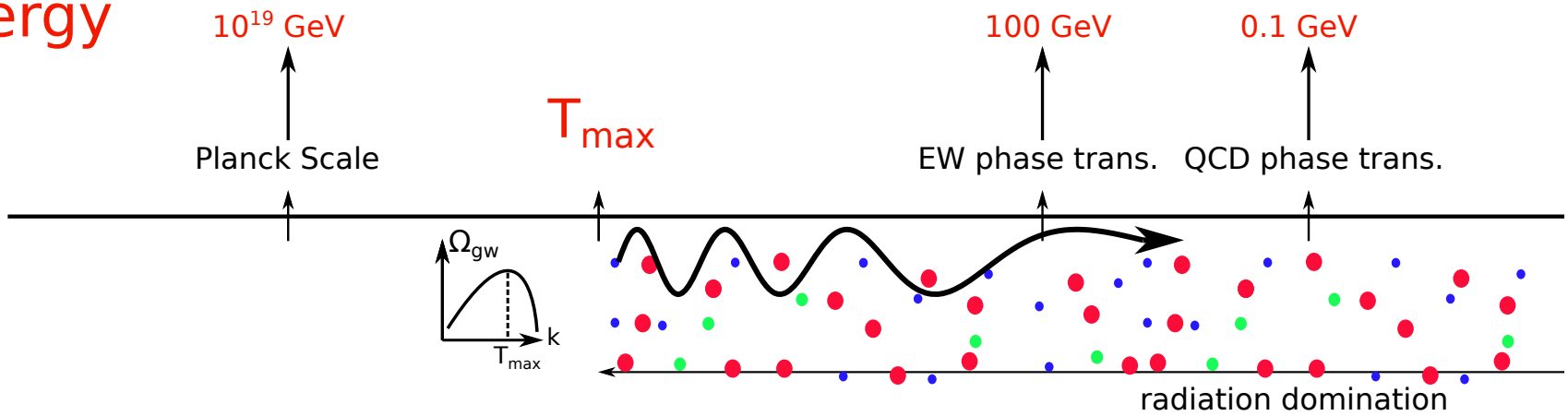
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Planck Scale
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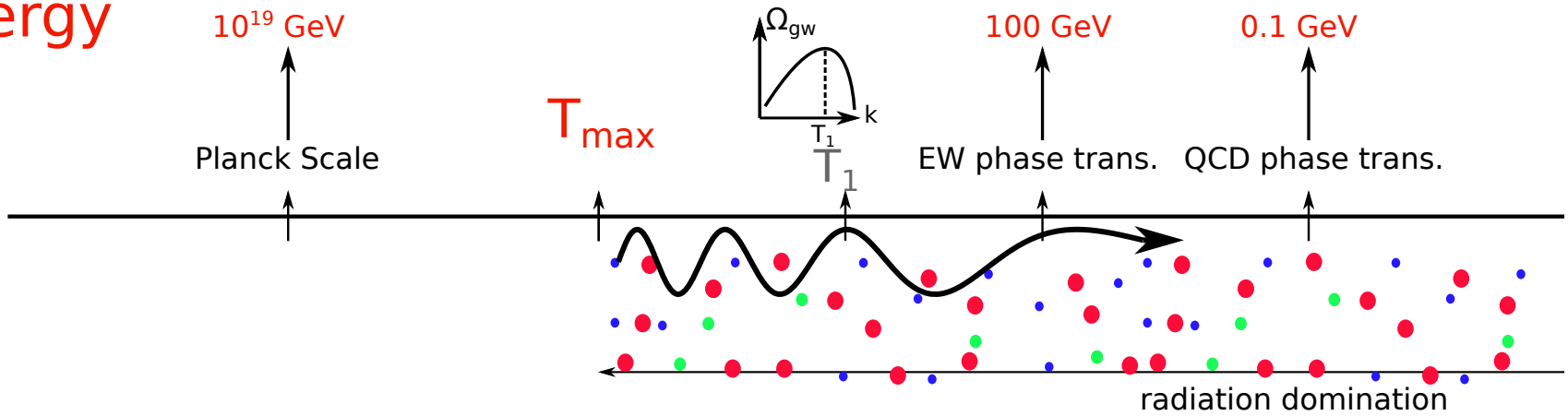
QCD phase trans.
Ω_{gw}
k
T_{max}
radiation domination

Energy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k
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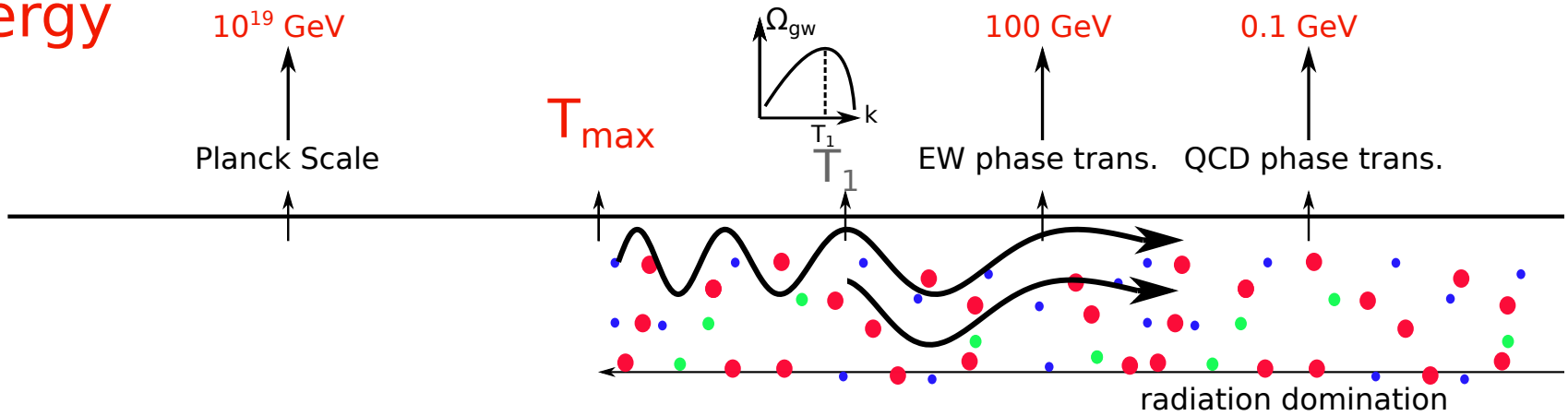
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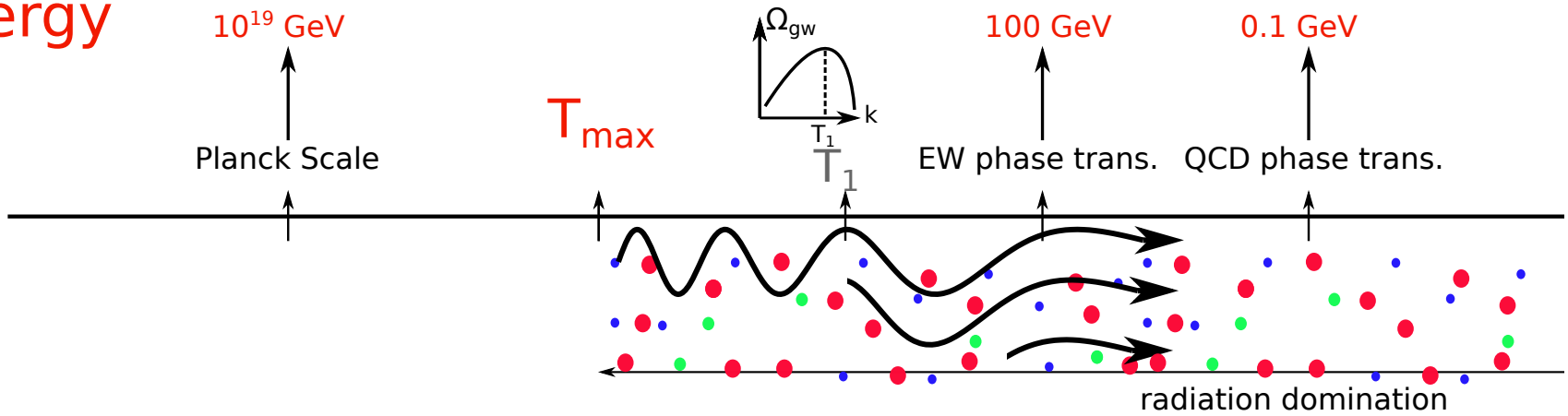
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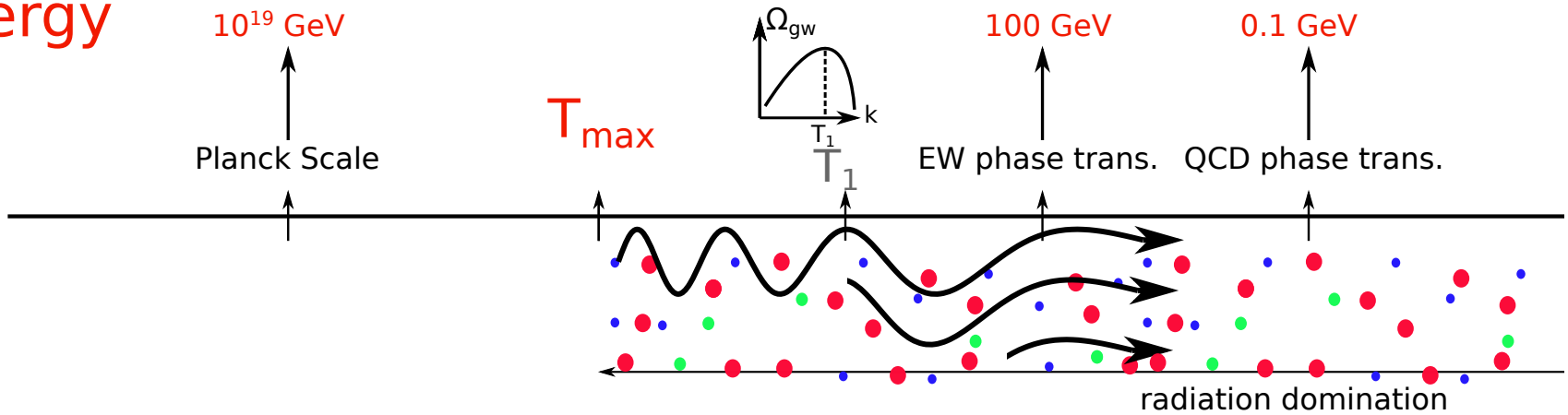
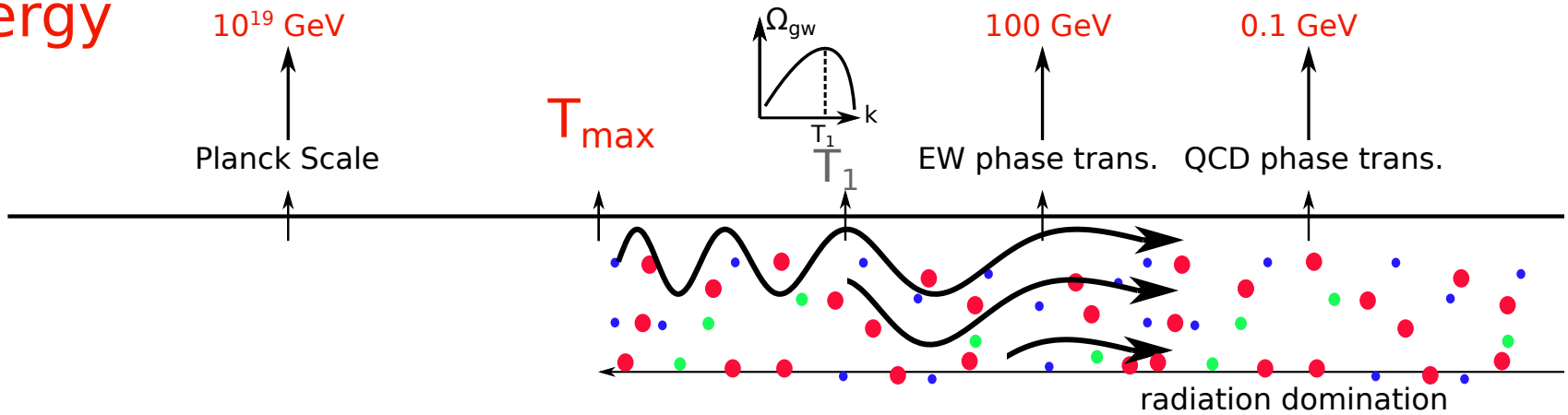
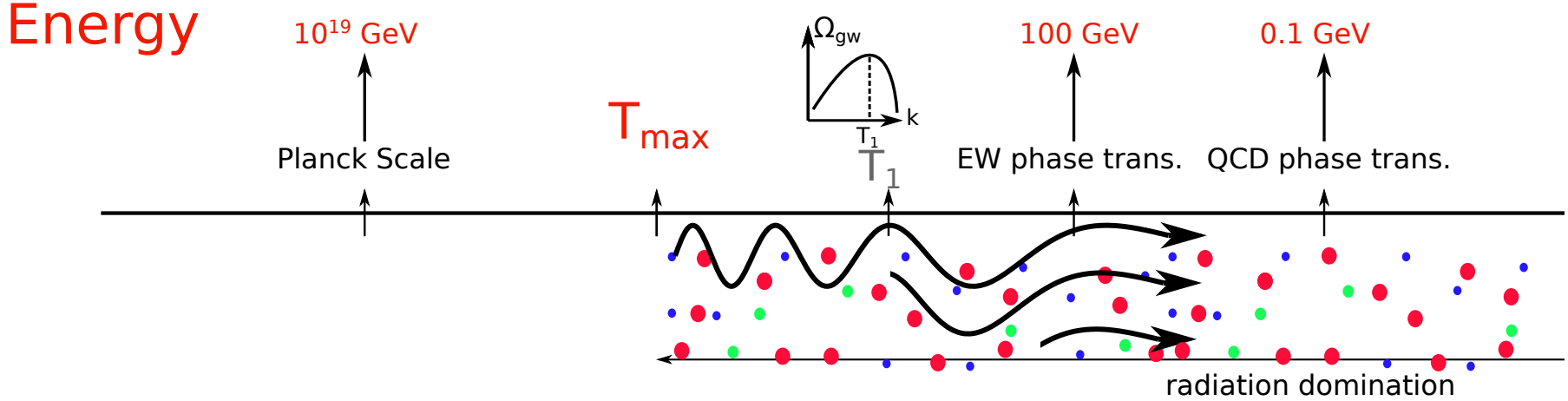


$$\omega_g^{\text{today}} = \left(\frac{g_{*s}(T_0)}{g_{*s}(T_{\max})}\right)^{\frac{1}{3}} \frac{k_{\max}}{T_{\max}} T_0$$

Energy



$$\omega_g^{\text{today}} = \left(\frac{g_{*s}(T_0)}{g_{*s}(T_{\max})}\right)^{\frac{1}{3}} \frac{k_{\max}}{T_{\max}} T_0 \approx \left(\frac{g_{*s}(T_0)}{g_{*s}(T_1)}\right)^{\frac{1}{3}} \frac{k_1}{T_1} T_0$$



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Cosmic Gravitational Microwave Background (CGMB)

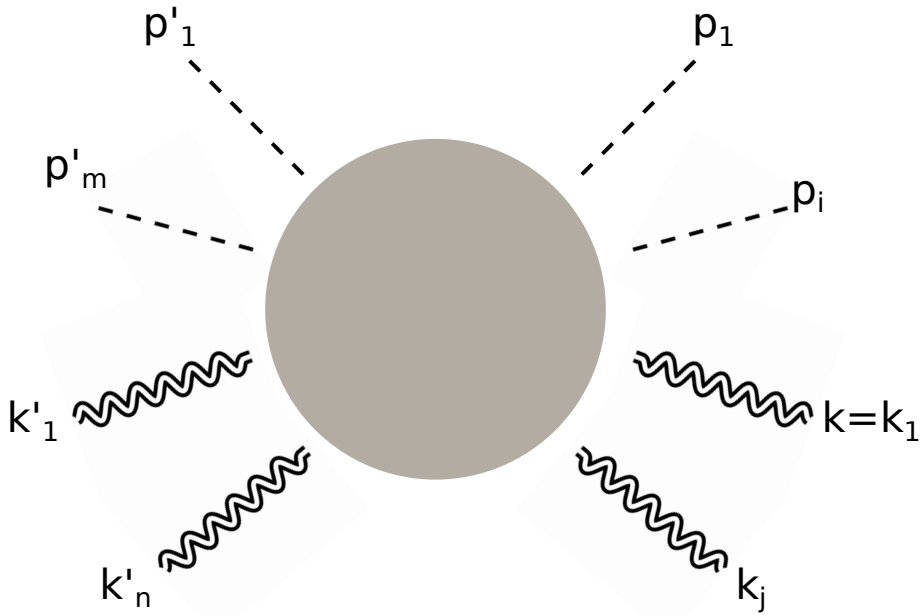
Distribution functions

$$f_\phi(t,k) \equiv \frac{\text{Number of } \phi\text{-states with momentum } k \text{ in } d^3k \text{ interval}}{V\, d^3k/(2\pi)^3},$$

$$f_h(t,k) \equiv \frac{\text{Number of gravitons with momentum } k \text{ in } d^3k \text{ interval}}{V\, d^3k/(2\pi)^3},$$

Evolution equations

$$\begin{aligned} \dot{f}_\phi(t,k) &= G_\phi(t,k) - L_\phi(t,k), \\ \dot{f}_h(t,k) &= G_h(t,k) - L_h(t,k), \end{aligned}$$



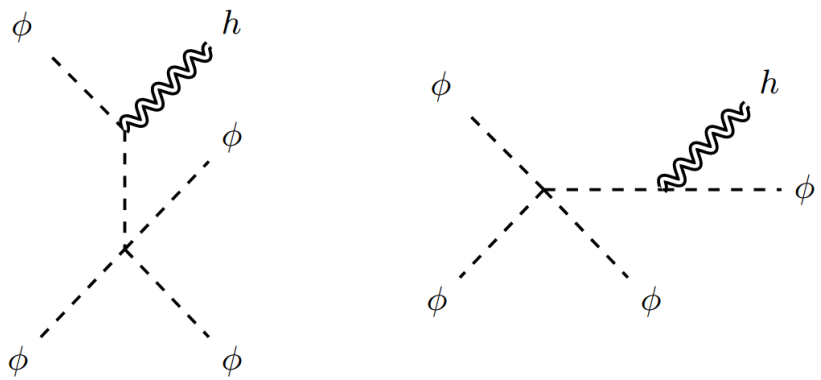
$$G_h(t,k) = \frac{1}{4k} \sum_{\substack{\text{all processes } r \\ \text{with at least one} \\ \text{final state graviton}}} S_r \int d\Omega_r \; |\mathcal{M}_r|^2 \times f_\phi(p_1') \cdots f_\phi(p_m')\, f_h(k_1') \cdots f_h(k_n') \times$$

$$\times \left(1 + f_\phi(p_1)\right) \cdots \left(1 + f_\phi(p_i)\right) \left(1 + f_h(k)\right) \cdots \left(1 + f_h(k_j)\right).$$

Lowest order processes

$$f_h(k) = 0 + f_h^{(2,2)}(k) + f_h^{(0,4)}(k),$$

Single graviton production



$$|\mathcal{M}|^2 \sim \lambda^2 \left(\frac{1}{m_p}\right)^2$$

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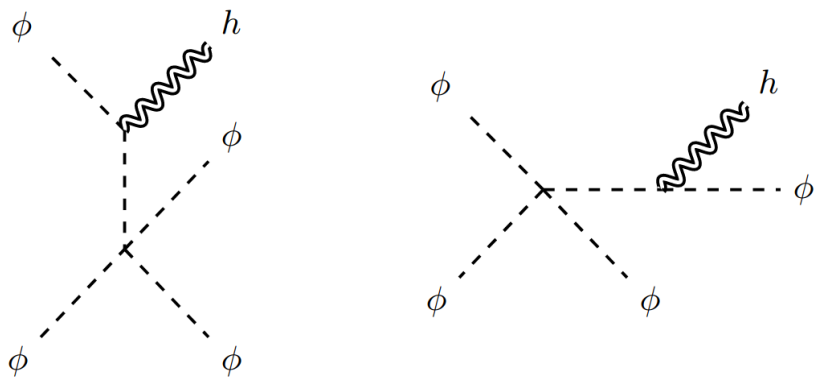

$$|\mathcal{M}|^2 \sim \lambda^2 \left(\frac{T_{\max}}{m_p}\right)^2$$

Lowest order processes

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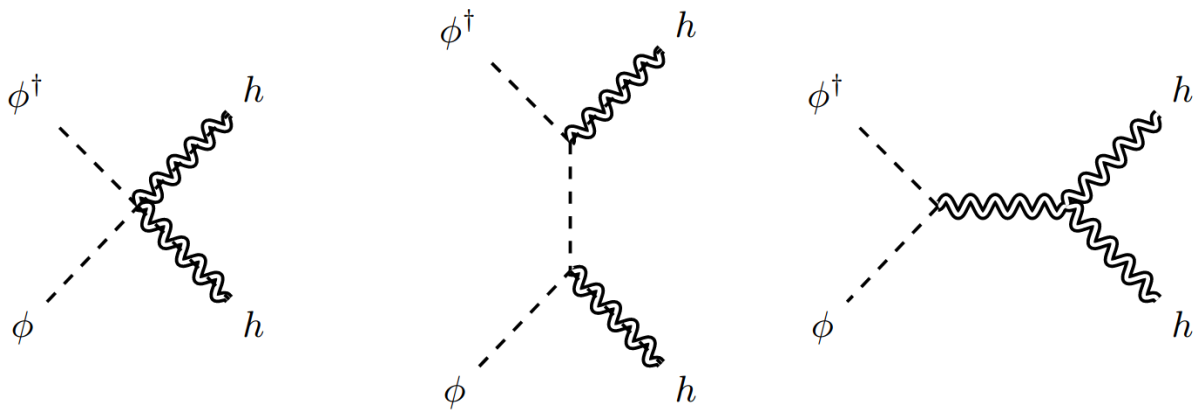
graviton pair production



$$|\mathcal{M}|^2 \sim \left(\frac{1}{m_p}\right)^4$$

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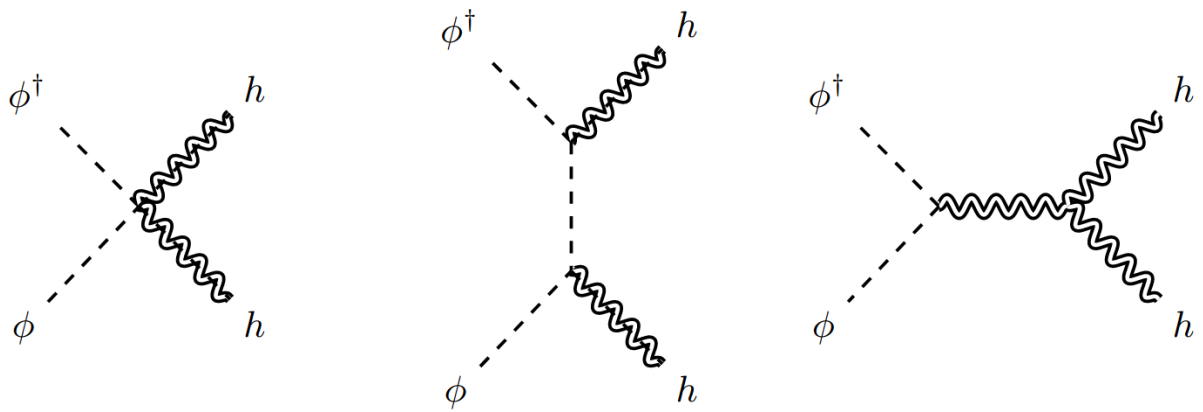

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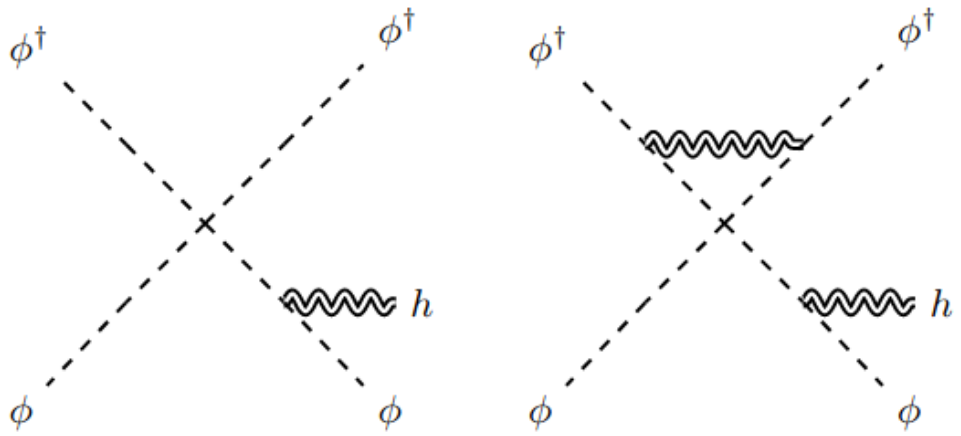
Quantum Gravity effects

An example:



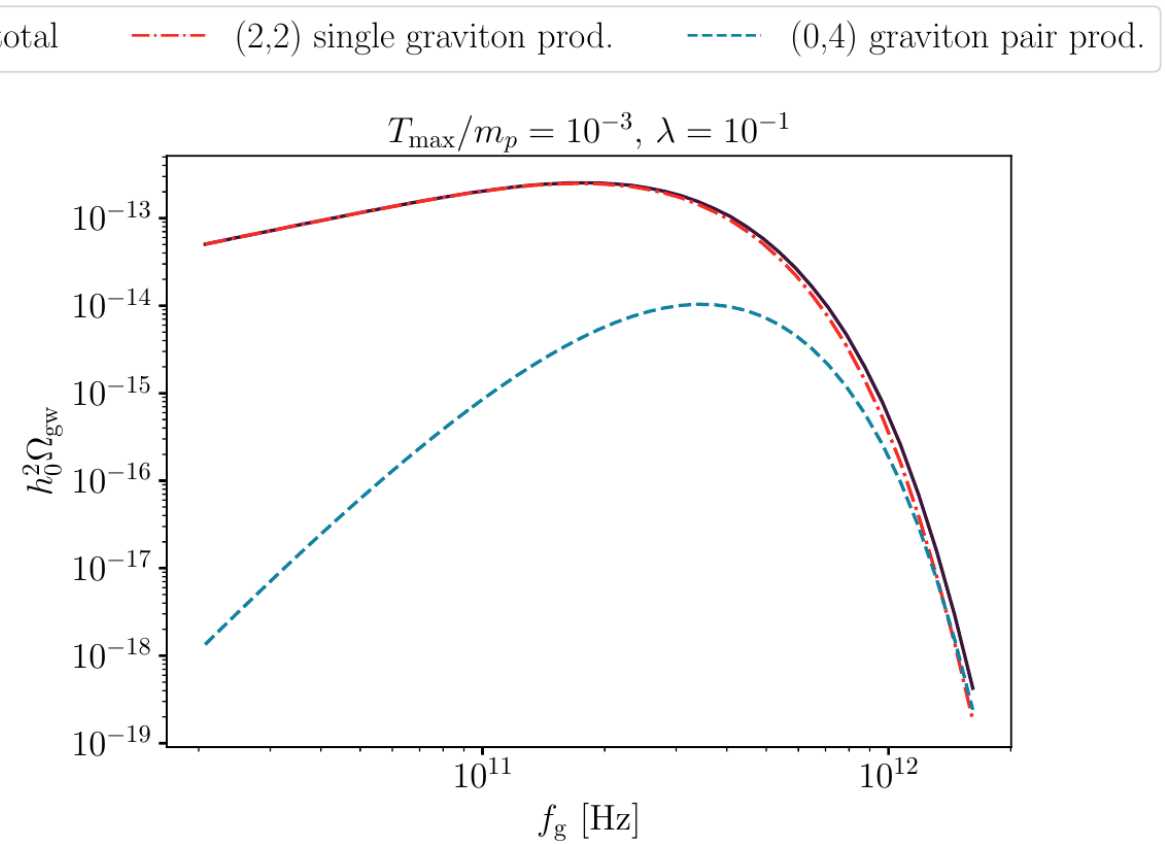
Interference

$$|\mathcal{M}|^2 \sim \lambda^2 \left(\frac{T_{\max}}{m_p}\right)^4$$

[Ghiglieri, **JSE**, Speranza, in progress]

$$h_0^2 \Omega_{\rm gw}(f_{\rm g}) \sim \frac{T_{\rm max}}{m_{\rm p}} n_{\rm B}(y_{\rm max}) \left(\lambda^2 \psi^{(2,2)}(y_{\rm max}) + \frac{1}{3} \left(\frac{T_{\rm max}}{m_{\rm p}} \right)^2 \psi^{(0,4)}(y_{\rm max}) + \cdots \right)$$

with $y_{\rm max} \equiv \frac{2\pi f_{\rm g}}{T_{\rm today}} \left(\frac{g_{*\rm s}(T_{\rm max})}{g_{*\rm s}(T_{\rm today})} \right)^{1/3} = 0.14 \left(\frac{f_{\rm g}}{10^{10}\ {\rm Hz}} \right) \left(\frac{g_{*\rm s}(T_{\rm max})}{2} \right)^{1/3}$



[Ghiglieri, **JSE**, Speranza 22]

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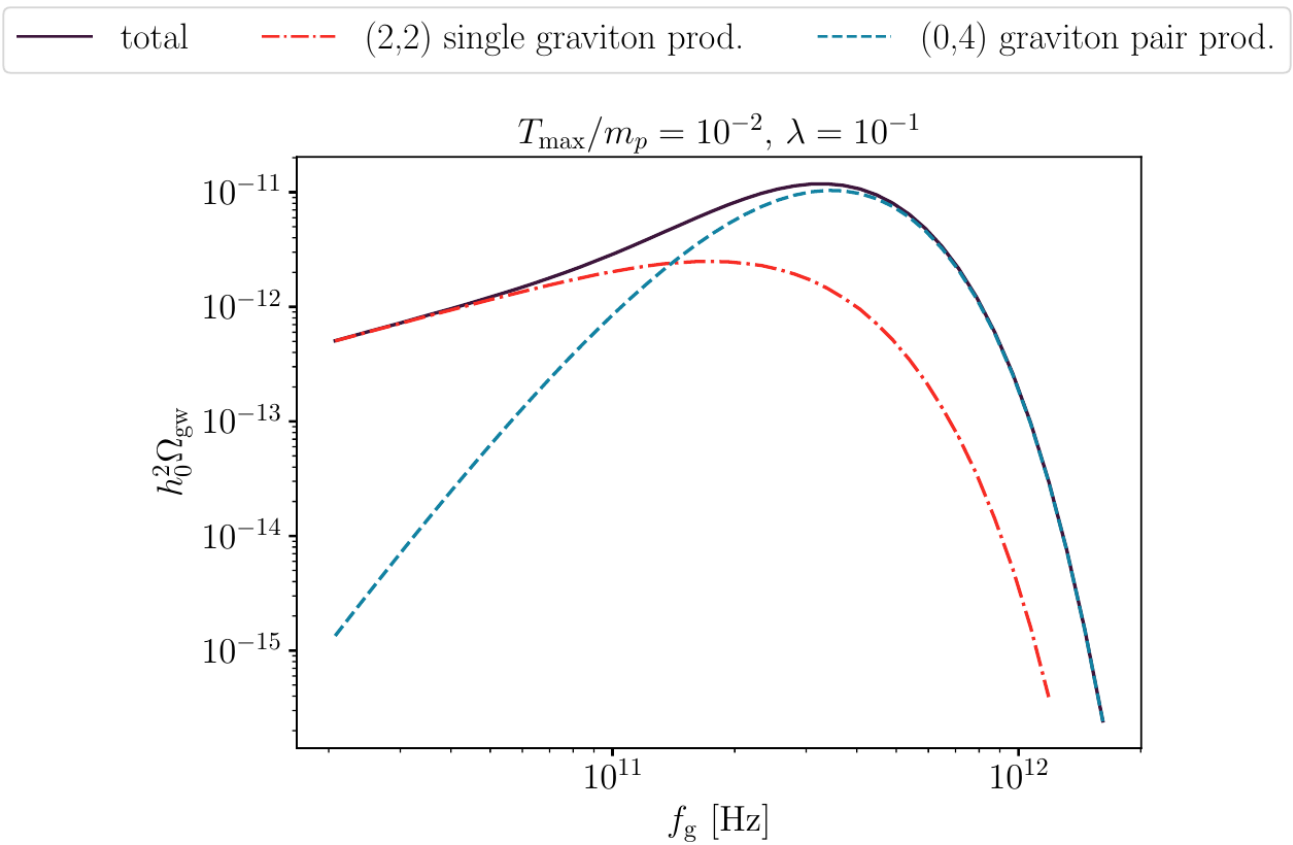

[Ghiglieri, **JSE**, Speranza 22]

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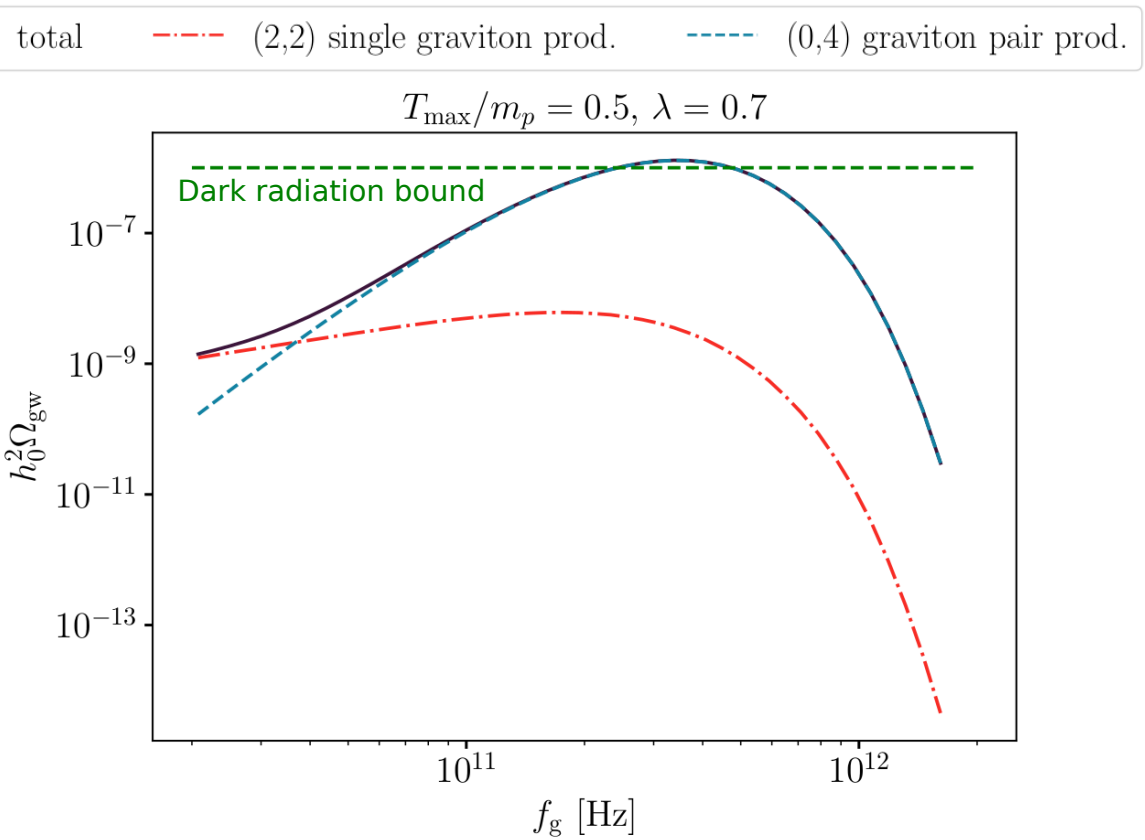
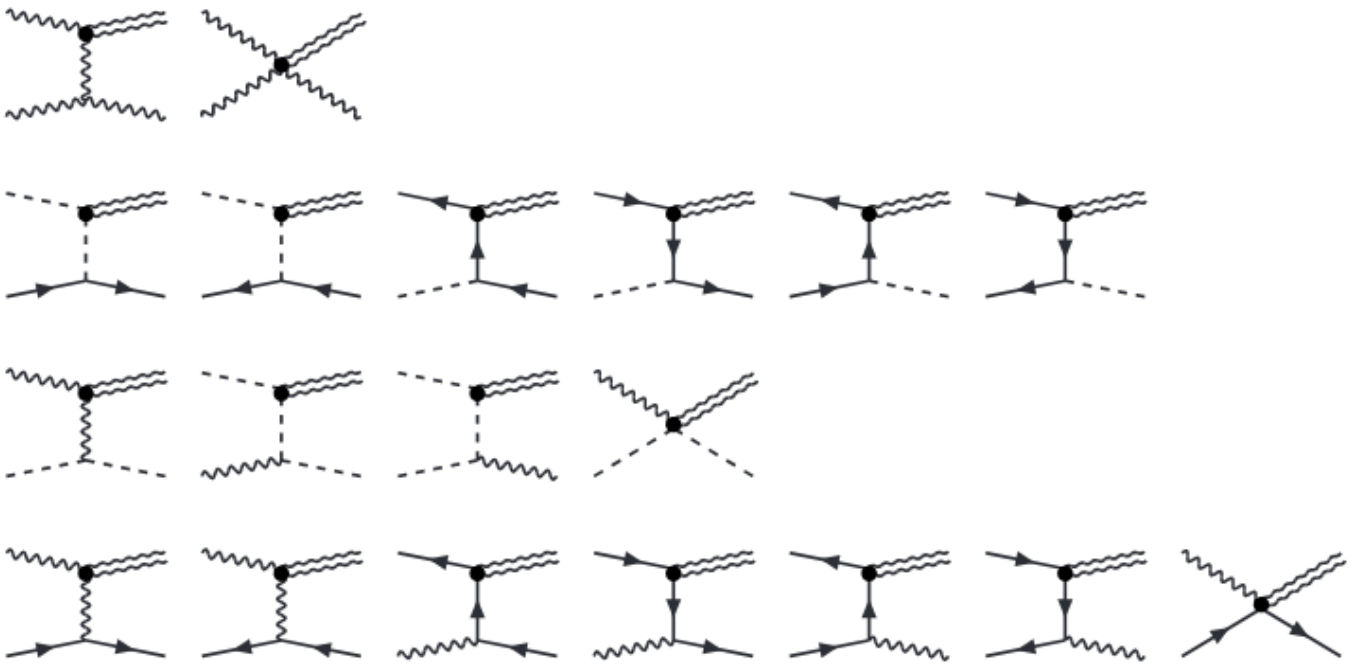


[Ghiglieri, **JSE**, Speranza 22]

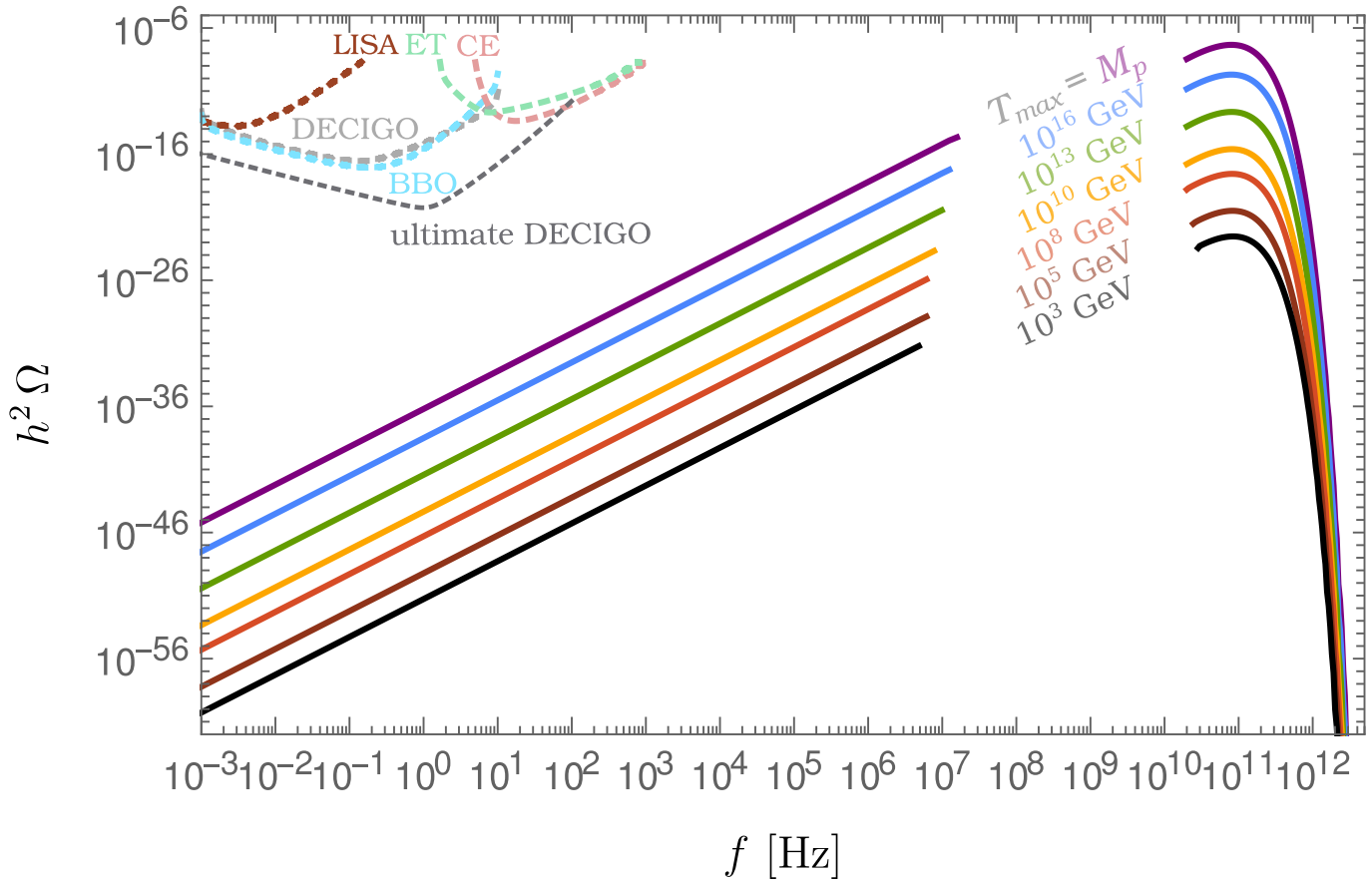
The CGMB spectrum in the Standard Model



[Ghiglieri, Laine 15],[Ghiglieri, Jackson, Laine, Zhu 20]

(only single graviton production processes included)

The CGMB spectrum in the SM and beyond



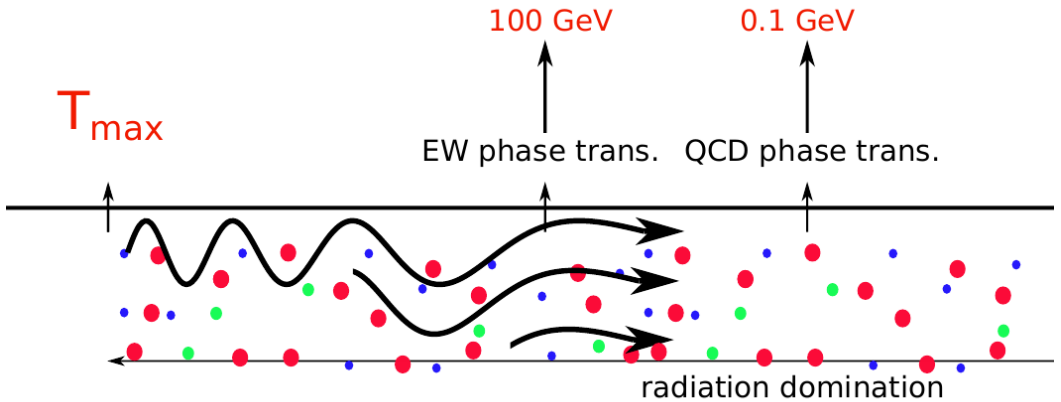
[Ghiglieri, Jackson, Laine, Zhu 20], [Ringwald, **JSE**, Tamarit 20]

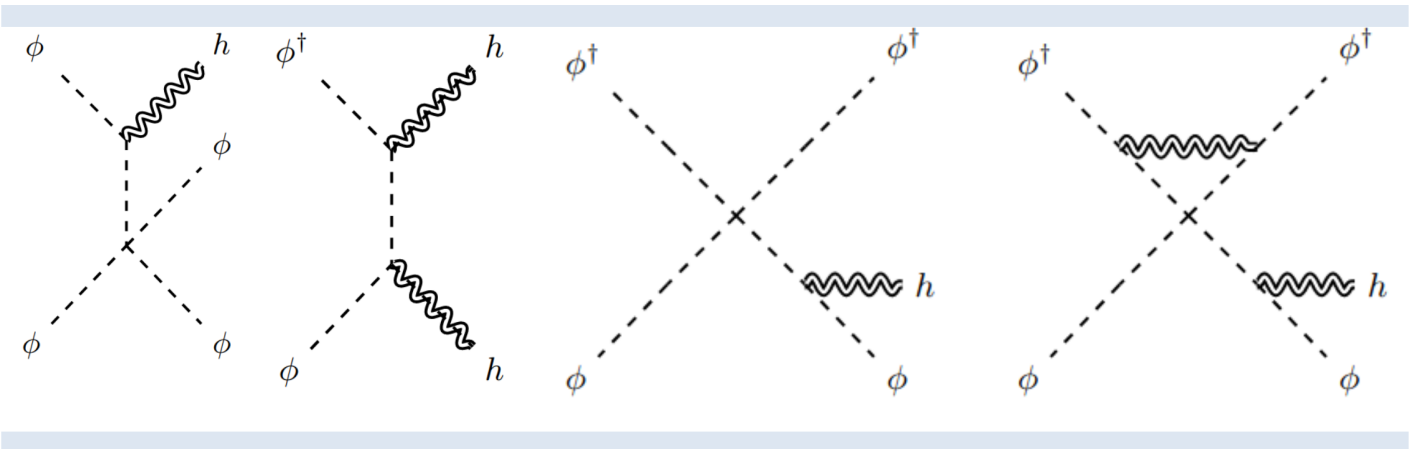
In BSM theories:

$$f_{\rm peak}(T_{\rm max}) \sim \left[\frac{106.75}{g_{*s}(T_{\rm max})}\right]^{\frac{1}{3}}$$

	SM	MSSM
$g_{*s}(T_{\rm max})$	106.75	228

Conclusions





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