Learning Linear Cosmological Physics

Cosmology From Home 2023

Jamie Sullivan
(work w/ Zack Li)

Bolt - The Differentiable Boltzmann Solver

Linear Einstein-Boltzmann solver in Julia

Automatic Differentiation (AD) gradients

~2 seconds for a redshift zero linear power spectrum!

Within a factor of 2 of camb/CLASS!

See last-year's CFH talk for more: https://www.cosmologyfromhome.com/wp-content/uploads/2022/06/cfh2022_Sullivan_James.pdf

Also github: https://github.com/xzackli/Bolt.jl

Motivation: Theoretical Progress

Cosmology in the decadal survey:

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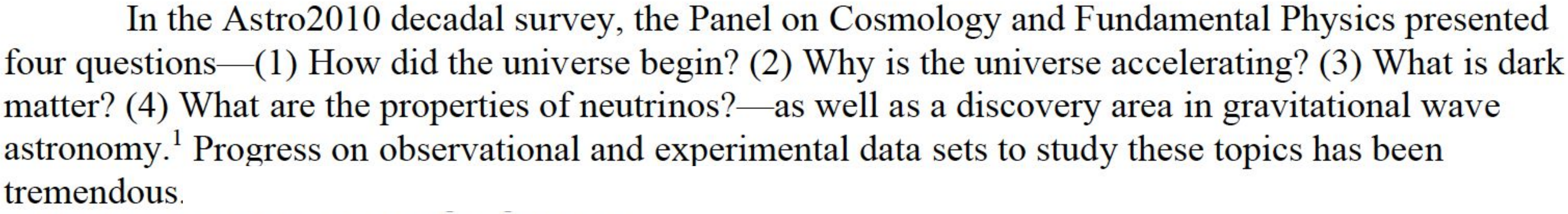

In the Astro2010 decadal survey, the Panel on Cosmology and Fundamental Physics presented four questions—(1) How did the universe begin? (2) Why is the universe accelerating? (3) What is dark matter? (4) What are the properties of neutrinos?—as well as a discovery area in gravitational wave astronomy.[1] Progress on observational and experimental data sets to study these topics has been tremendous.

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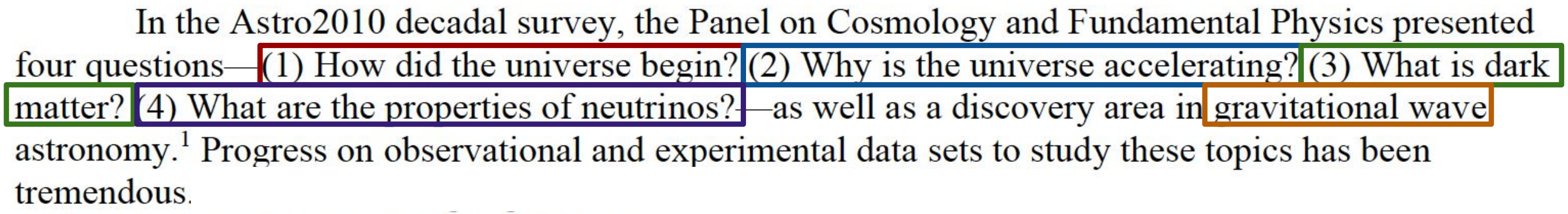

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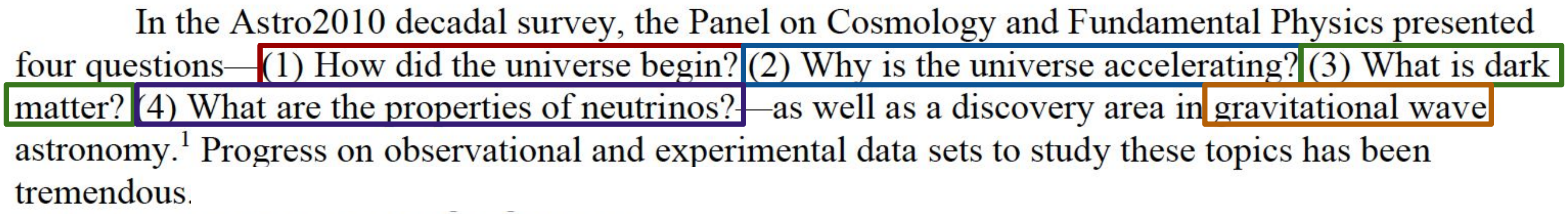

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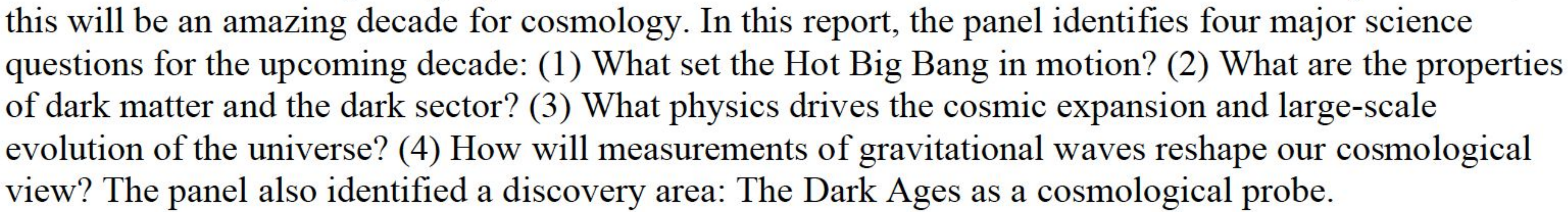

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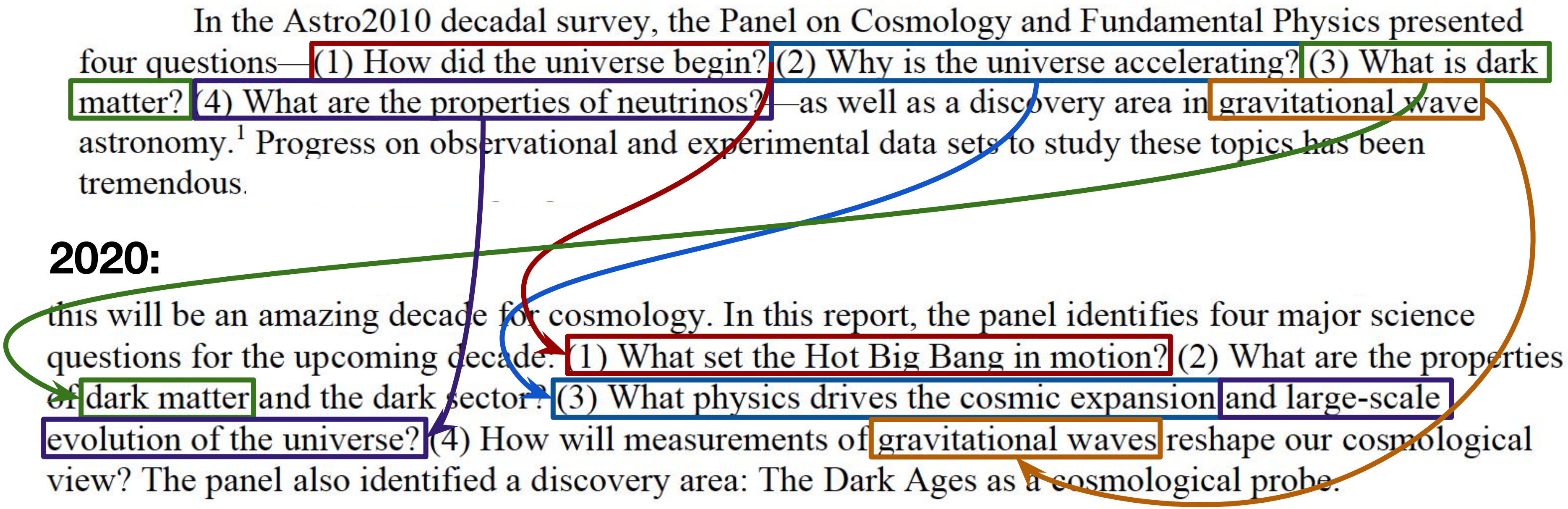

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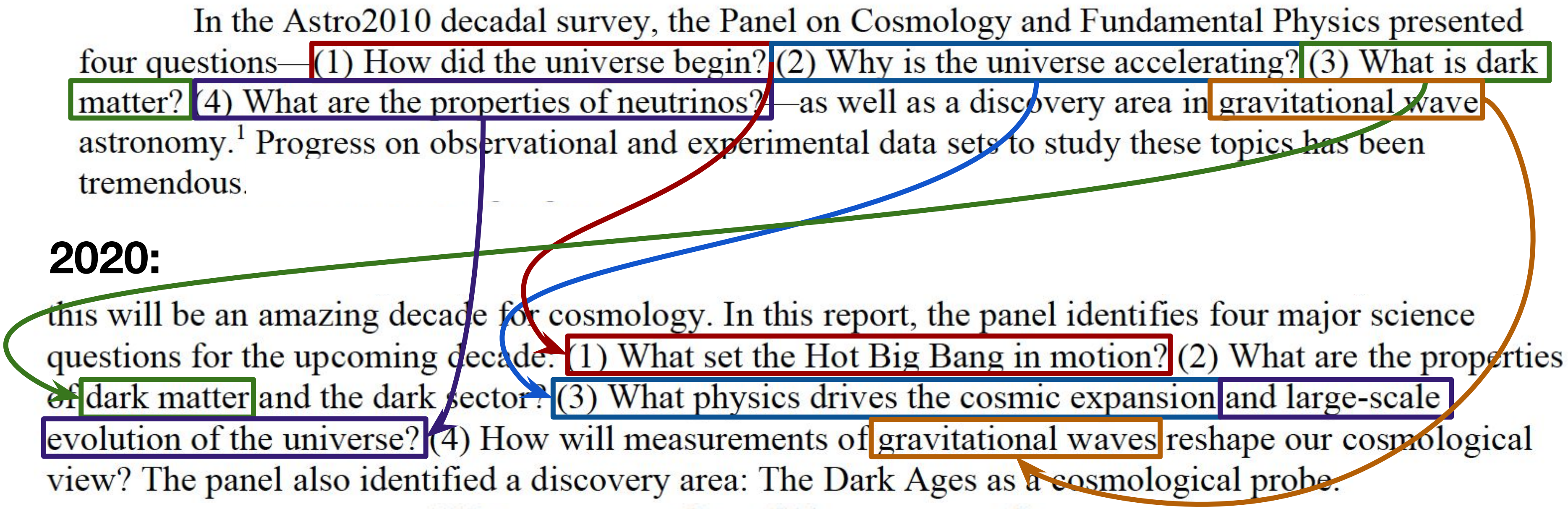

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These are familiar questions.

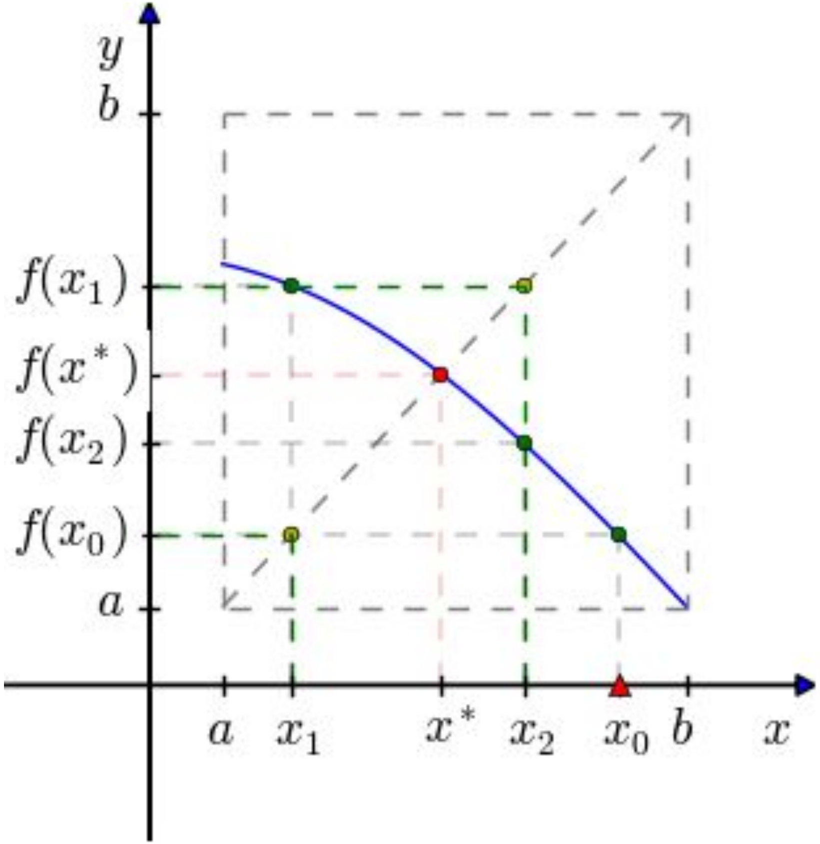
Motivation: Theoretical Progress

Current data makes model extensions *hard*!

How do we facilitate new model development?

1. Remove standard assumptions
2. Aim for (nuanced) “model discovery” with ML

Iterative Methods in Bolt



To Infinity…and Beyond!

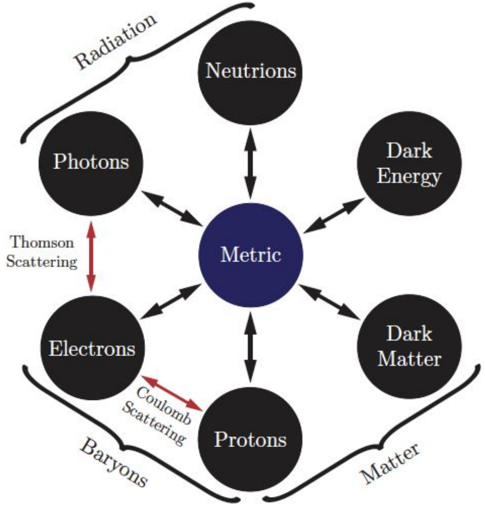
Fully-general E-B system is **stiff**

Stiff solvers require accurate Jacobians

Achieve this with AD,
but linear solve bottleneck

Linear solve scales like
number of perturbations

Set by **infinite** species boltzmann hierarchies



Baumann (adapted)

Iterative Methods

Instead of infinite hierarchy of photons + neutrinos can write:

Kamionkowski 21, Ji++22

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Instead of infinite hierarchy of photons + neutrinos can write:

$$\begin{aligned}\mathcal{M}_\ell(k,\tau,q) = {} & \sum_{\ell'}(-1)^{\ell'}(2\ell'+1)W_{\ell\ell'}\left(k\chi(\tau_i,\tau)\right)\Psi_{\ell'} \\ & + \boxed{\int_{\tau_0}^{\tau} d\tau' \frac{d\ln f_0}{d\ln q}\left(\dot{\Phi} j_\ell(k\chi(\tau',\tau)) + k\frac{\epsilon}{q}\Psi j'_\ell(k\chi(\tau',\tau))\right)}\end{aligned}$$

$$\Theta_\ell(k,\tau) = \boxed{\begin{aligned}\int_{\tau_i}^{\tau} d\tau' \frac{g(\tau,\tau')}{\dot{\kappa}(\tau')}\Big(& \left[\dot{\kappa}(\tau')\Theta_0(k,\tau') - \dot{\Phi}(k,\tau')\right] j_\ell(x) \\ & + \left[\dot{\kappa}(\tau')v_b(k,\tau') - k\Psi\right] j'_\ell(x) \\ & - \frac{1}{2}\dot{\kappa}(\tau')\Pi(k,\tau')R_\ell(x)\Big)\end{aligned}}$$

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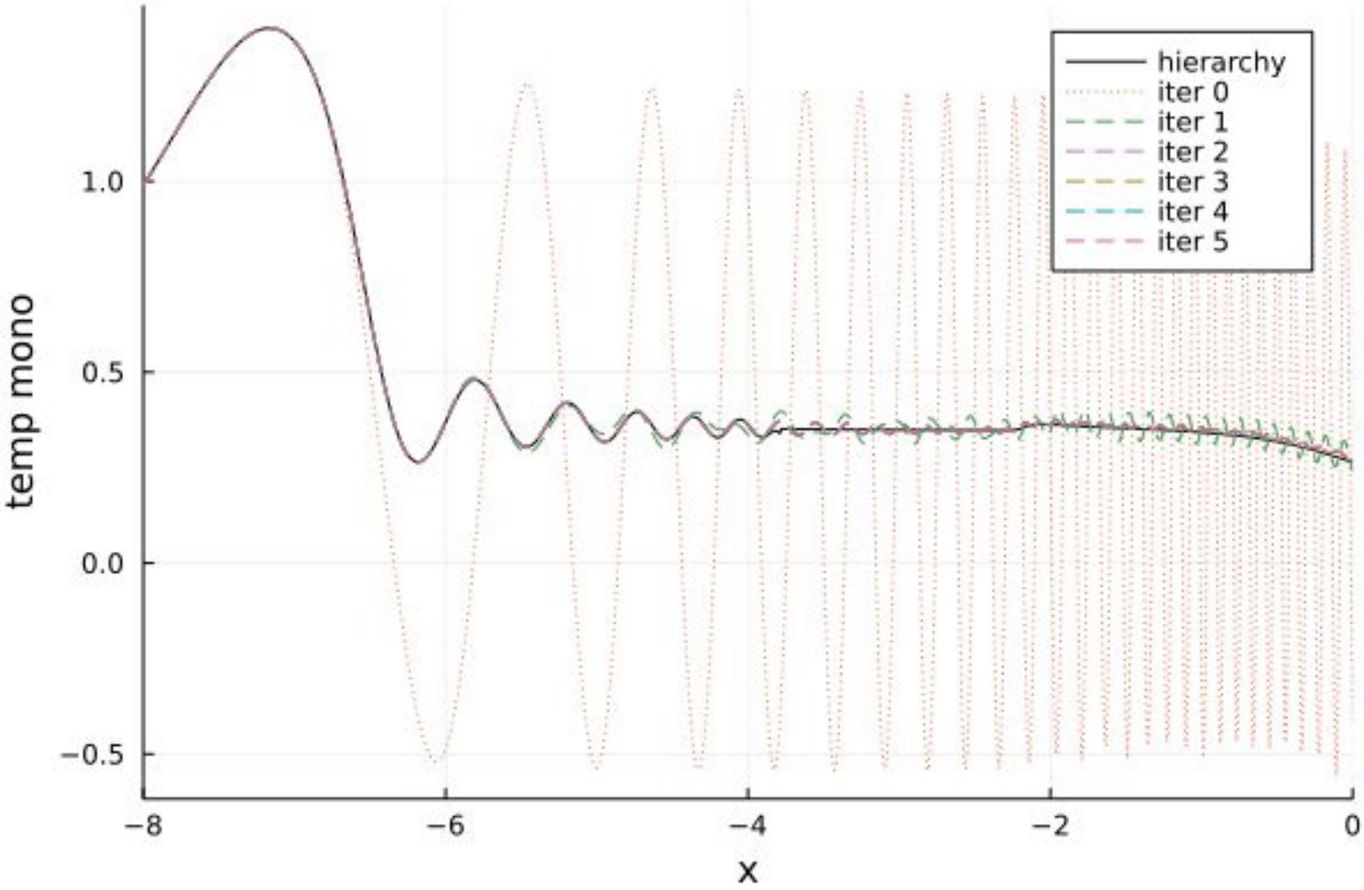
Replace ODE evolution with FFT and quadrature

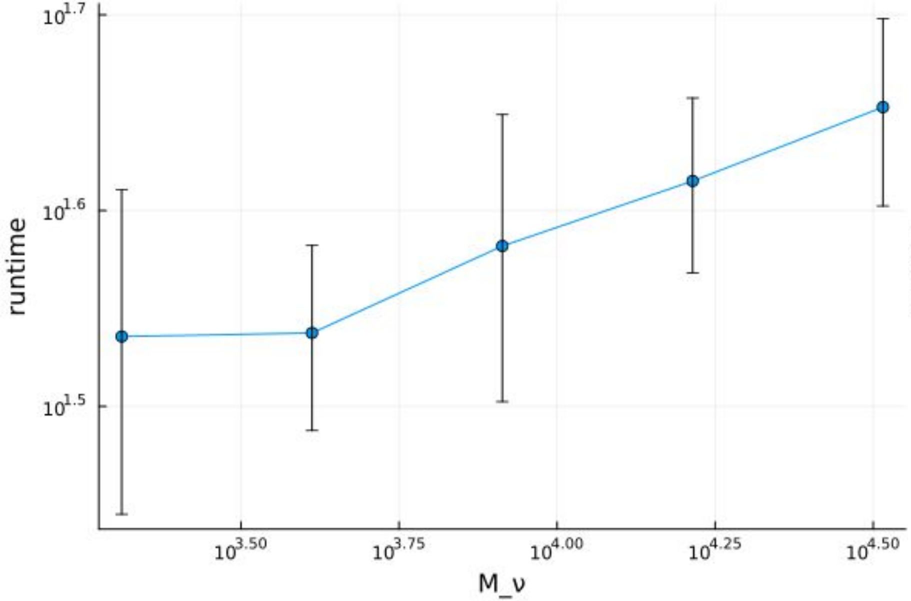
Kamionkowski 21, Ji++22

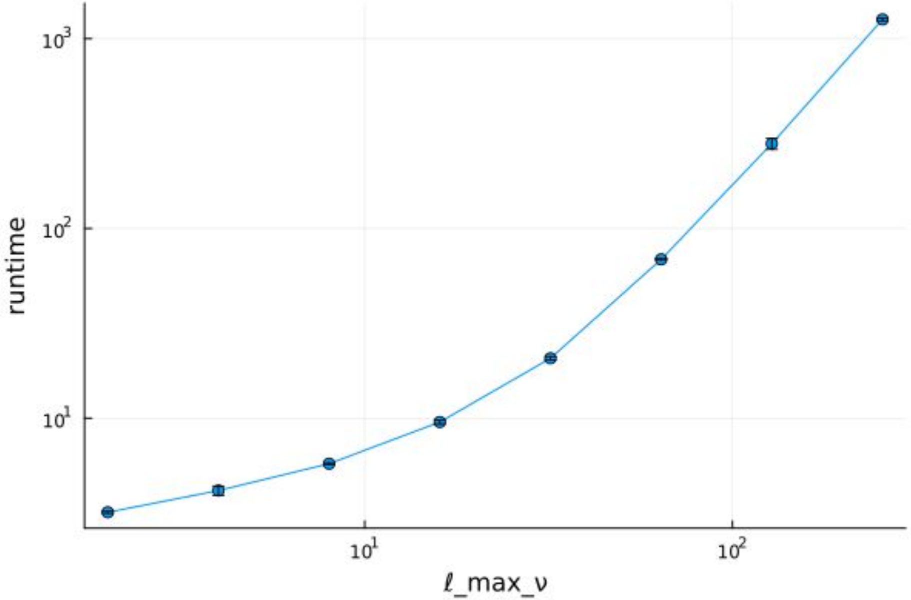
Picard Iteration Converges!



Picard Iteration Converges!

Runtime scaling is as expected!





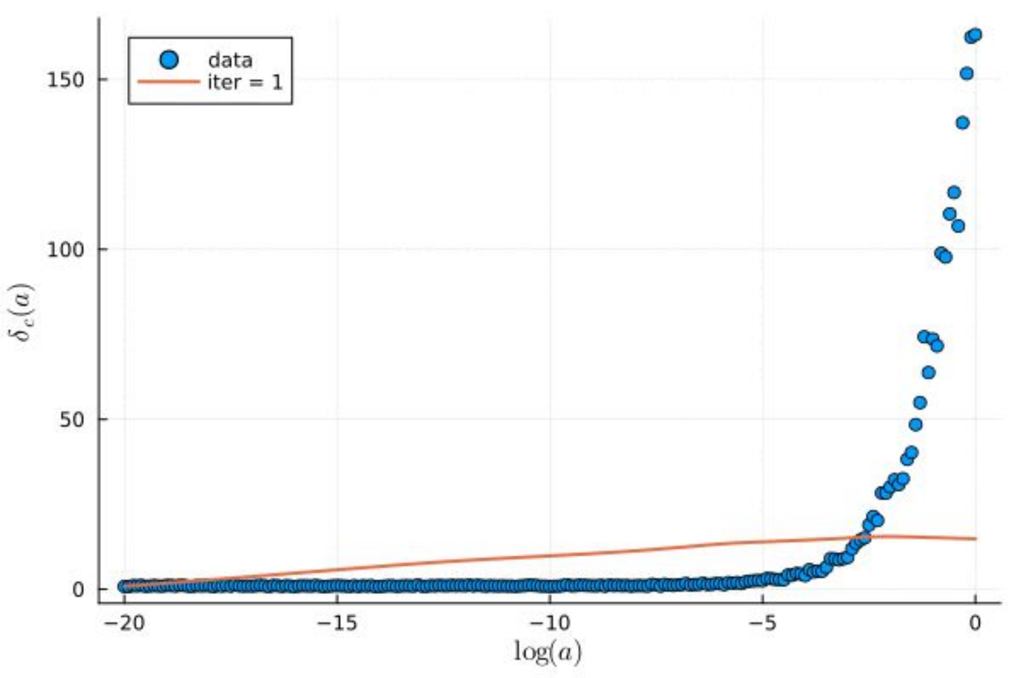
The Next Iteration of Bolt

For fiducial choices, iterative time is ~ same as hierarchy

But this can be improved!

- Optimizations (e.g. photon window)
- Quantifying convergence rate
- MCMC nearby ansatz

Learning Cosmological Physics



A new workflow for linear cosmology?

How do we learn unknown physics?

Background has been explored, but now preturbations!

Embed neural network *inside* ODE function:

$$\delta_i'(k,u,x) = NN_1^{(\theta)}(u,x)$$
$$v_i'(k,u,x) = NN_2^{(\theta)}(u,x)$$

As flexible as you desire! (or dare)

Farhang++12,Liu++19,Hart+Chluba20,Lee++23

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Pretend we “forgot” CDM linear theory - can it be learned?

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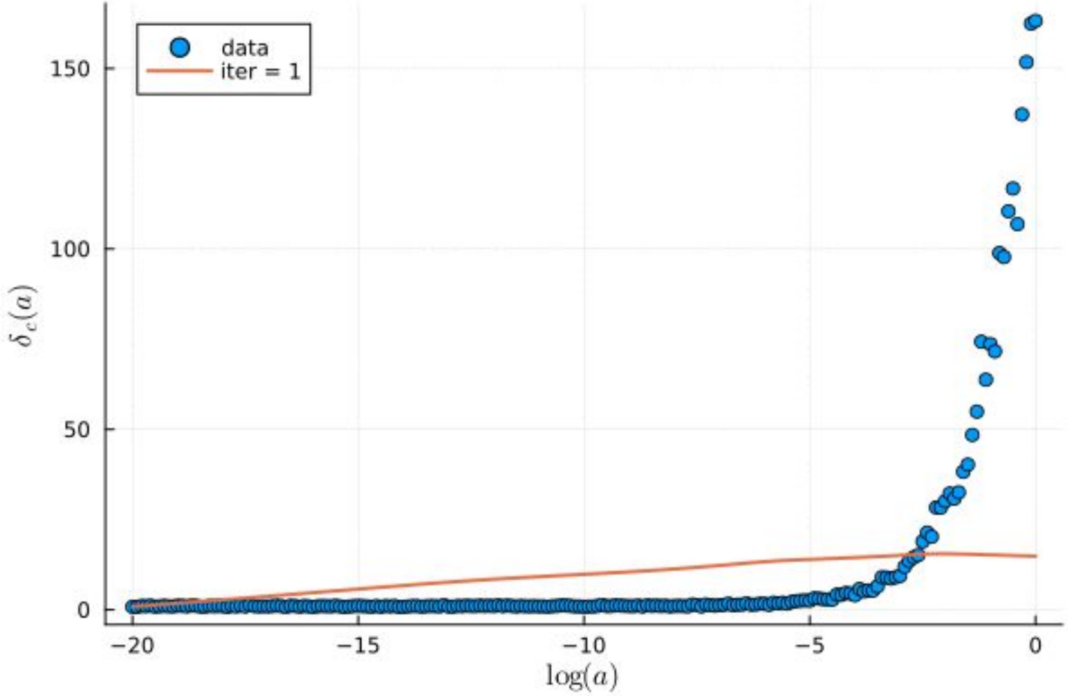

Yes!*

Technical Challenges of NNs in ODEs

Reverse mode vs forward mode automatic differentiation

Prefer to use adjoint state method

High-dimensional optimization of NNs

Software framework: SimpleChains vs Flux vs Lux vs …

Pontryagin62, Griewank+Walther08, Rackauckas20

What’s next? - NN Uncertainty!

NNs in ODEs a step forward in flexibility

But does **not** tell you **where** to focus model-building efforts

Workflow goal is to guide **human** model building

Luckily, we can obtain uncertainty estimates for neural network predictions - current work!

Further development

Apply to (multiple) realistic datasets

When does NN predictive power end?

Where does “theory” want more data?

Stay tuned!

Summary

Bolt **differentiably solves** the stiff E-B system

Leverage this to **relax standard assumptions**

Iterative methods can compete with the Boltzmann hierarchy

Flexible neural network models will **guide model builders** toward **missing physics**