Tracking the validity of the quasi-static and sub-horizon approximations in modified gravity

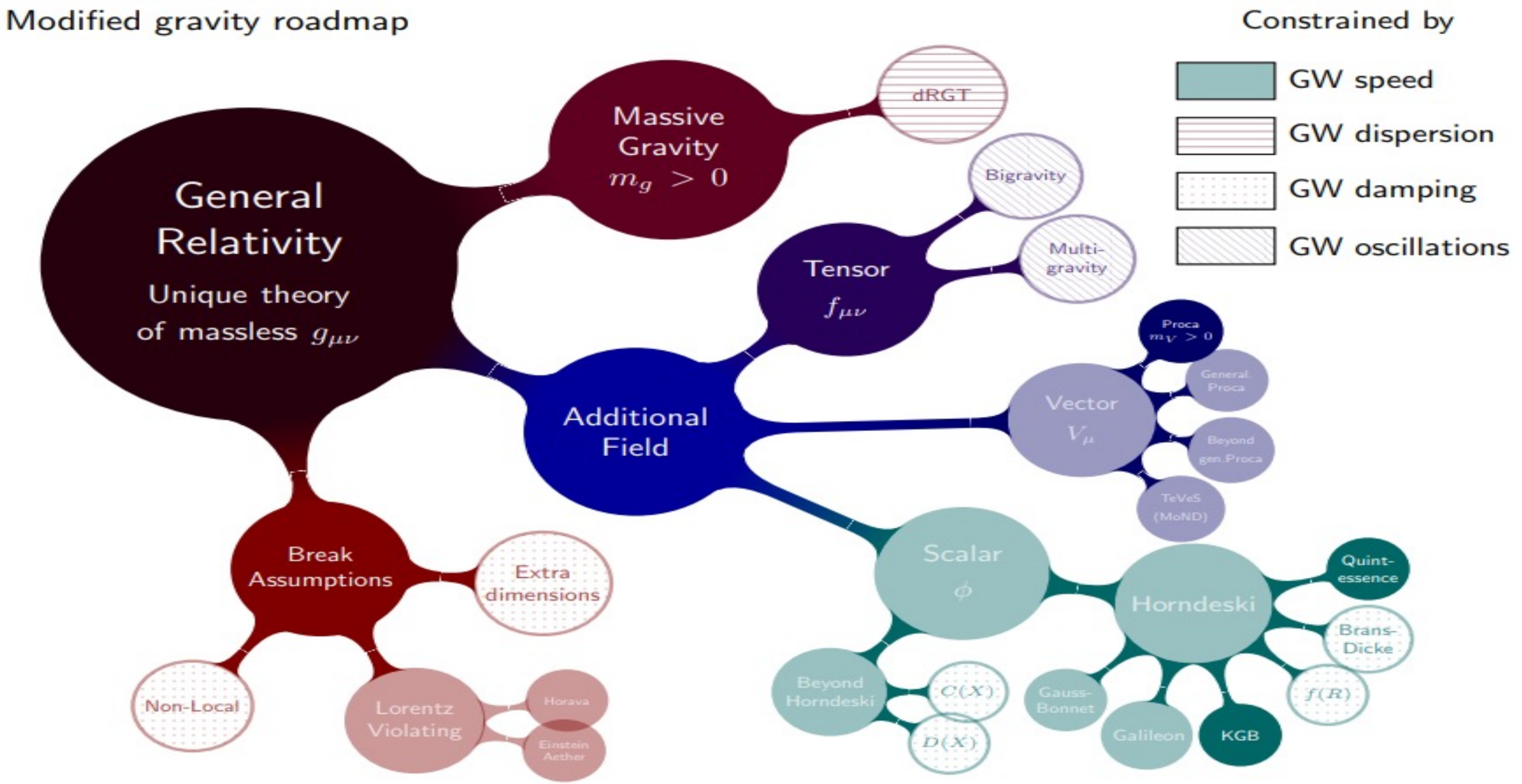
J. Bayron Orjuela- Quintana

Universidad del Valle

Savvas Nesseris

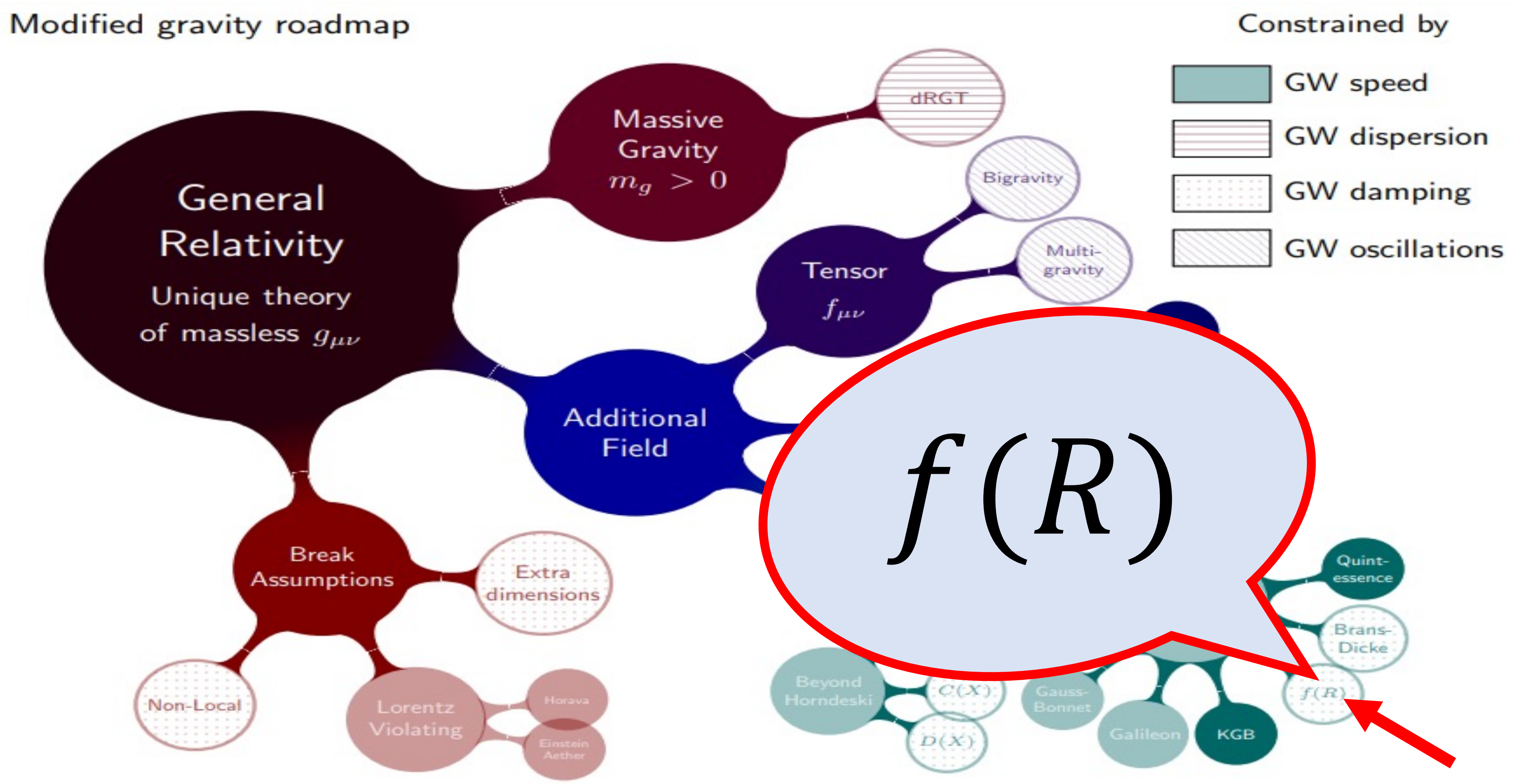
Instituto de Física Teórica UAM-CSIC

Modified Gravity Theories



- Ezquiaga, Zumalacarregui: [1807.09241](1807.09241)

Modified Gravity Theories



- Ezquiaga, Zumalacarregui: [1807.09241](1807.09241)

Modified Gravity ↔ Dark Energy

MG here

$$\mathcal{G}_{\mu\nu} = \frac{1}{2} T^{(m)}_{\mu\nu}$$

DE here

$$G_{\mu\nu} = \kappa \left(T^{(m)}_{\mu\nu} + T^{(\mathrm{DE})}_{\mu\nu} \right)$$

DE Tensor: $$\kappa T^{(\mathrm{DE})}_{\mu\nu} = G_{\mu\nu} - 2\kappa \mathcal{G}_{\mu\nu}$$

Dark Energy Density and Pressure: $$\rho_{\mathrm{DE}}, \quad P_{\mathrm{DE}}$$

DE Perturbations:

$$\delta\rho_{\mathrm{DE}} = (\cdots)\dot{\Psi} + (\cdots)\dot{\Phi} + (\cdots)\Psi + (\cdots)\Phi + (\cdots)\text{Other Fields},$$
$$\delta P_{\mathrm{DE}} = (\cdots)\ddot{\Psi} + (\cdots)\ddot{\Phi} + (\cdots)\dot{\Psi} + (\cdots)\dot{\Phi} + (\cdots)\Psi + (\cdots)\Phi + (\cdots)\text{Other Fields},$$
$$V_{\mathrm{DE}} = (\cdots)\dot{\Psi} + (\cdots)\dot{\Phi} + (\cdots)\Psi + (\cdots)\Phi + (\cdots)\text{Other Fields},$$

$f(R)$ Theories

$$\kappa T_{\mu\nu}^{(\mathrm{DE})} = (1-F)\,G_{\mu\nu} + \frac{1}{2}(f - FR)\,g_{\mu\nu} - (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)\,F.$$

$$\kappa\bar{\rho}_{\mathrm{DE}} = -\frac{f}{2} + 3H^2(1+F) + 3F\dot{H} - 3H\dot{F},$$

$$\kappa\bar{P}_{\mathrm{DE}} = \frac{f}{2} - 3H^2(1+F) - \dot{H}(2+F) + 2H\dot{F} + \ddot{F},$$

$f(R)$ Theories

$$\kappa T_{\mu\nu}^{(\mathrm{DE})} = (1-F)\,G_{\mu\nu} + \frac{1}{2}\,(f - FR)\,g_{\mu\nu} - (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)\,F.$$

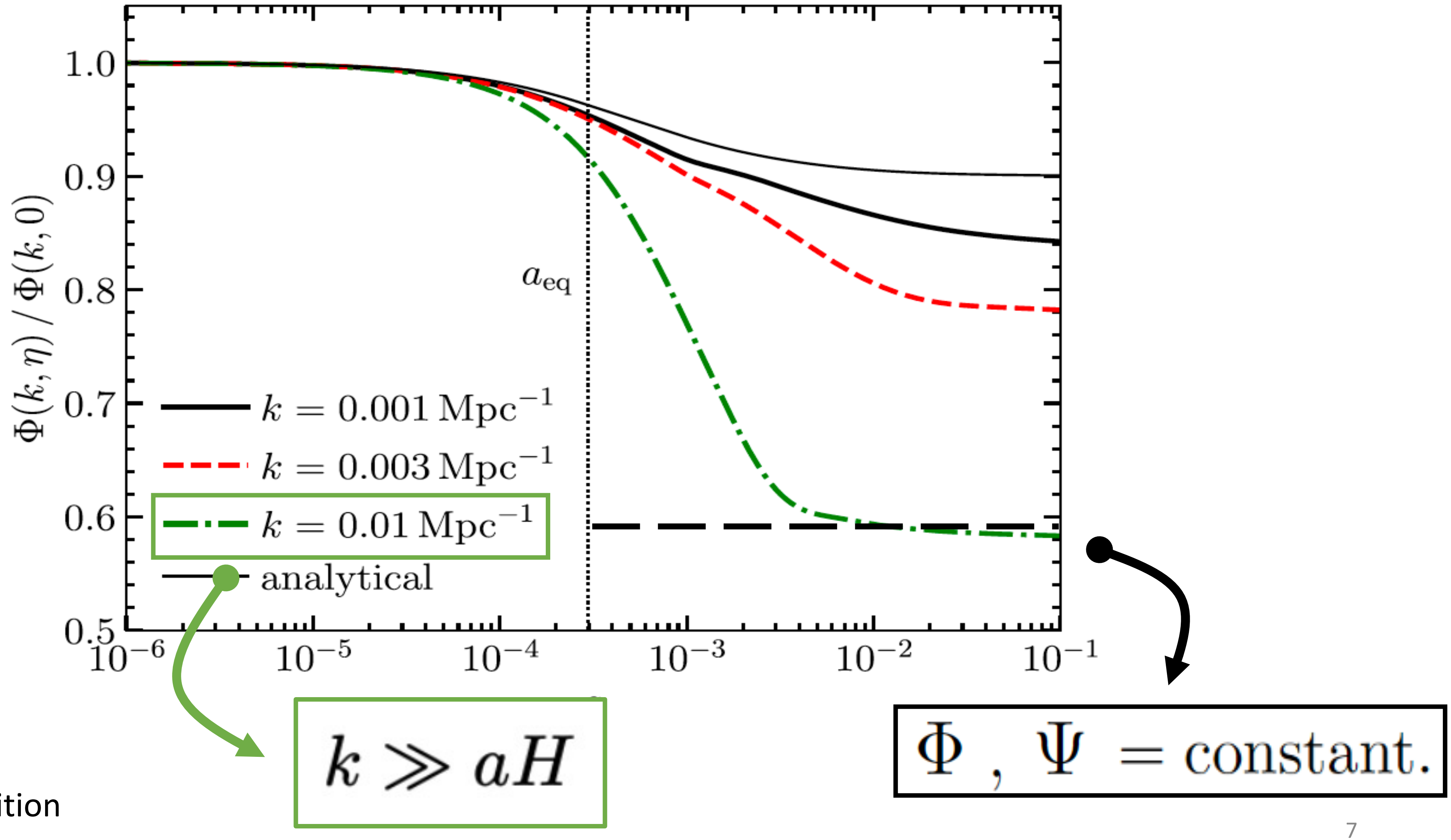
$$\kappa\delta\rho_{\mathrm{DE}} = W_1\dot{\Phi} + W_2\dot{\Psi} + \left(W_3 + W_4\frac{k^2}{a^2}\right)\Phi + \left(W_5 + W_6\frac{k^2}{a^2}\right)\Psi,$$

$$\kappa\delta P_{\mathrm{DE}} = Y_1\ddot{\Phi} + Y_2\ddot{\Psi} + Y_3\dot{\Phi} + Y_4\dot{\Psi} + \left(Y_5 + Y_6\frac{k^2}{a^2}\right)\Phi + \left(Y_7 + Y_8\frac{k^2}{a^2}\right)\Psi,$$

$$\frac{a\kappa\bar{\rho}_{\mathrm{DE}}}{k^2}V_{\mathrm{DE}} = Z_1\dot{\Phi} + Z_2\dot{\Psi} + Z_3\Phi + Z_4\Psi,$$

$$\kappa\bar{\rho}_{\mathrm{DE}}\pi_{\mathrm{DE}} = -\frac{k^2}{a^2}\,(\Phi + \Psi)\,,$$

Quasi-Static and Sub-Horizon Approximations



- Dodelson 2nd Edition

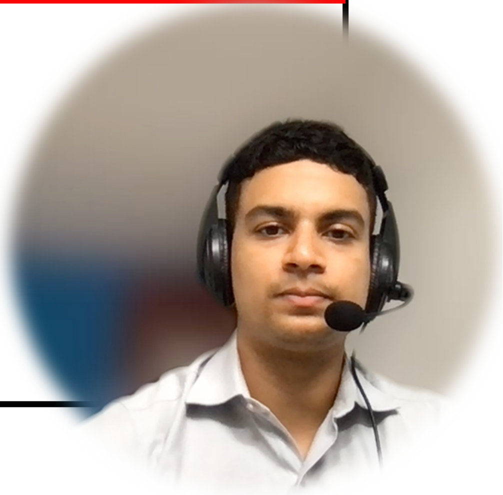
Standard Procedure

Applying QSA-SHA:

$$\delta R = 6\cancel{\ddot{\Phi}} + 24H\cancel{\dot{\Phi}} - 6H\cancel{\dot{\Psi}} + 4\frac{k^2}{a^2}\Phi + \left(2\frac{k^2}{a^2} - 12\dot{H} + 24H^2\right)\Psi$$

$$\overset{\text{QSA}}{\approx} 4\frac{k^2}{a^2}\Phi + \left(2\frac{k^2}{a^2} - 12\cancel{\dot{H}} + 24\cancel{H^2}\right)\Psi$$

$$\overset{\text{SHA}}{\approx} 4\frac{k^2}{a^2}\Phi + 2\frac{k^2}{a^2}\Psi.$$



$$\frac{k^2}{a^2}\Phi = \frac{F + 2\frac{k^2}{a^2}F_R}{2F^2 + 6\frac{k^2}{a^2}FF_R}\kappa\bar{\rho}_m\delta_m, \qquad \frac{k^2}{a^2}\Psi = -\frac{F + 4\frac{k^2}{a^2}F_R}{2F^2 + 6\frac{k^2}{a^2}FF_R}\kappa\bar{\rho}_m\delta_m.$$

Standard Procedure

Applying QSA-SHA:

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$$\overset{\text{QSA}}{\approx} 4\frac{k^2}{a^2}\Phi + \left(2\frac{k^2}{a^2} - 12\dot{H} + 24H^2\right)\Psi$$

$$\overset{\text{SHA}}{\approx} 4\frac{k^2}{a^2}\Phi + 2\frac{k^2}{a^2}\Psi.$$

$$\delta_{\text{DE}} = \frac{(1-F)F + (2-3F)\frac{k^2}{a^2}F_R}{F(F+3\frac{k^2}{a^2}F_R)}\frac{\bar{\rho}_m}{\bar{\rho}_{\text{DE}}}\delta_m,$$

$$\frac{\delta P_{\text{DE}}}{\bar{\rho}_{\text{DE}}} = \frac{1}{3F}\frac{2\frac{k^4}{a^4}F_R + 15\frac{k^2}{a^2}F_R\ddot{F} + 3F\ddot{F}}{3\frac{k^2}{a^2}F_R + F}\frac{\bar{\rho}_m}{\bar{\rho}_{\text{DE}}}\delta_m,$$

$$V_{\text{DE}} = \frac{a\dot{F}}{2F}\frac{F + 6\frac{k^2}{a^2}F_R}{3\frac{k^2}{a^2}F_R + F}\frac{\bar{\rho}_m}{\bar{\rho}_{\text{DE}}}\delta_m.$$

$$\pi_{\text{DE}} = \frac{\frac{k^2}{a^2}F_R}{F^2 + 3\frac{k^2}{a^2}FF_R}\frac{\bar{\rho}_m}{\bar{\rho}_{\text{DE}}}\delta_m$$

$$\frac{\delta P_{\rm DE}}{\bar{\rho}_{\rm DE}} = \frac{1}{3F}\frac{2\frac{k^4}{a^4}F_R + 15\frac{k^2}{a^2}F_R\ddot{F} + 3F\ddot{F}}{3\frac{k^2}{a^2}F_R + F}\frac{\bar{\rho}_m}{\bar{\rho}_{\rm DE}}\delta_m,$$

$$V_{\rm DE} = \frac{a\dot{F}}{2F}\frac{F + 6\frac{k^2}{a^2}F_R}{3\frac{k^2}{a^2}F_R + F}\frac{\bar{\rho}_m}{\bar{\rho}_{\rm DE}}\delta_m.$$

$$\dot{F} = F_R\dot{R}$$

$$R = 6(\dot{H} + 2H^2),$$

$$\dot{F} \approx 0,$$

$$\frac{\delta P_{\rm DE}}{\bar{\rho}_{\rm DE}} \approx \frac{1}{3F}\frac{2\frac{k^2}{a^2}F_R}{3\frac{k^2}{a^2}F_R + F}\frac{\bar{\rho}_m}{\bar{\rho}_{\rm DE}}\delta_m, \qquad V_{\rm DE} \approx 0.$$

Where is the mistake here?

Tracking the Accuracy of the QSA and the SHA

"Slow-Roll" Parameters:

$$\varepsilon \equiv \frac{aH}{k}, \qquad \delta \equiv \frac{\dot{\varepsilon}}{\varepsilon H}, \qquad \xi \equiv \frac{\ddot{\varepsilon}}{\varepsilon H^2}, \qquad \chi \equiv \frac{\dddot{\varepsilon}}{\varepsilon H^3},$$

$$\varepsilon_\Phi \equiv \frac{\dot{\Phi}}{\Phi H}, \qquad \varepsilon_\Psi \equiv \frac{\dot{\Psi}}{\Psi H}, \qquad \chi_\Phi \equiv \frac{\dot{\varepsilon}_\Phi}{\varepsilon_\Phi H}, \qquad \chi_\Psi \equiv \frac{\dot{\varepsilon}_\Psi}{\varepsilon_\Psi H}.$$

SHA: $k \gg aH$ → $\varepsilon \ll 1,$

QSA: $\dot{\Phi} \sim \dot{\Psi} \approx 0$ → $\varepsilon_\Phi \sim \varepsilon_\Psi \ll 1.$

Tracking the Accuracy of the QSA and the SHA

"Slow-Roll" Parameters:

$$\varepsilon \equiv \frac{aH}{k}, \qquad \delta \equiv \frac{\dot{\varepsilon}}{\varepsilon H}, \qquad \xi \equiv \frac{\ddot{\varepsilon}}{\varepsilon H^2}, \qquad \chi \equiv \frac{\dddot{\varepsilon}}{\varepsilon H^3},$$

$$\varepsilon_\Phi \equiv \frac{\dot{\Phi}}{\Phi H}, \qquad \varepsilon_\Psi \equiv \frac{\dot{\Psi}}{\Psi H}, \qquad \chi_\Phi \equiv \frac{\dot{\varepsilon}_\Phi}{\varepsilon_\Phi H}, \qquad \chi_\Psi \equiv \frac{\dot{\varepsilon}_\Psi}{\varepsilon_\Psi H}.$$

$$\delta R = 6\ddot{\Phi} + 24H\dot{\Phi} - 6H\dot{\Psi} + 4\frac{k^2}{a^2}\Phi + \left(2\frac{k^2}{a^2} - 12\dot{H} + 24H^2\right)\Psi,$$

$$\overset{\text{QSA}}{=} \left(4\frac{k^2}{a^2} + (24H^2 + 6\dot{H})\varepsilon_\Phi + 3H^2\varepsilon_\Phi^2 + 3H^2\varepsilon_\Phi\chi_\Phi\right)\Phi + \left(2\frac{k^2}{a^2} - 24H^2 - 12\dot{H} - 6H^2\varepsilon_\Psi\right)\Psi,$$

$$\overset{\text{SHA}}{=} \left(4 + 6\varepsilon^2[\varepsilon_\Phi\{3+\delta\} + \varepsilon_\Phi^2 + \varepsilon_\Phi\chi_\Phi]\right)\frac{k^2}{a^2}\Phi + \left(2 - 6\varepsilon^2[2 + 2\delta + \varepsilon_\Psi]\right)\frac{k^2}{a^2}\Psi.$$

2nd Order in SHA – 0th Order in QSA

$$\frac{k^2}{a^2}\Phi = \frac{F + 2\frac{k^2}{a^2}F_R}{2F^2 + 6\frac{k^2}{a^2}FF_R}\kappa\bar{\rho}_m\delta_m$$

$$+ \frac{3\kappa\bar{\rho}_m\delta_m\varepsilon^2}{2F\left(F + 3\frac{k^2}{a^2}F_R\right)^2}\left\{F^2 + \frac{k^2}{a^2}(\delta+8)FF_R + 2\frac{k^4}{a^4}(3\delta+8)F_R^2\right\},$$

$$\frac{k^2}{a^2}\Psi = -\frac{F + 4\frac{k^2}{a^2}F_R}{2F^2 + 6\frac{k^2}{a^2}FF_R}\kappa\bar{\rho}_m\delta_m$$

$$+ \frac{3\kappa\bar{\rho}_m\delta_m\varepsilon^2}{2F\left(F + 3\frac{k^2}{a^2}F_R\right)^2}\left\{F^2 - \frac{k^2}{a^2}(3\delta-6)FF_R - 2\frac{k^4}{a^4}(6\delta-4)F_R^2\right\}.$$

Deepest modes

2nd order

$$\delta_{\rm DE} = \frac{(1-F)F + \frac{k^2}{a^2}(2-3F)F_R}{F(F+3\frac{k^2}{a^2}F_R)}\frac{\bar\rho_m}{\bar\rho_{\rm DE}}\delta_m$$

$$-\frac{3\frac{k^2}{a^2}F_R\varepsilon^2}{F\left(F+3\frac{k^2}{a^2}F_R\right)^2}\left\{(\delta+1)F + 2\frac{k^2}{a^2}(3\delta+2)F_R\right\}\frac{\bar\rho_m}{\bar\rho_{\rm DE}}\delta_m,$$

$$\frac{\delta P_{\rm DE}}{\bar\rho_{\rm DE}} = \frac{1}{3F}\frac{2\frac{k^2}{a^2}F_R}{3\frac{k^2}{a^2}F_R+F}\frac{\bar\rho_m}{\bar\rho_{\rm DE}}\delta_m + \frac{3\frac{\bar\rho_m}{\bar\rho_{\rm DE}}\delta_m\varepsilon^2}{F\left(F+3\frac{k^2}{a^2}F_R\right)^2}\left\{F^2(F-1)(1+2\delta)\right.$$

$$\left.+\frac{k^2}{a^2}(5F-10\delta+13F\delta-5)FF_R + \frac{k^4}{a^4}(6F-6\delta+21F\delta-4)F_R^2\right\},$$

$$V_{\rm DE} = \frac{a}{F\left(F+3\frac{k^2}{a^2}F_R\right)}\left\{F(F-1)+\frac{k^2}{a^2}(3F-4)F_R\right\}\frac{\bar\rho_m}{\bar\rho_{\rm DE}}\delta_m\frac{k}{a}\varepsilon,$$

$$\pi_{\rm DE} = \frac{\frac{k^2}{a^2}F_R}{F^2+3\frac{k^2}{a^2}FF_R}\frac{\bar\rho_m}{\bar\rho_{\rm DE}}\delta_m + \frac{3F_R\frac{\bar\rho_m}{\bar\rho_{\rm DE}}\delta_m\frac{k^2}{a^2}\varepsilon^2}{F\left(F+3\frac{k^2}{a^2}F_R\right)^2}\left\{F(1+2\delta)+\frac{k^2}{a^2}F_R(4+9\delta)\right\}.$$

Specific Models

fDES:

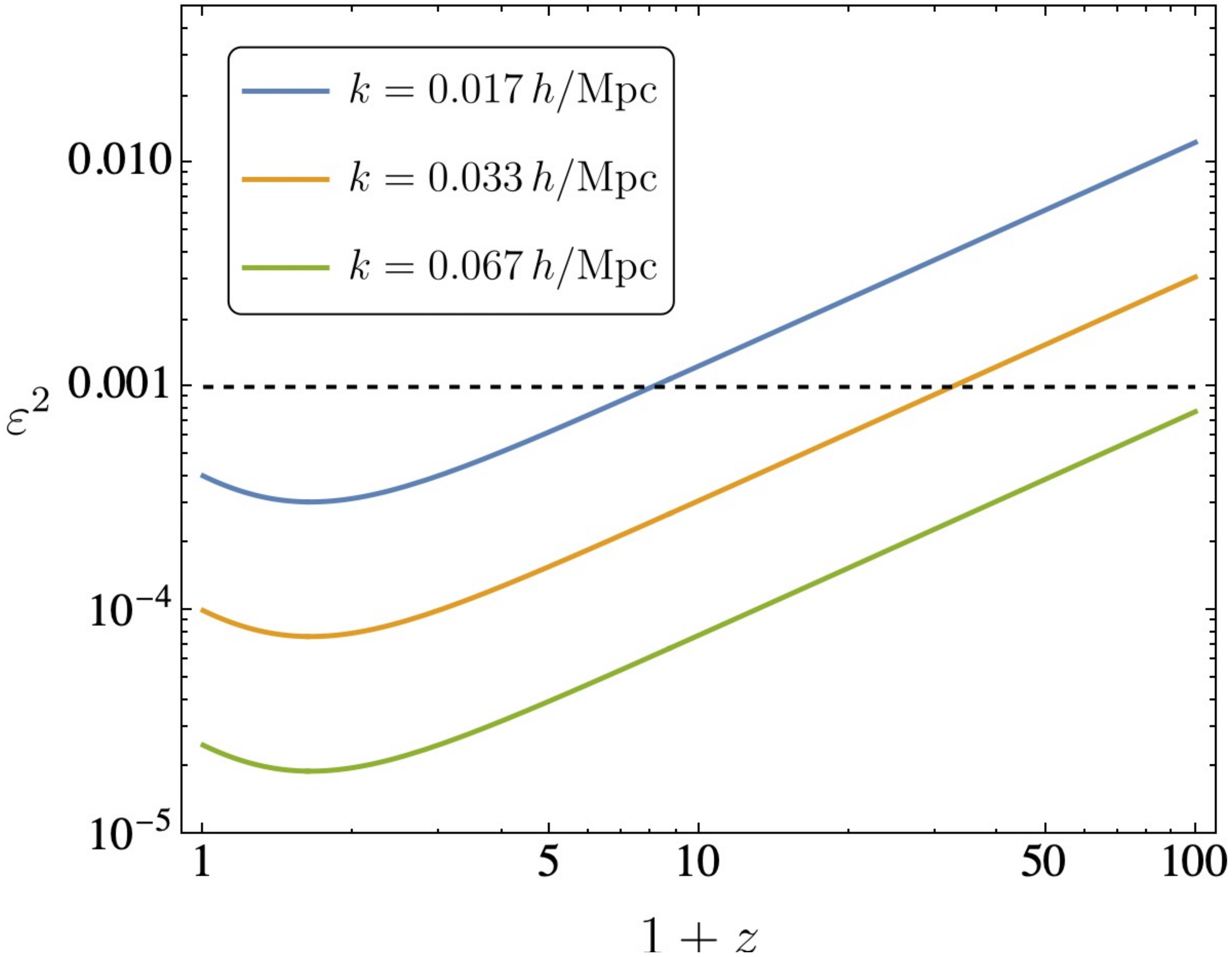
$$f(R) = R - 2\Lambda + \alpha H_0^2 \left(\frac{\Lambda}{R-3\Lambda}\right)^{c_0} {}_2F_1\left(c_0, \frac{3}{2} + c_0, \frac{13}{6} + 2c_0, \frac{\Lambda}{R-3\Lambda}\right),$$

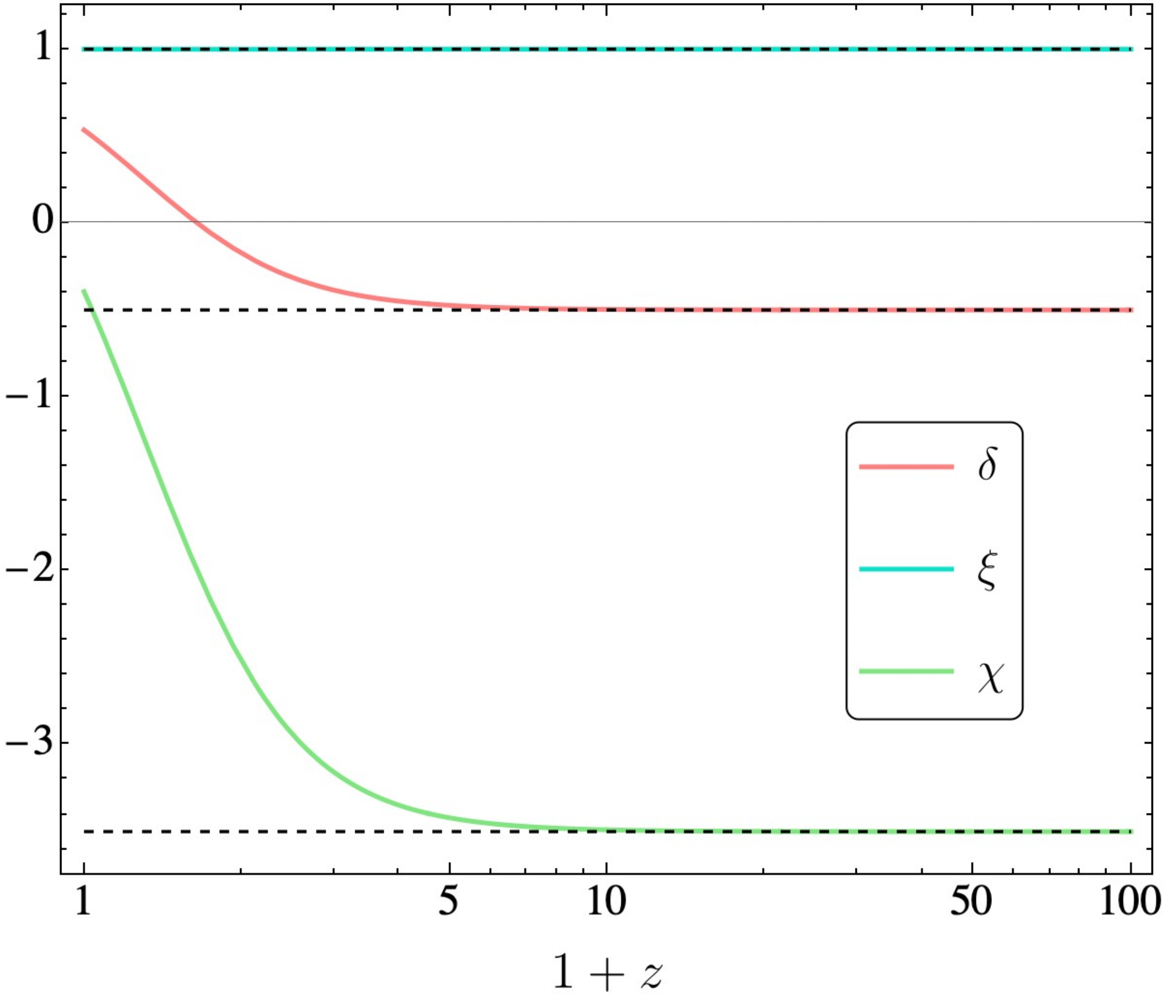
Hu-Sawicki:

$$f(R) = R - \frac{2\Lambda}{1 + \left(\frac{b\Lambda}{R}\right)^n},$$

- Nojiri, Odintsov: 0608008
- Hu, Sawicki: 0708.1190

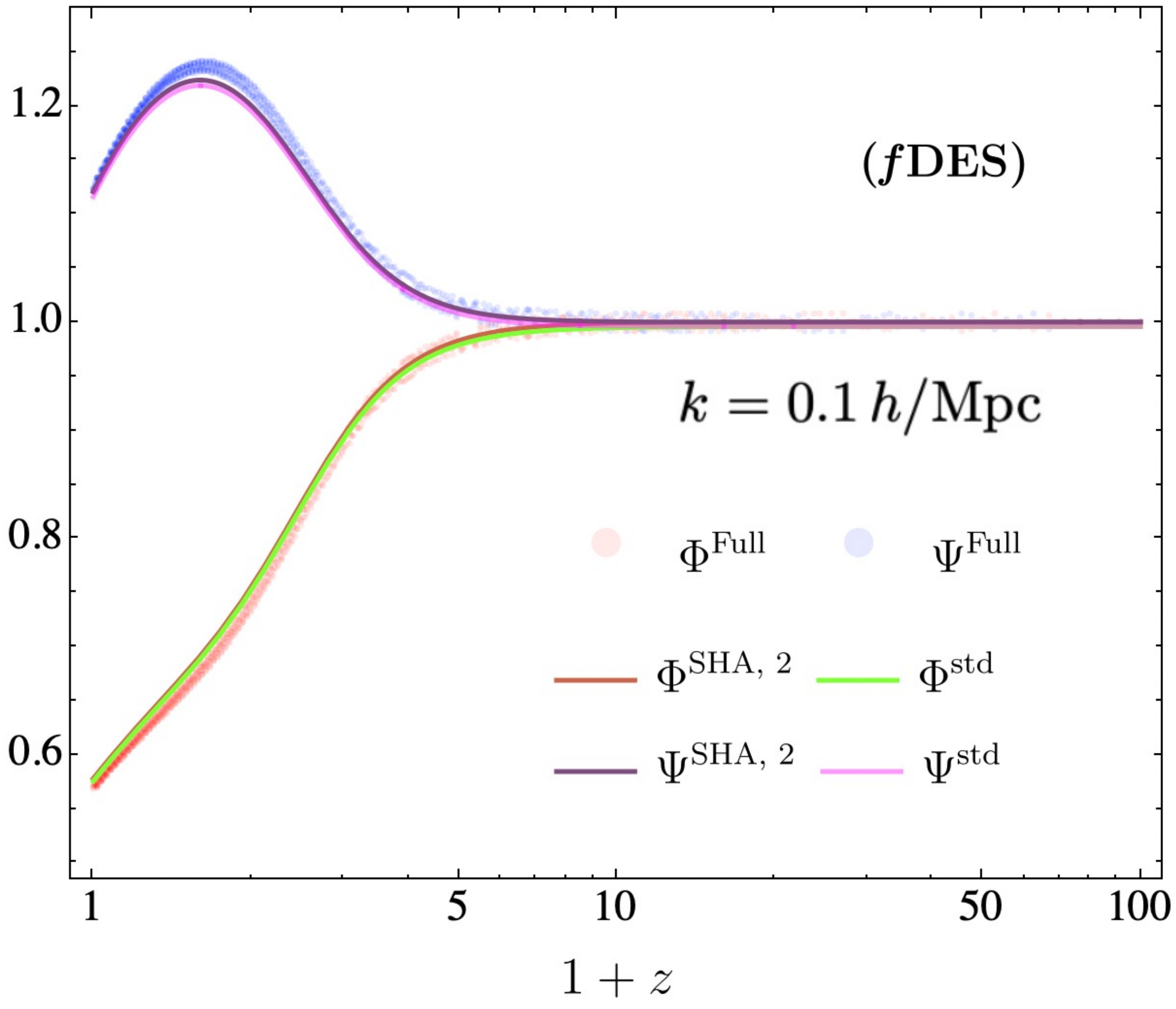
Specific Models

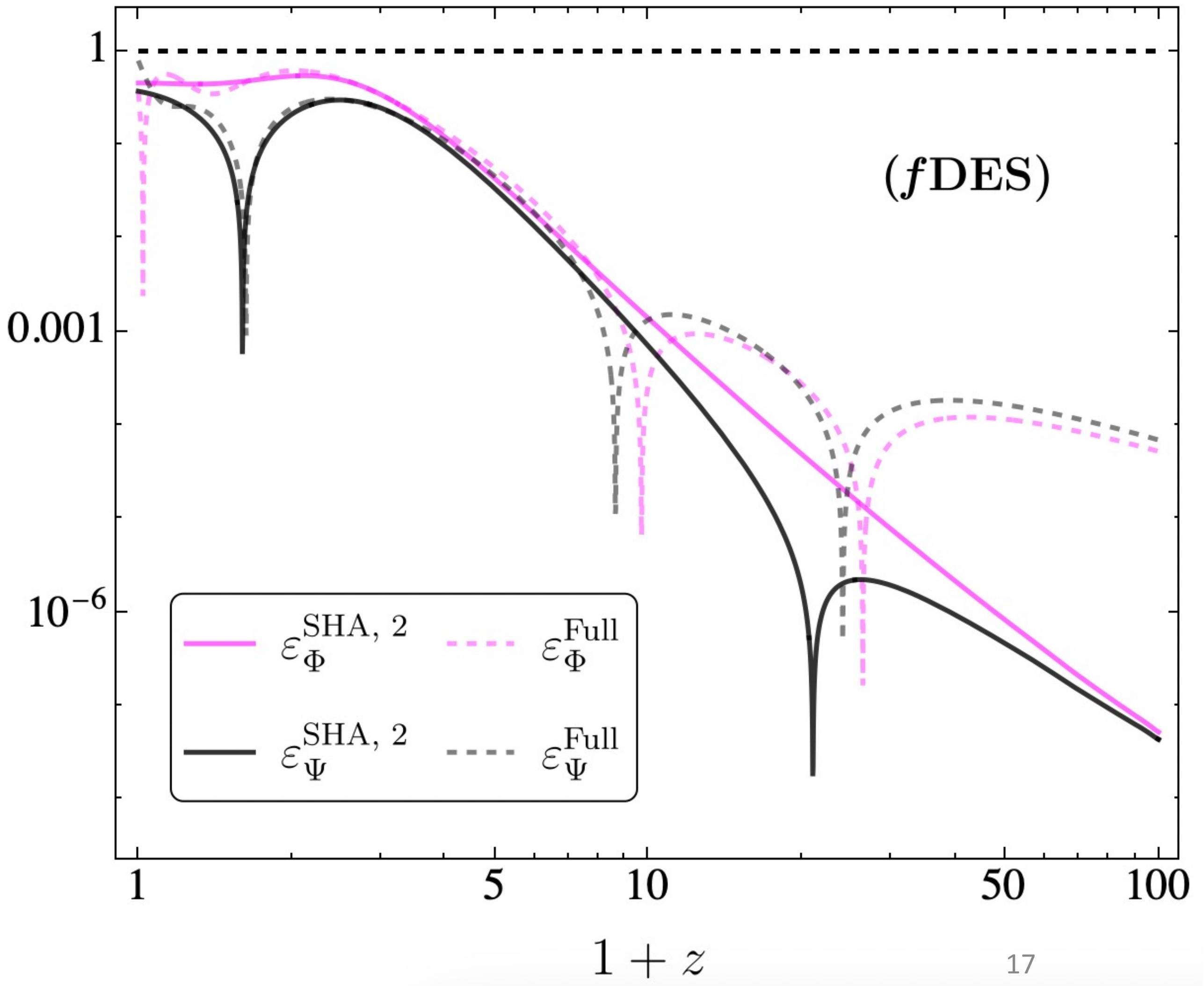


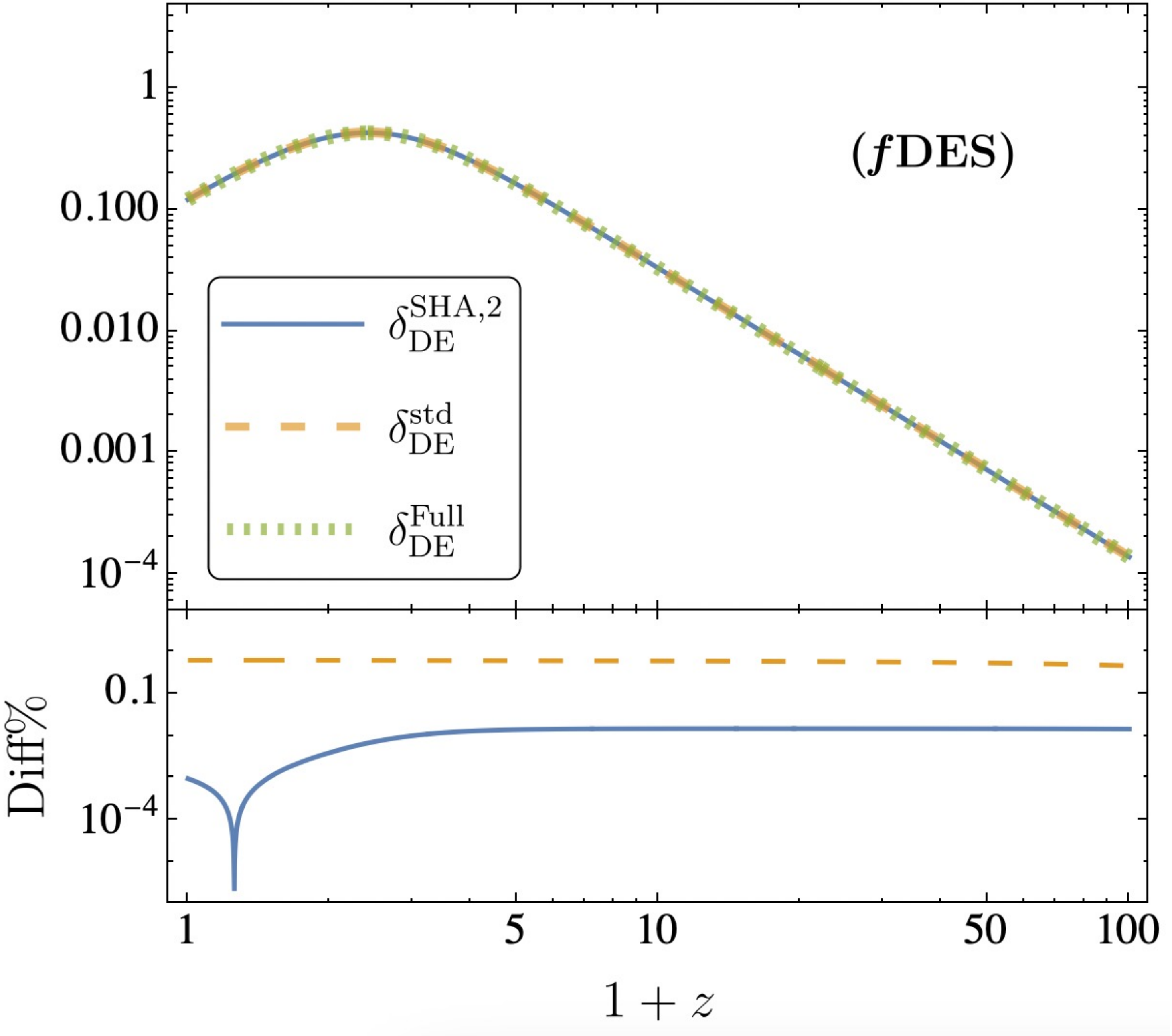


- Nojiri, Odintsov: [0608008]
- Hu, Sawicki: [0708.1190]

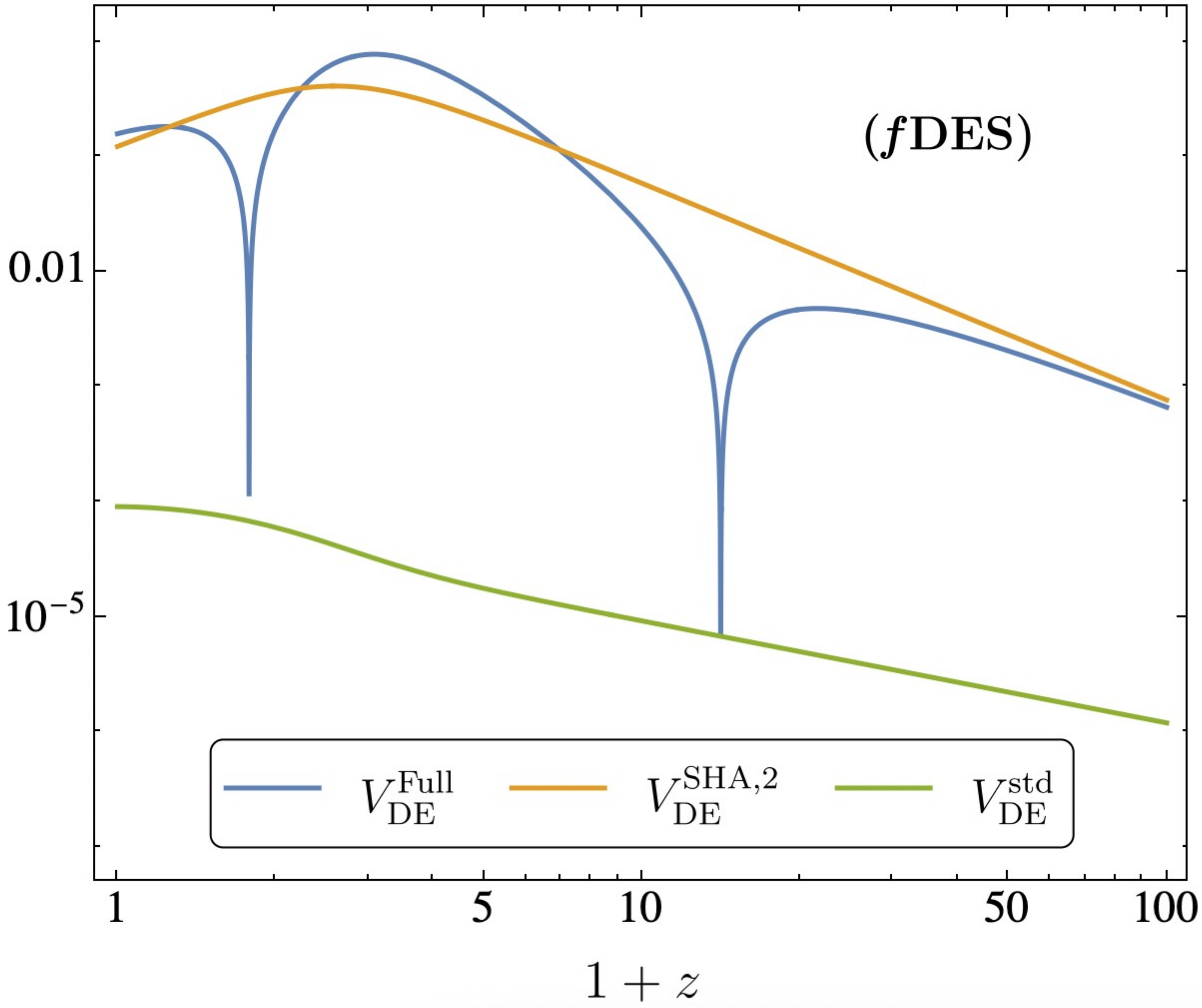
Comparison with Numerical Solutions

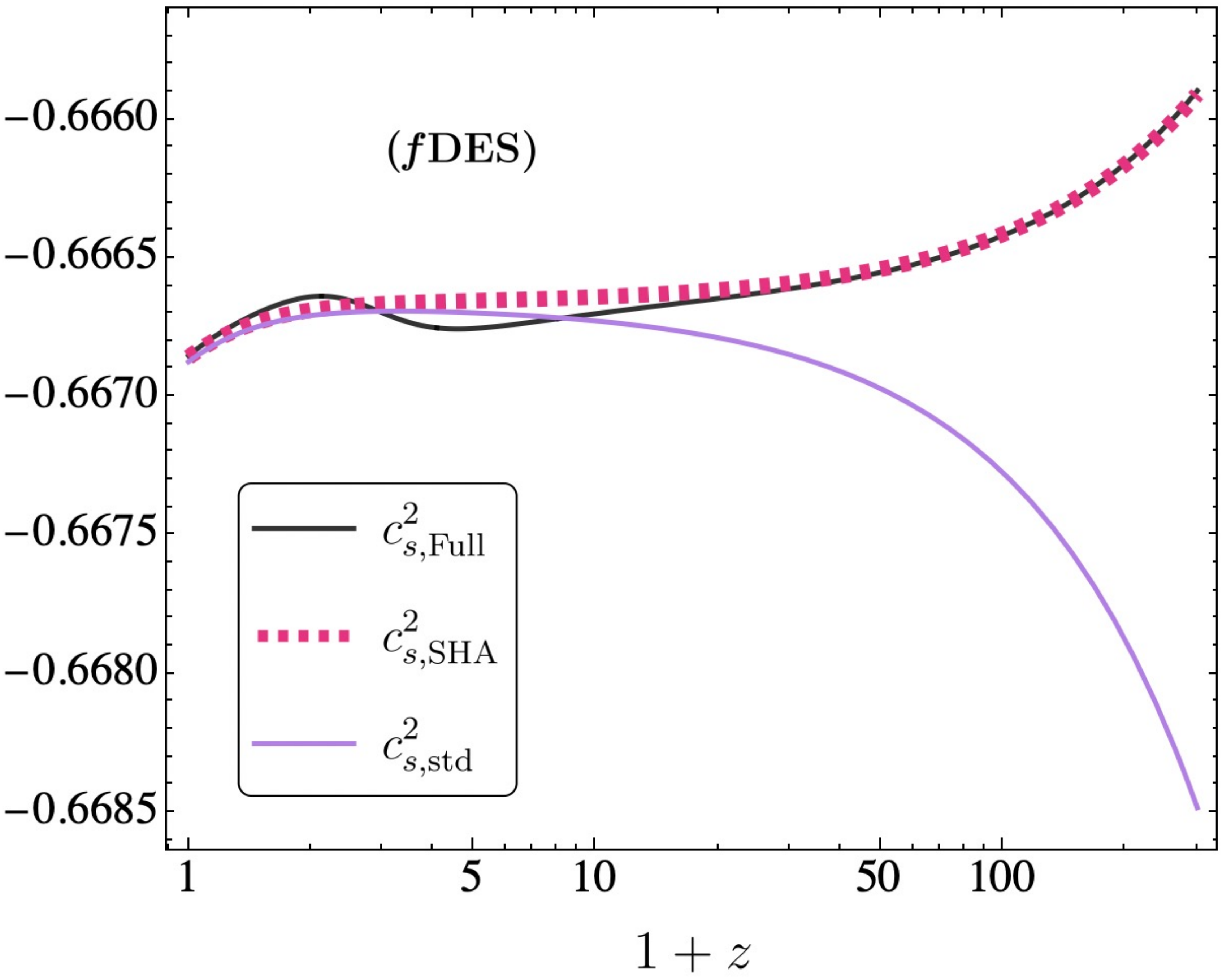




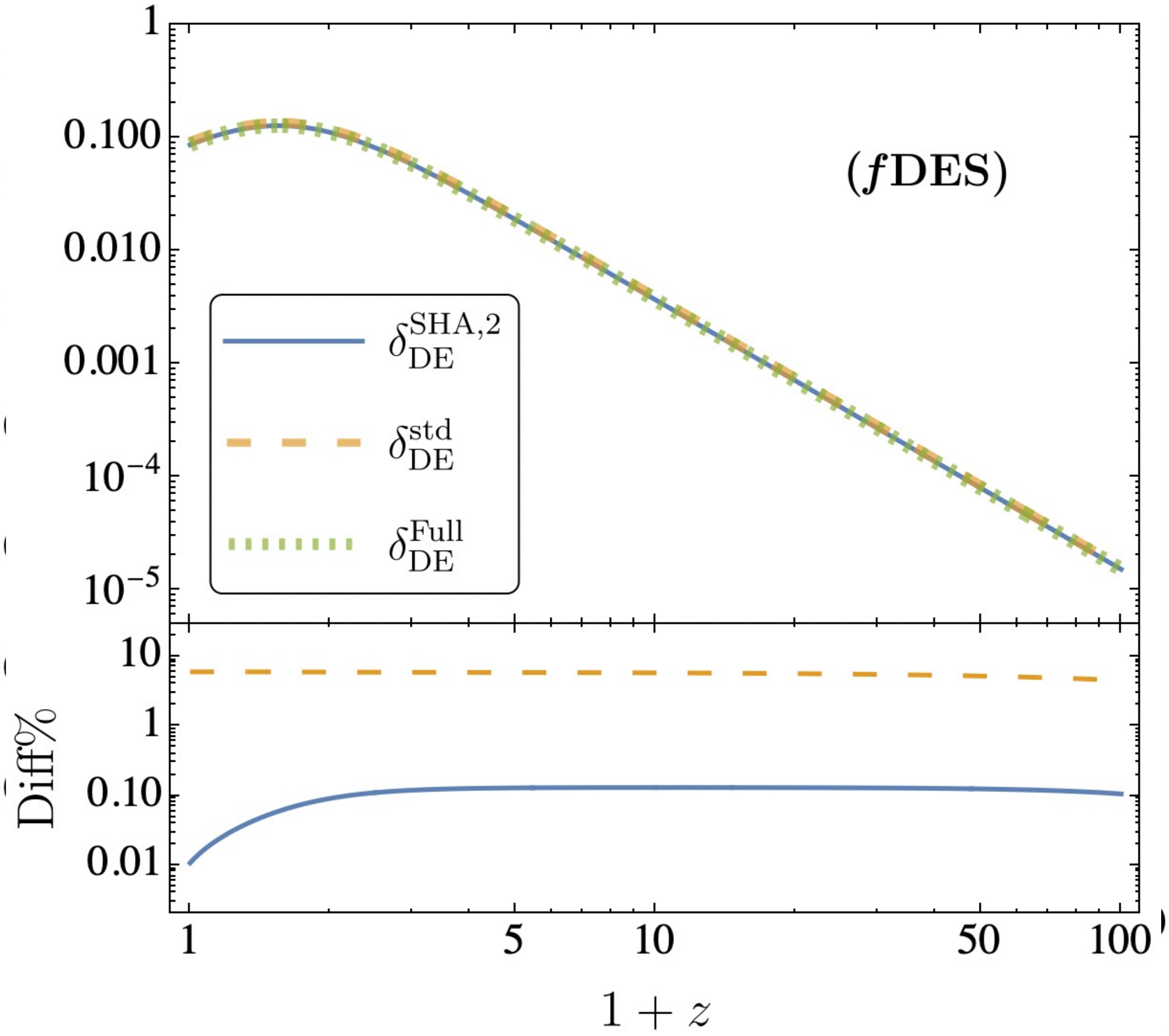


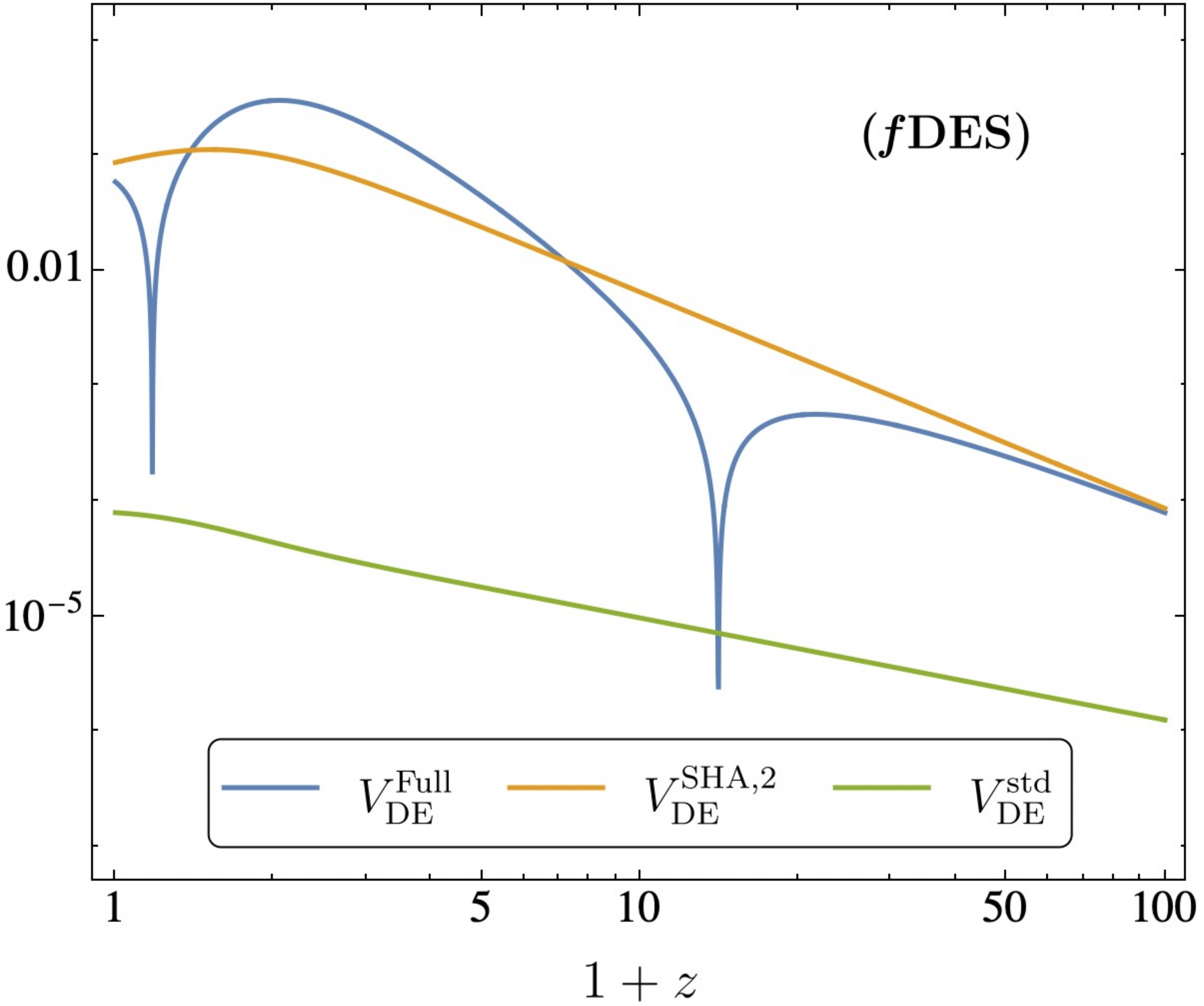
(fDES)
$\delta_{DE}^{SHA,2}$
δ_{DE}^{std}
δ_{DE}^{Full}
1
0.100
0.010
0.001
10^{-4}
Diff%
0.1
10^{-4}
1
5
10
50
100
$1+z$


(fDES)
0.01
10^{-5}
$V_{\rm DE}^{\rm Full}$
$V_{\rm DE}^{\rm SHA,2}$
$V_{\rm DE}^{\rm std}$
1
5
10
50
100
$1+z$

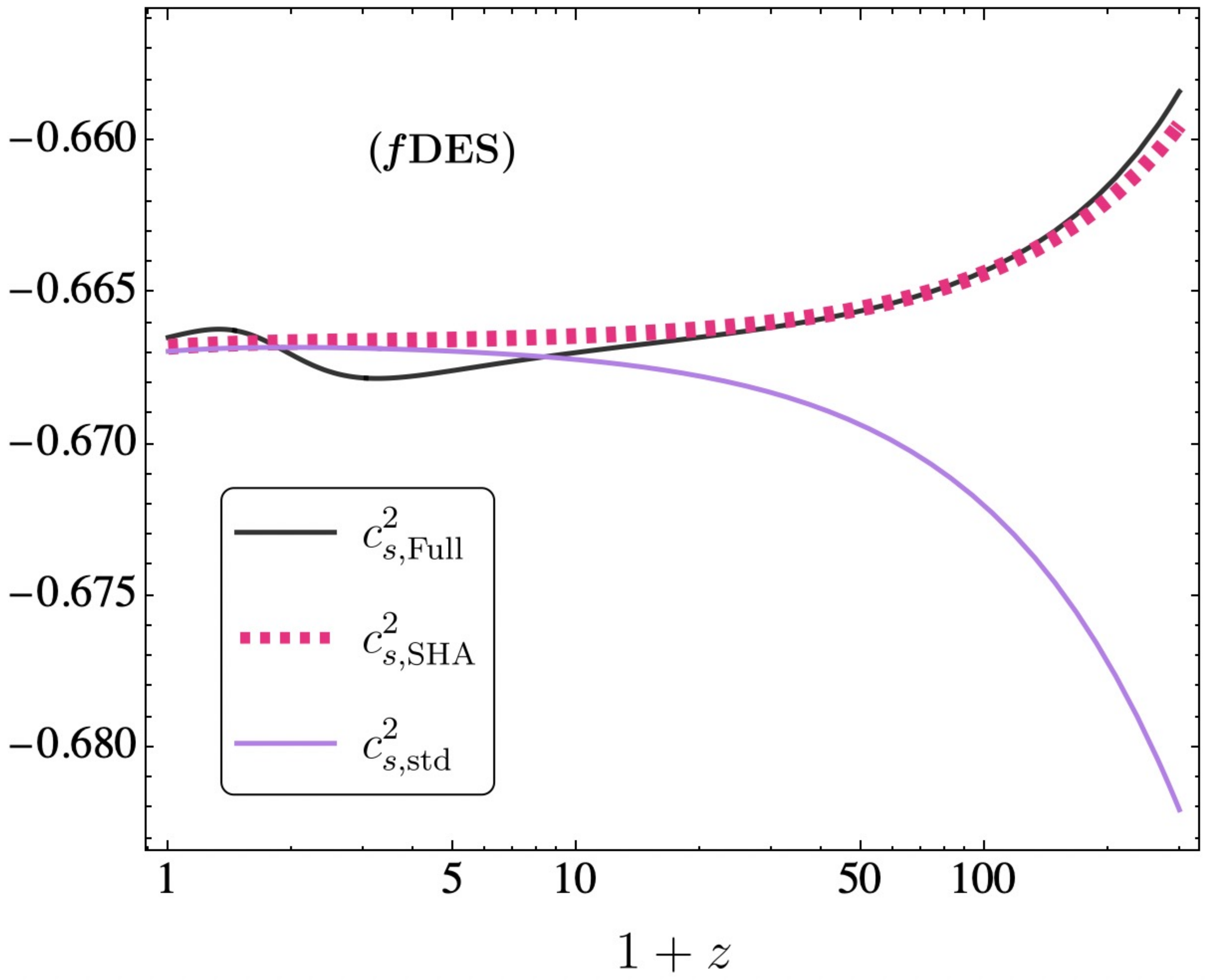


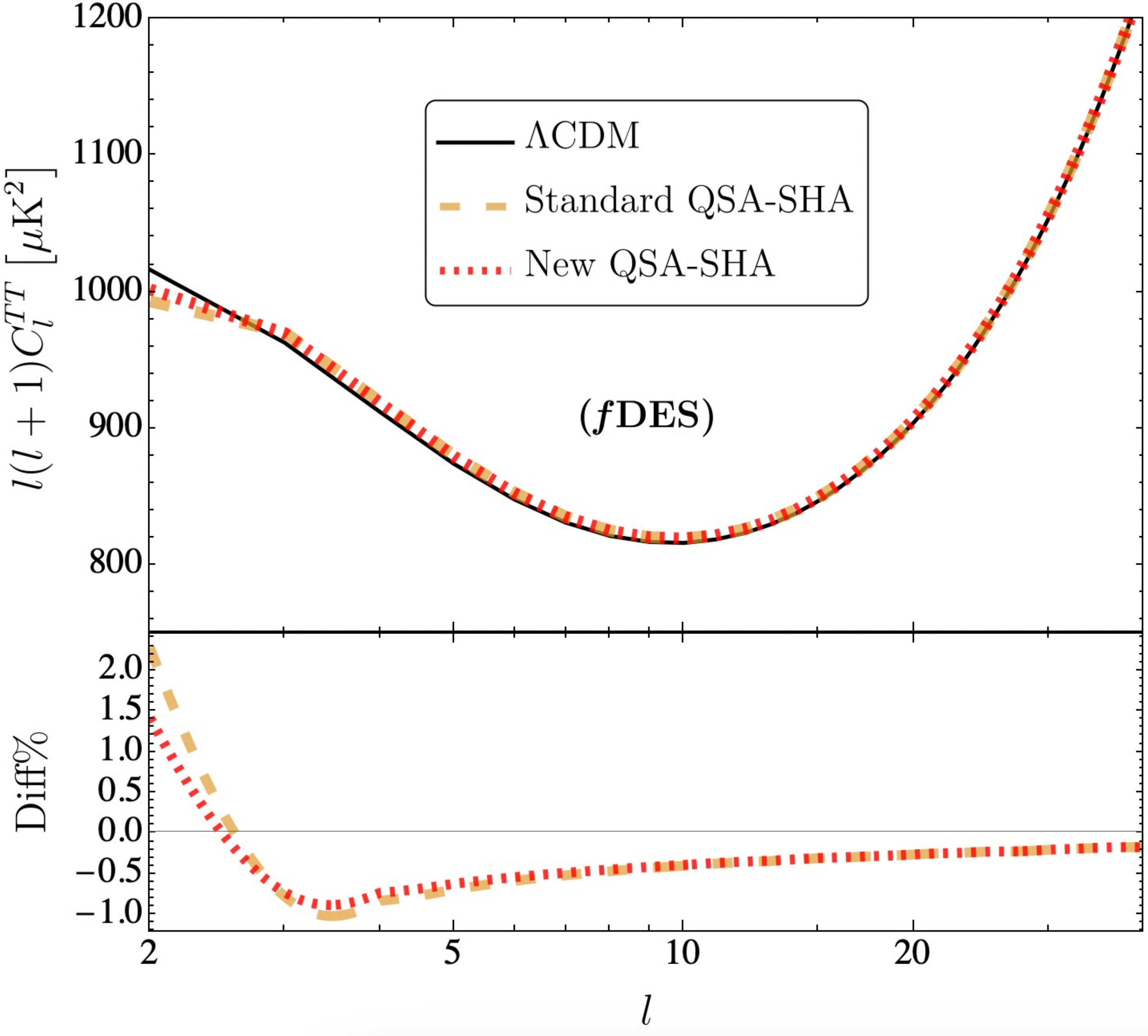
−0.6660
−0.6665
−0.6670
−0.6675
−0.6680
−0.6685
(fDES)
$c^2_{s,\mathrm{Full}}$
$c^2_{s,\mathrm{SHA}}$
$c^2_{s,\mathrm{std}}$
1
5
10
50
100
$1+z$


(fDES)
$\delta_{\rm DE}^{\rm SHA,2}$
$\delta_{\rm DE}^{\rm std}$
$\delta_{\rm DE}^{\rm Full}$
1
0.100
0.010
0.001
10^{-4}
10^{-5}
Diff%
10
1
0.10
0.01
1
5
10
50
100
$1+z$

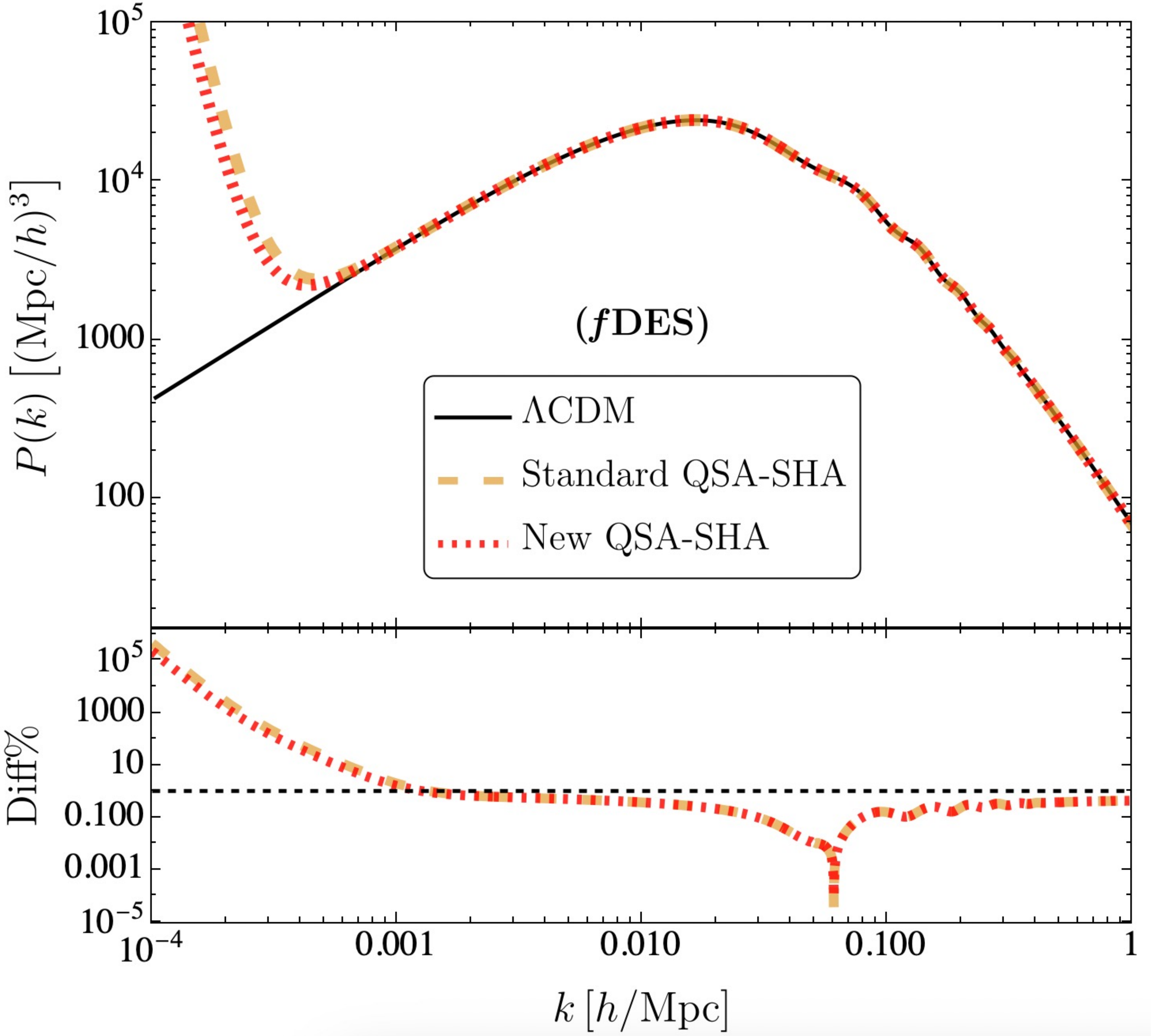


(fDES)
0.01
10^{-5}
$V_{\rm DE}^{\rm Full}$
$V_{\rm DE}^{\rm SHA,2}$
$V_{\rm DE}^{\rm std}$
1
5
10
50
100
$1+z$


(fDES)
−0.660
−0.665
−0.670
−0.675
−0.680
$c^2_{s,\mathrm{Full}}$
$c^2_{s,\mathrm{SHA}}$
$c^2_{s,\mathrm{std}}$
1
5
10
50
100
$1+z$



1200
1100
1000
900
800
$l(l+1)C_l^{TT}$ [μK^2]
ΛCDM
Standard QSA-SHA
New QSA-SHA
(fDES)
Diff%
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
2
5
10
20
l

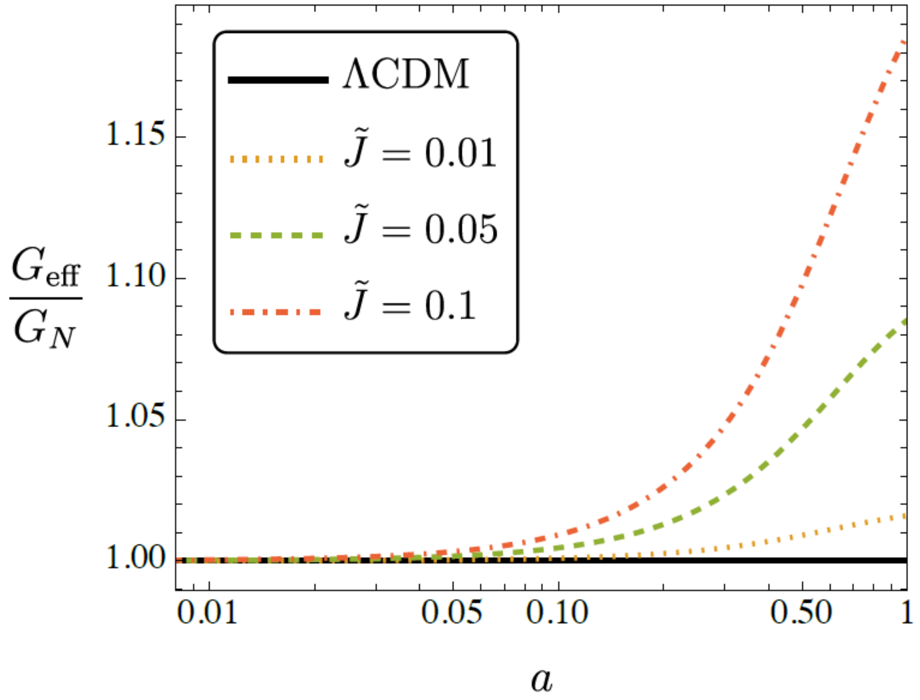


10^5
10^4
1000
100
$P(k)\ [(\mathrm{Mpc}/h)^3]$
(*f*DES)
ΛCDM
Standard QSA-SHA
New QSA-SHA
10^5
1000
10
0.100
0.001
10^{-5}
Diff%
10^{-4}
0.001
0.010
0.100
1
$k\ [h/\mathrm{Mpc}]$

Preliminary Results: Designer Horndeski

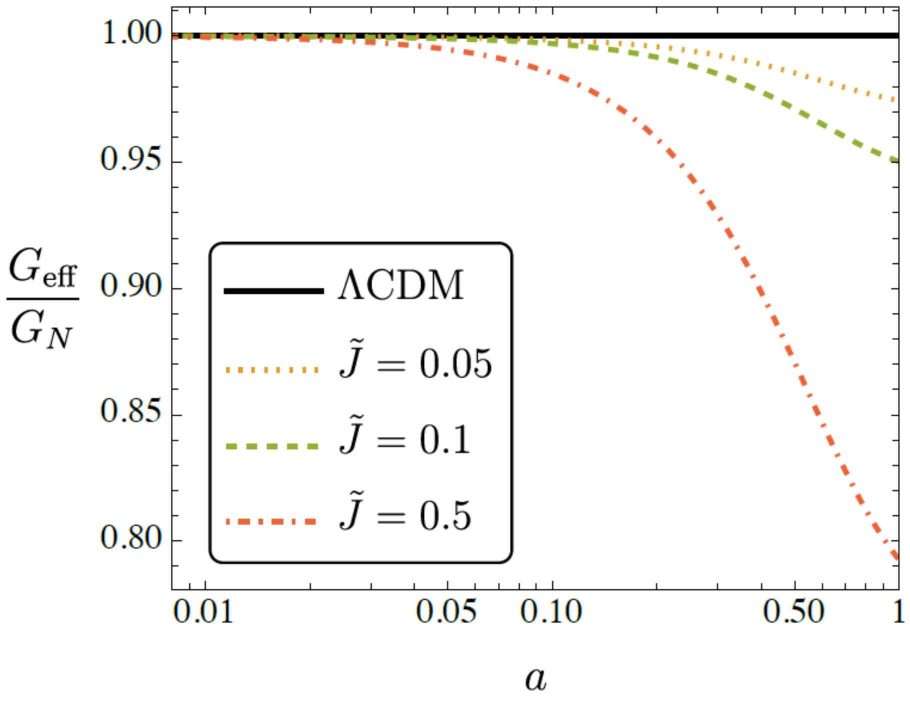
Standard

$$G_{\rm eff}/G_N = 1 + \frac{\sqrt{2}\tilde{J}_c}{3\Omega_{m,0}H(a)/H_0}$$



Non-Standard

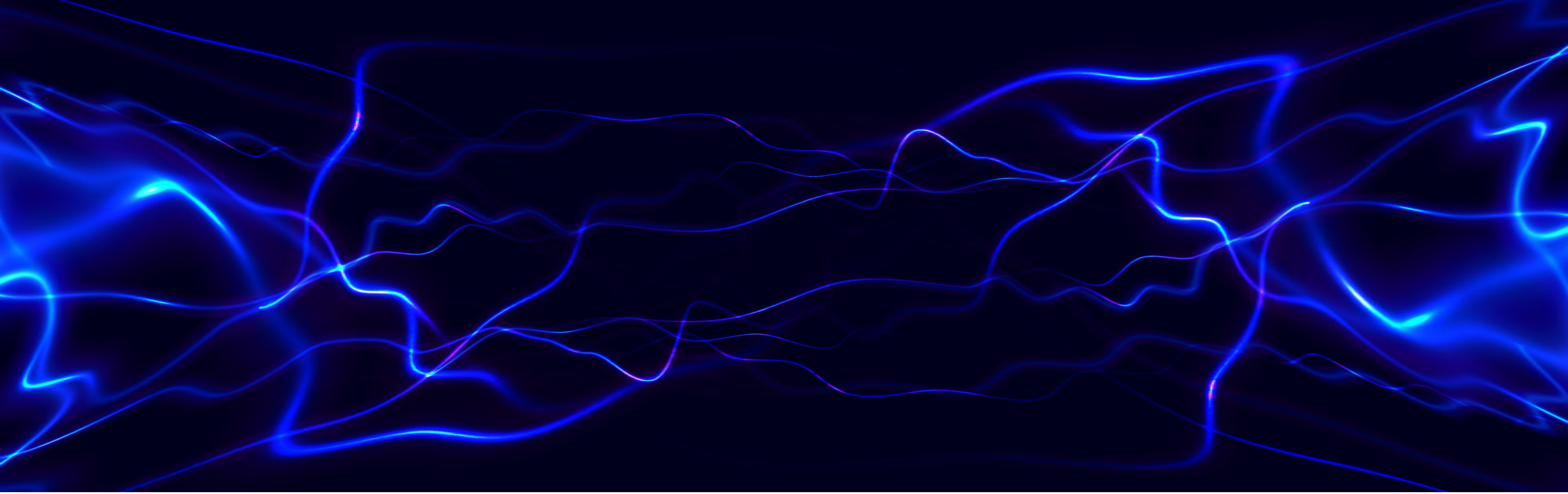
$$G_{\rm eff}/G_N = 1 - \frac{\sqrt{2}\tilde{J}_c}{9\Omega_{m,0}H(a)/H_0}$$



- Arjona, Cardona, Nesseris: [1904.06294](1904.06294)

Take-Home Points

- Novel parameterization to make transparent the QSA-SHA.
- Some terms are neglected in the standard approach → What are their relevance?
- No much improvement for viable $f(R)$ theories → What about other MG theories?
- Changes could be dramatic in general → See “Preliminary results”.
- Further details: 2303.14251 (Orjuela-Quintana & Nesseris)… Soonly available in JCAP.



Thank you!