Non-Gaussian imprints on Cosmic Microwave Background

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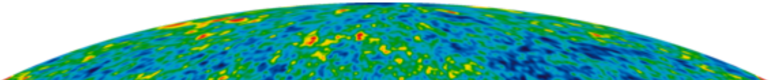
based on

Indirect imprints of primordial non-Gaussianity on cosmic microwave background

Barnali Das and H. V. Ragavendra, arXiv:2304.05941 [astro-ph.CO]



Overview

- Introduction
- Non-Gaussianity
  - Non-Gaussianity in CMB
  - Non-Gaussianity in single field inflationary models
  - Results
    - Simple templates
    - Realistic models
- Conclusion and Outlook

Introduction

- The period shortly after Big-Bang is inflation where universe expands exponentially.
- Inflation solves horizon and flatness problem of the ΛCDM cosmology.
- It is driven by a scalar field, Inflaton (ϕ).
- Inflation generates primordial perturbations.
- The scalar perturbation $\mathcal{R} = \Psi + \frac{H}{\dot{\phi}_0}\delta\phi$ where $\delta\phi$ and Ψ are the perturbation terms from the inflaton field and metric respectively, $\dot{\phi}_0$ is time derivative of inflaton field without perturbation and H is the Hubble parameter during inflation.

- Scalar perturbations follow the equation:

$$\boxed{\mathcal{R}''_{\boldsymbol{k}} + 2\frac{z'}{z}\mathcal{R}'_{\boldsymbol{k}} + k^2\mathcal{R}_{\boldsymbol{k}} = 0} \tag{1}$$

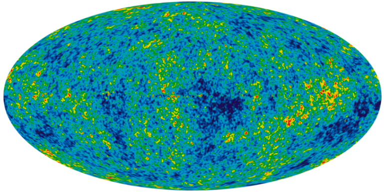
- Scalar power spectrum:

$$\boxed{\mathcal{P}_{\rm S} = \frac{k^3}{2\pi^2}\left|\mathcal{R}_{\boldsymbol{k}}\right|^2\Big|_{\eta\to 0}} \tag{2}$$

where $()' = \frac{d()}{d\eta}$ and η is the conformal time $\left(\eta = \int \frac{dt}{a(t)}\right)$.

Non-Gaussianity in CMB

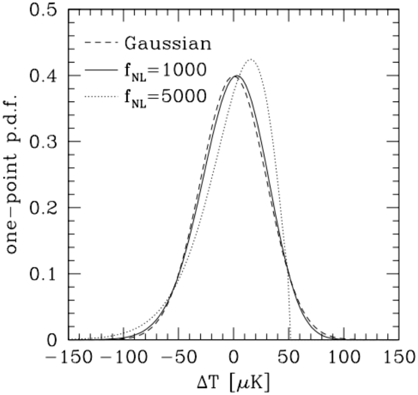
Non-Gaussianity in Cosmic Microwave Background



Mean = 0, Variance $\propto C_\ell^{TT}$

Skewness = 0 if distribution is Gaussian

$$C_\ell^{TT} = \frac{2}{\pi}\int k^2 dk \underbrace{P_R(k)}_{\text{inflation}} \underbrace{\Delta_{T\ell}(k)\Delta_{T\ell}(k)}_{\text{anisotropies}} \qquad (3)$$



$f_{\rm NL}$ is measure of non-Gaussianity.

[1] E. Komatsu, The Pursuit of Non-Gaussian Fluctuations in the Cosmic Microwave Background, PhD thesis, arXiv:astro-ph/0206039 (2002)

Non-Gaussianity in single field inflationary models and our methodology

Scalar power spectrum and bispectrum

Scalar power spectrum:

$$\langle \mathcal{R}^{\mathrm{G}}_{\boldsymbol{k}_1} \mathcal{R}^{\mathrm{G}}_{\boldsymbol{k}_2} \rangle = \frac{2\pi^2}{k_1^3} \mathcal{P}_{\mathrm{S}}(k_1) \delta^{(3)}(\boldsymbol{k}_1 + \boldsymbol{k}_2) \,. \quad (4)$$

Scalar bispectrum:

$$\langle \mathcal{R}_{\boldsymbol{k}_1} \mathcal{R}_{\boldsymbol{k}_2} \mathcal{R}_{\boldsymbol{k}_3} \rangle = (2\pi)^3 \, \mathcal{B}(k_1, k_2, k_3) \delta^{(3)}(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3) \,. \quad (5)$$

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[1] J. Maldacena, Non-gaussian features of primordial fluctuations in single field inflationary models, Journal of High Energy Physics (2003)

Correction to the power spectrum

Scalar perturbation with a Gaussian and non-Gaussian component:

$$\mathcal{R}_{\boldsymbol{k}}(\eta) \;=\; \mathcal{R}^{\mathrm{G}}_{\boldsymbol{k}}(\eta) - \frac{3}{5}\int \frac{\mathrm{d}^3\boldsymbol{k}_1}{(2\,\pi)^{3/2}} \mathcal{R}^{\mathrm{G}}_{\boldsymbol{k_1}}(\eta)\mathcal{R}^{\mathrm{G}}_{\boldsymbol{k}-\boldsymbol{k_1}}(\eta)\, f_{\mathrm{NL}}[\boldsymbol{k}, (\boldsymbol{k}_1 - \boldsymbol{k}), -\boldsymbol{k}_1] \tag{6}$$

Power spectrum with correction:

$$\mathcal{P}^{\mathrm{Total}}_{\mathrm{S}}(k) \;=\; \underbrace{\mathcal{P}^{\mathrm{G}}_{\mathrm{S}}(k)}_{\mathcal{P}_{\mathrm{S}}(k)} + \underbrace{\frac{9}{50\pi}k^3 \int \mathrm{d}^3\boldsymbol{k}_1\, \frac{\mathcal{P}^{\mathrm{G}}_{\mathrm{S}}(k_1)}{k_1^3}\frac{\mathcal{P}^{\mathrm{G}}_{\mathrm{S}}(|\boldsymbol{k}-\boldsymbol{k}_1|)}{|\boldsymbol{k}-\boldsymbol{k}_1|^3}\, f^2_{\mathrm{NL}}[k, |\boldsymbol{k}_1 - \boldsymbol{k}|, k_1]}_{\mathcal{P}_{\mathrm{C}}(k)} \tag{7}$$

[1] H. V. Ragavendra, Accounting for scalar non-Gaussianity in secondary gravitational waves, Physical Review D (2022)

[2] I. Agullo, D. Kranas, V. Sreenath, Anomalies in the Cosmic Microwave Background and their Non-Gaussian Origin in Loop Quantum Cosmology, Frontiers in Astronomy and Space Sciences (2021)

[3] F. Schmidt, M. Kamionkowski, Halo clustering with nonlocal non-Gaussianity, Physical Review D (2010)

Definition of f_{NL}

Non-Gaussianity parameter f_{NL}:

$$f_{\mathrm{NL}}(k_1, k_2, k_3) = -\frac{10\sqrt{2\pi}}{3} \frac{(k_1 k_2 k_3)^3 \mathcal{B}(k_1, k_2, k_3)}{\left[k_1^3 \mathcal{P}_{\mathrm{S}}^{\mathrm{G}}(k_2)\mathcal{P}_{\mathrm{S}}^{\mathrm{G}}(k_3) + k_2^3 \mathcal{P}_{\mathrm{S}}^{\mathrm{G}}(k_1)\mathcal{P}_{\mathrm{S}}^{\mathrm{G}}(k_3) + k_3^3 \mathcal{P}_{\mathrm{S}}^{\mathrm{G}}(k_1)\mathcal{P}_{\mathrm{S}}^{\mathrm{G}}(k_2)\right]}. \quad (8)$$

- Knowing the Gaussian power spectrum, $\mathcal{P}_{\mathrm{S}}(k)$, and the bispectrum, $\mathcal{B}(k_1, k_2, k_3)$, for a given model, will provide f_{NL} and subsequently the correction the power spectrum, $\mathcal{P}_{\mathrm{C}}$.

Results

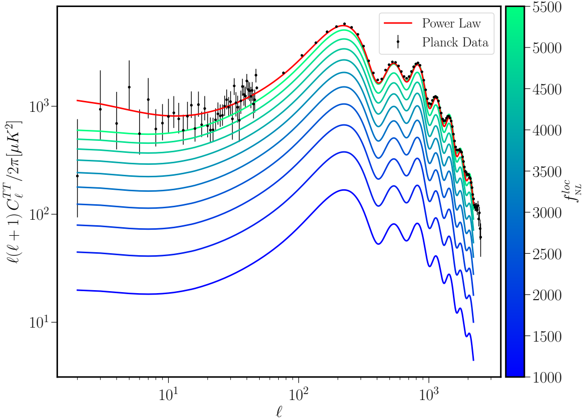
Local type

$$\mathcal{P}_{\mathrm{S}}^{\mathrm{loc}}(k) = A_s(k)\left(\frac{k}{k_*}\right)^{n_{\mathrm{S}}-1}.$$

$$\mathcal{B}^{\mathrm{loc}}(k_1,k_2,k_3) = -\frac{3}{10\sqrt{2\pi}}{A_S}^2 f_{\mathrm{NL}}^{loc}\left[\frac{1}{k_1^3k_2^3} + 2\text{ perm}\right]$$

Correction to power spectrum

$$\mathcal{P}_{\mathrm{C}}^{\mathrm{loc}}(k) = -\frac{18}{25}\mathcal{P}_{\mathrm{S}}^2(k)\,(f_{\mathrm{NL}}^{\mathrm{loc}})^2 \times \left(1 + 2\lim_{k_{\mathrm{min}}\to 0}\ln\frac{k_{\mathrm{min}}}{k}\right).$$



$$f_{\mathrm{NL}}^{loc} = -0.9 \pm 5.1$$

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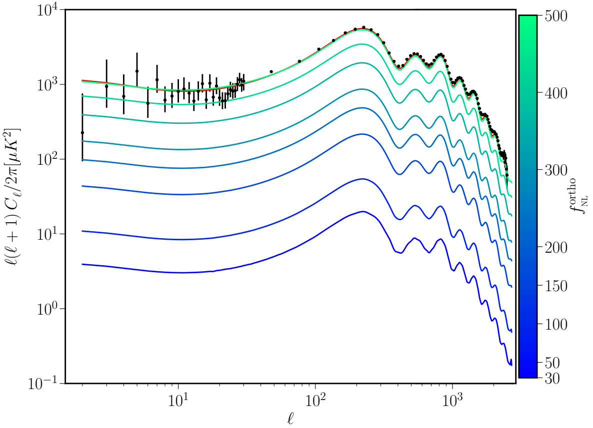
[1] Planck Collaboration, Planck 2018 results. IX. Constraints on primordial non-Gaussianity, Astronomy and Astrophysics (2020)

Orthogonal type

$$\mathcal{P}_{\mathrm{S}}^{\mathrm{ortho}}(k) = A_s(k)\left(\frac{k}{k_*}\right)^{n_{\mathrm{S}}-1}.$$

$$\mathcal{B}^{\mathrm{ortho}}(k_1,k_2,k_3) = \frac{6A_s^2 f_{\mathrm{NL}}^{\mathrm{ortho}}}{k_*^{2(n_{\mathrm{S}}-1)}}\left\{-\frac{8}{(k_1k_2k_3)^{\frac{2}{3}(4-n_{\mathrm{S}})}} - \frac{3}{(k_1k_2)^{4-n_{\mathrm{S}}}} - \frac{3}{(k_2k_3)^{4-n_{\mathrm{S}}}} - \frac{3}{(k_3k_1)^{4-n_{\mathrm{S}}}} + \left[\frac{3}{k_1^{(4-n_{\mathrm{S}})/3}k_2^{2(4-n_{\mathrm{S}})/3}k_3^{4-n_{\mathrm{S}}}} + 5\text{ perms}\right]\right\}.$$

$$\mathcal{P}_{\mathrm{C}}^{\mathrm{ortho}}(k) \simeq 1.71\times 10^3\,\mathcal{P}_{\mathrm{S}}^2(k)\,(f_{\mathrm{NL}}^{\mathrm{ortho}})^2.$$



$f_{\mathrm{NL}}^{ortho} = -38 \pm 24$

[1] Planck Collaboration, Planck 2018 results. IX. Constraints on primordial non-Gaussianity, Astronomy and Astrophysics (2020)

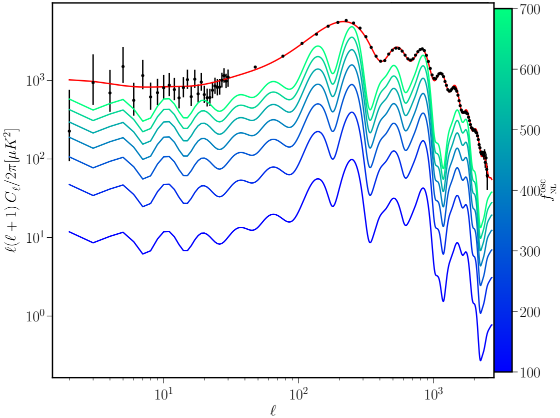
Oscillatory type

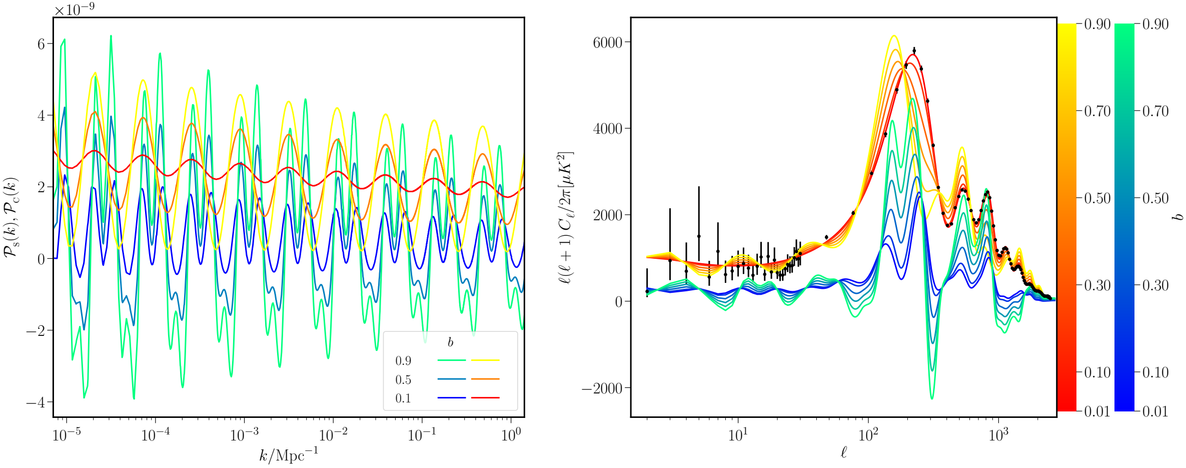
$$\mathcal{P}_{\rm S}^{\rm osc}(k) = A_{\rm S}\left(\frac{k}{k_*}\right)^{n_{\rm S}-1}\left\{1 + b\,\sin\left[\omega\ln\left(\frac{k}{k_o}\right)\right]\right\},$$

$$\mathcal{B}^{\rm osc}(k_1,k_2,k_3) = \frac{6A_{\rm S}^2 f_{\rm NL}^{\rm osc}}{(k_1k_2k_3)^2}\sin\left[\omega\ln\left(\frac{k_1+k_2+k_3}{k_o}\right)\right].$$

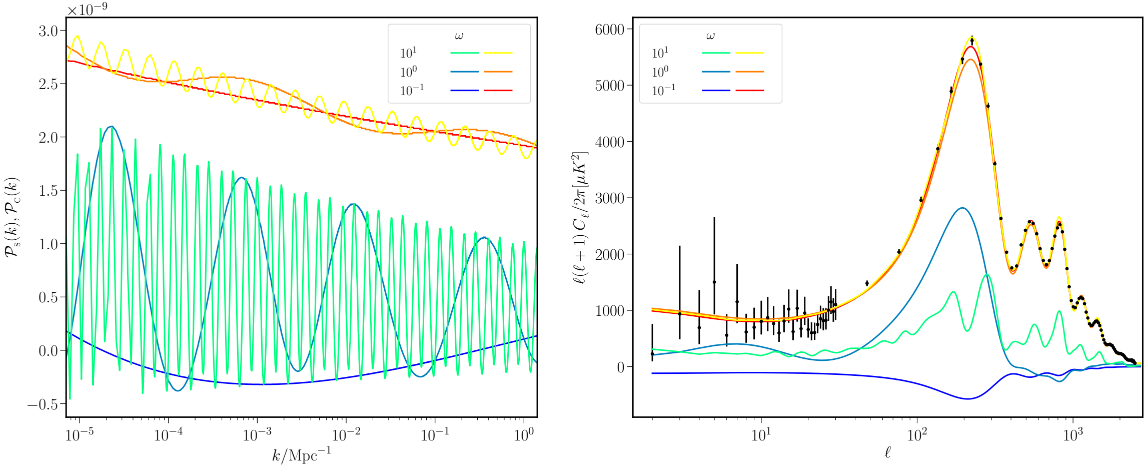
$$\mathcal{P}_{\rm C}^{\rm osc}(k) = 288\pi\left[A_{\rm S}\left(\frac{k}{k_*}\right)^{n_{\rm S}-1} f_{\rm NL}^{\rm osc}\right]^2\int_0^\infty {\rm d}x\int_{|1-x|}^{1+x}{\rm d}y\,x\,y$$
$$\times F(x)\,F(y)\sin^2\left[\omega\ln\left(\frac{k}{k_o}(1+x+y)\right)\right]$$
$$\times\left\{F(x)\,F(y) + x^3\,F(1)\,F(y) + y^3\,F(1)\,F(x)\right\}^{-1}.$$

where $F(z) = \left[1 + b\sin\left(\omega\ln\frac{kz}{k_o}\right)\right], z = 1, x, y$





[1]B. Das, H. V. Ragavendra, Indirect imprints of primordial non-Gaussianity on cosmic microwave background, arXiv:2304.05941 (2023)



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[1] B. Das, H. V. Ragavendra, Indirect imprints of primordial non-Gaussianity on cosmic microwave background, arXiv:2304.05941 (2023)

Starobinsky model

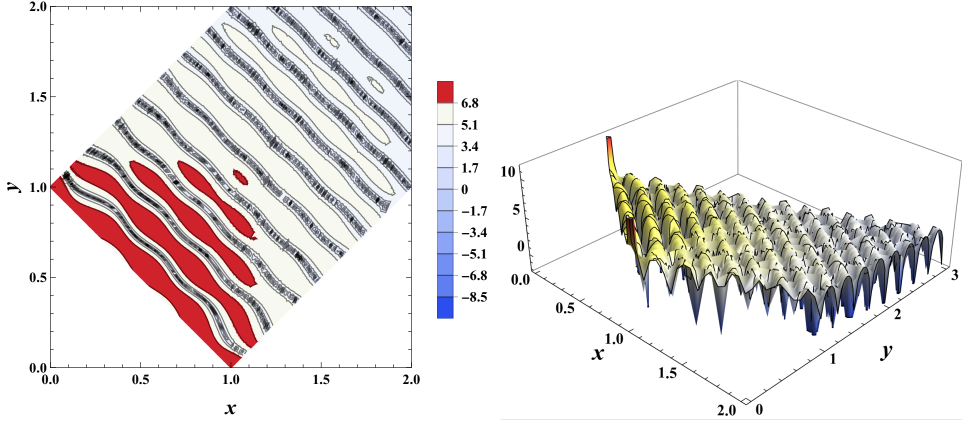
$$V(\phi) = \begin{cases} V_0 + A_+(\phi - \phi_0), & \text{for } \phi > \phi_0\,, \\ V_0 + A_-(\phi - \phi_0), & \text{for } \phi < \phi_0, \end{cases}$$

$$\begin{aligned}\mathcal{P}_{\mathrm{S}}(k) &= \frac{1}{12\pi^2}\left(\frac{V_0}{M_{\mathrm{Pl}}^4}\right)\left(\frac{V_0}{A_- M_{\mathrm{Pl}}}\right)^2 \left\{1 - \frac{3\Delta A}{A_+}\frac{k_0}{k}\left[\left(1 - \frac{k_0^2}{k^2}\right)\sin\left(\frac{2k}{k_0}\right) + \frac{2k_0}{k}\cos\left(\frac{2k}{k_0}\right)\right]\right. \\ &\left. + \frac{9\Delta A^2}{2A_+^2}\frac{k_0^2}{k^2}\left(1 + \frac{k_0^2}{k^2}\right)\left[1 + \frac{k_0^2}{k^2} - \frac{2k_0}{k}\sin\left(\frac{2k}{k_0}\right) + \left(1 - \frac{k_0^2}{k^2}\right)\cos\left(\frac{2k}{k_0}\right)\right]\right\}.\end{aligned}$$

$$\begin{aligned}\mathcal{P}_{\mathrm{C}}(k) &\simeq \frac{9}{16}\frac{k_0^2}{k^2}\left(\frac{A_-}{A_+}\right)^2\left(1 - \frac{A_-}{A_+}\right)^2\left(\mathcal{P}_{\mathrm{S}}^0\right)^2\int_0^\infty \mathrm{d}x \int_{|1-x|}^{1+x}\mathrm{d}y \frac{|\alpha_{kx} - \beta_{kx}|^2}{x^2}\frac{|\alpha_{ky} - \beta_{ky}|^2}{y^2} \\ &\times \left(\frac{Z(k,x,y)}{|\alpha_{kx} - \beta_{kx}|^2|\alpha_{ky} - \beta_{ky}|^2 + y^3|\alpha_k - \beta_k|^2|\alpha_{kx} - \beta_{kx}|^2 + x^3|\alpha_k - \beta_k|^2|\alpha_{ky} - \beta_{ky}|^2}\right)^2.\end{aligned}$$

where $\mathcal{P}_{\mathrm{S}}^0 \simeq \frac{1}{12\pi^2}\frac{V_0}{M_{\mathrm{Pl}}^4}\left(\frac{V_0}{A_- M_{\mathrm{Pl}}}\right)^2$, α_k and β_k are the Bogoliubov coefficients of the mode functions and $Z(k,x,y)$ function captures shape of the bispectrum.

Integrand of $\mathcal{P}_{\mathrm{C}}$ for Starobinsky



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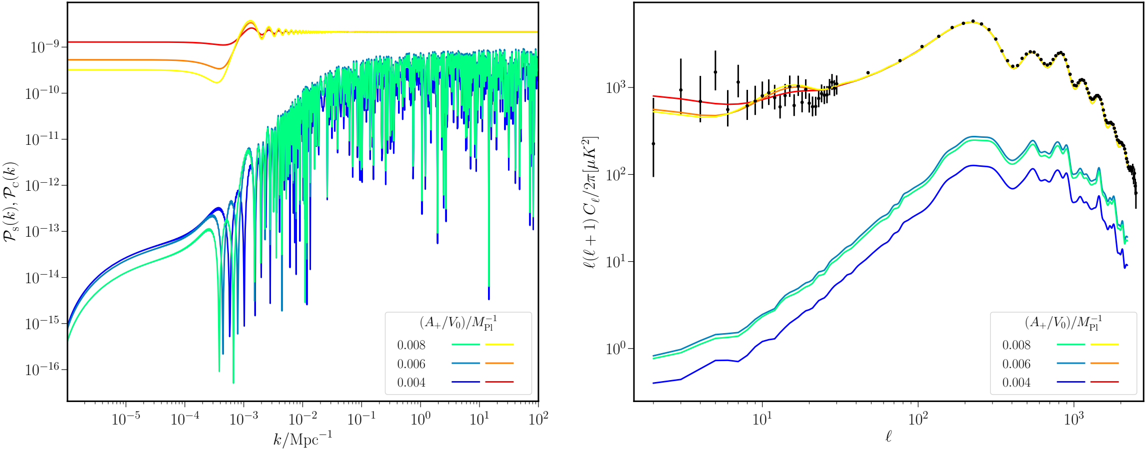

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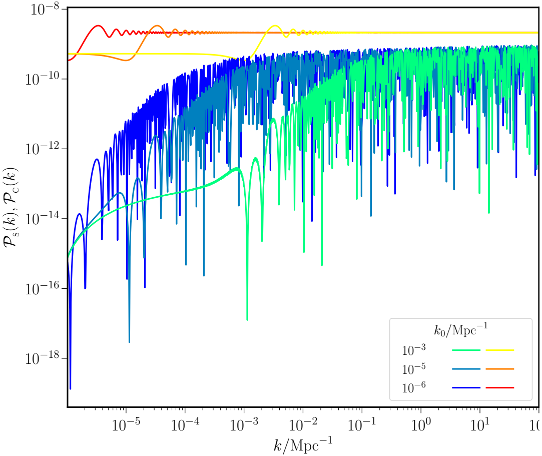

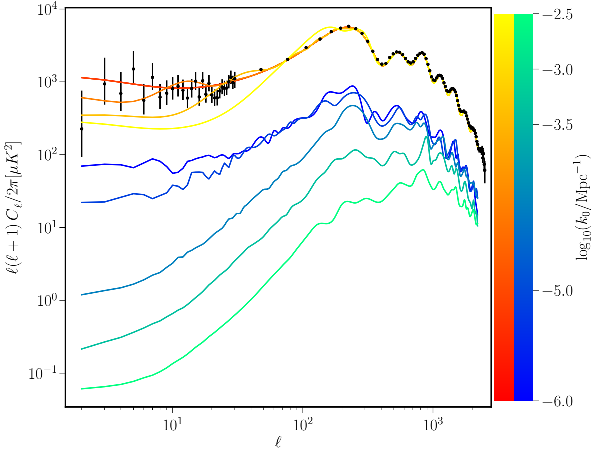

[1] B. Das, H. V. Ragavendra, Indirect imprints of primordial non-Gaussianity on cosmic microwave background, arXiv:2304.05941 (2023)

Conclusion and Outlook

Conclusion

- Non-Gaussianity can arise from non-linear dynamics because of the self-interaction of inflaton field.
- There has been ways to constrain $f_{\rm NL}$ directly. But, it involves computing three point function which are numerically time taking for non-trivial models.
- In our paper, we used a new method of constraining $f_{\rm NL}$ indirectly. It introduces a correction term, $\mathcal{P}_{\rm C}(k)$, to the Gaussian power spectrum $\mathcal{P}_{\rm S}(k)$.
- We computed the corrected power spectrum for available templates as well as realistic models.
- We found significant non-Gaussian contribution for oscillatory template and Starobinsky model.

Outlook

- Researchers have tried to compute non-Gausssian corrections using loop-diagrams, especially over small scales. Our work uses method equivalent to it but over large scales.
- We will perform Monte-Carlo analysis of these models against Planck 2018 dataset as our future work.

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Thank you!

The talk was based on:

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