# Ultraviolet Unitarity Violations In Nonminimally Coupled Scalar-Starobinsky Inflation

This presentation outlines briefly the work leading up to the paper
JCAP01(2023)029 or arXiv:2205.12836

prepared for

Cosmology from Home 2023

# NAIVE PREDICTIONS AND THEIR FALLACY

- Consider the Lagrangian: $\mathcal{L} = \frac{1}{2}\phi\Box\phi + \frac{1}{\Lambda}\phi^2\Box\phi$
- The corresponding amplitude in the s-channel: $\mathcal{M}(\phi\phi \to \phi\phi) = \sim \frac{p^2}{\Lambda}\frac{1}{p^2}\frac{p^2}{\Lambda} \sim \frac{s}{\Lambda^2}$
- From this, we can estimate that the breakdown of the theory on grounds of unitarity happens as $E \to \Lambda$.
- This does <u>not</u> mean that the theory is not unitary, just that this diagram is insufficient to represent the physics of this process as $E \to \Lambda$.

Images and arguments taken from 'Quantum Field Theory and the Standard Model' by Matthew Schwartz

# HIGGS' INFLATION

- Now, consider a perturbative approach involving $H = \frac{1}{\sqrt{2}}(\Phi_1 + i\Phi_2)$ as the Higgs' singlet scalar for Higgs' inflation:

$$S = \int d^4x\sqrt{-g}\left[\left(1 + \frac{2\xi|H|^2}{M_{Pl}^2}\right)\frac{M_{Pl}^2 R}{2} - \partial_\mu H^\dagger \partial^\mu H - V(|H|)\right]$$

- Going to the Einstein frame by taking the Weyl transformation: $g_{\mu\nu} \to \left(1 + \frac{2\xi\,|H|^2}{M_P^2}\right) g_{\mu\nu}$

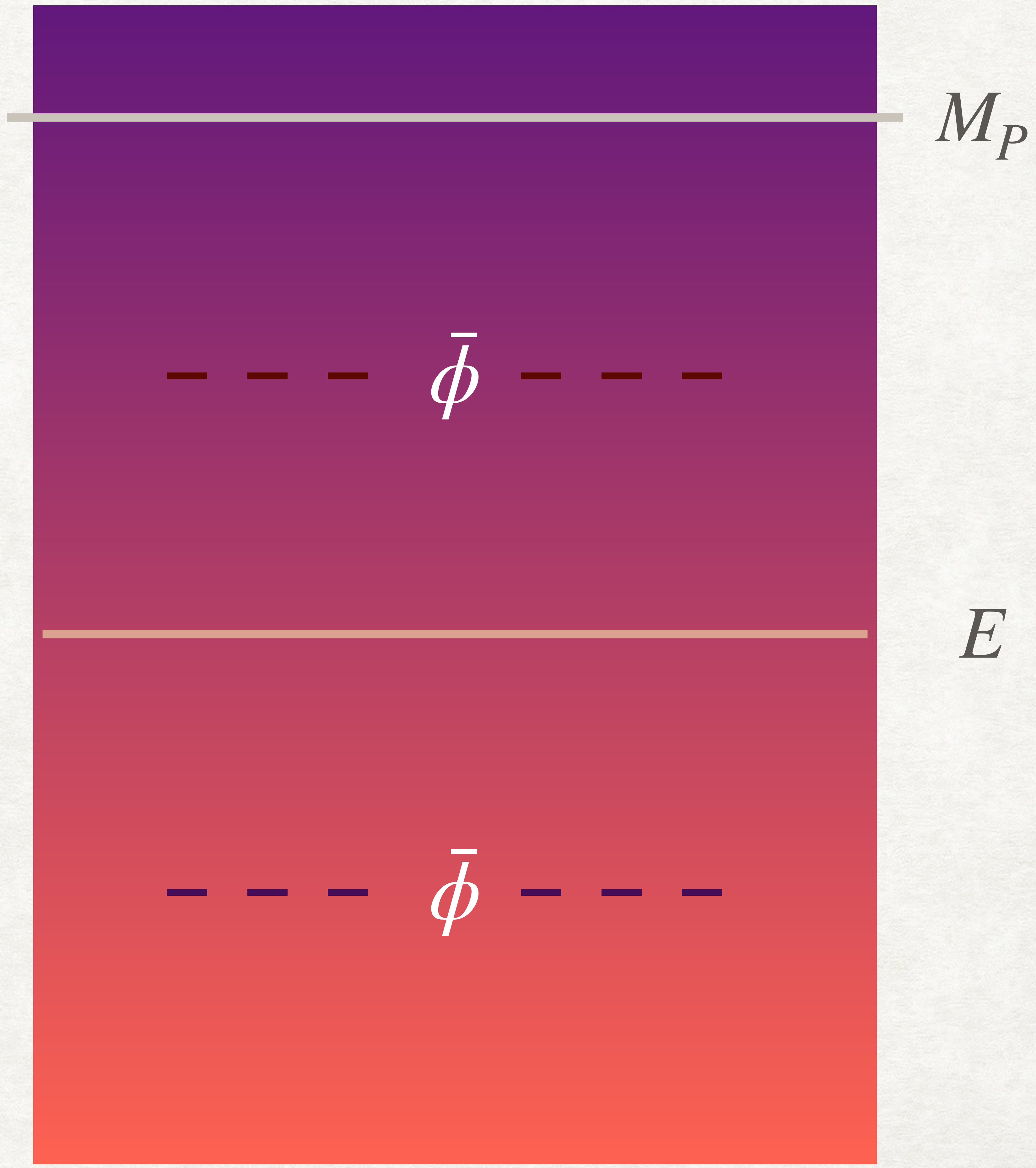
- Weyl transf. brings rescaling factors to denominator of matter terms and requires perturbative expansion and canonicalization.

- Scalars around a background $\left(\Phi = \bar{\phi} + \phi\right)$. Expansion hinges on how large or small $\bar{\phi}$ is compared to $M_P$ and $\phi$.

- Estimated unitarity violation close to $M_P/\sqrt{\xi}$ (Palatini formalism) for EW vacuum ($\bar{\phi} \to 0$).

# LARGE BACKGROUND, SMALL BACKGROUND

- The two regimes correspond to the two epochs in the cosmological paradigm.
- Inflation $\implies$ $\bar{\phi}$ is large.
- Reheating $\implies$ EW vacuum $\implies \bar{\phi} \sim 0$.
- Taylor expansion of $(1+x)^n$ depends on whether $x \gg 1$ or $x \ll 1$, and results are different unless we sum over all the terms without truncating the series.



# McDONALD’S PROPOSAL (Palatini Only)

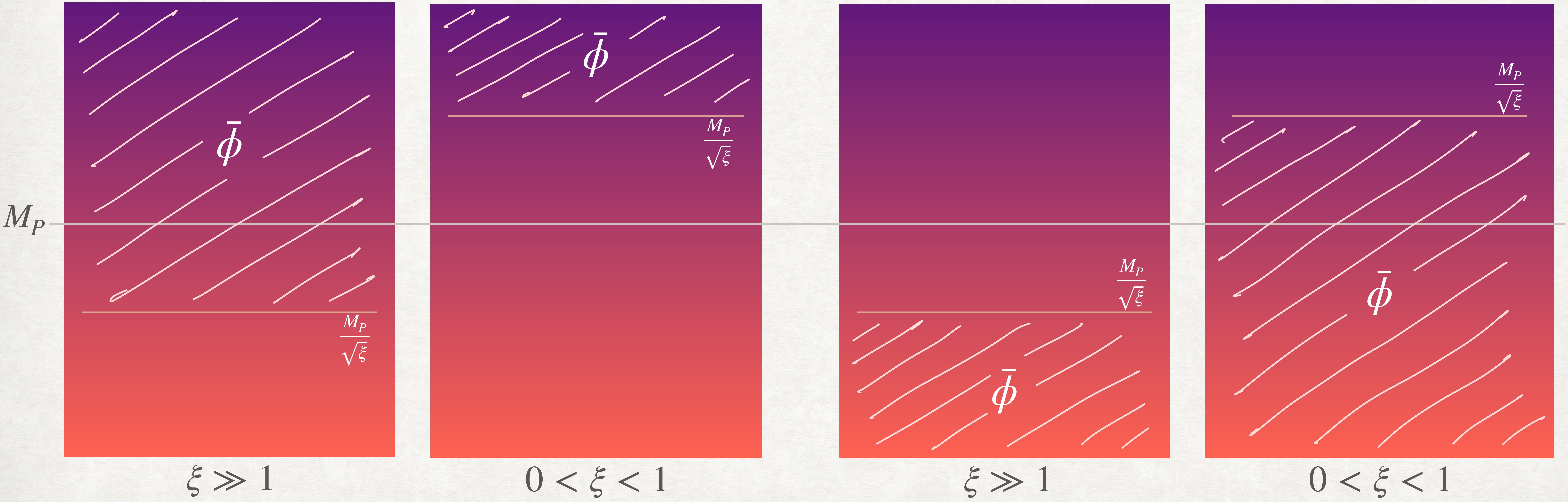
arXiv:2007.04111 [hep-ph]

- Such estimations are only applicable in specific regimes and cannot be extrapolated.
- Claimed that within the interaction volume: $\langle \phi \rangle \sim E$.
- Suggested working around $\bar{\phi} \ll E$; did not spoil predictions as we can safely take the limit $\bar{\phi} \to 0$.
- He also assumed that working in the $\bar{\phi} \gg E$ limit and later matching the two could give us the whole picture; better than the ‘naive predictions’.
- Also, proposed that predictions in the Jordan frame would be more accurate; could be recovered by looking at how fields transform between the frames.

# ENERGIES AND BACKGROUNDS

$\bar{\phi} \gg \frac{M_P}{\sqrt{\xi}}$

$\bar{\phi} \ll \frac{M_P}{\sqrt{\xi}}$




$E$ scales from 0 to $M_P$; shaded region shows range of $\bar{\phi}$

# ANTONIADIS ET AL.'S CORRECTION

arXiv:2203.10040 [hep-ph]

- The paper is actually an addendum to arXiv:2106.09390 [hep-th] which called out McDonald's proposals.
- Holds the key to circumventing one McDonald's problematic assumptions regarding matching.
- The authors essentially sum the infinite terms in the Taylor series using form factors:

$$\bar{G} = 1 + \frac{\xi\bar{\phi}^2}{M_P^2}, \qquad x^2 = \frac{\xi\bar{\phi}^2}{M_P^2\bar{G}}.$$

- Avoid any discrepancy between the large background and the small background regimes, at least at the level of perturbative expansion.

# SCALAR-STAROBINSKY COUPLING

$$S = \int d^4x \sqrt{g} \left[ \frac{M_P^2}{2} \left( 1 + \frac{\xi |\Phi|^2}{M_P^2} \right) \left( R + \frac{\alpha}{2M_P^2} R^2 \right) - |\partial \Phi|^2 \right]$$

- $\Phi^2 R^2$ coupling inspired by background behaviour in arXiv:1705.07945 and as one-loop correction in arXiv:2007.10395.
- Unitarity analysis performed for all limits of $\xi$ and $\alpha$. Later, matched with physical inflation models using observational constraints on parameters from different potentials.
- Example: arXiv:1705.07945 found that for safe exit from the inflationary epoch, for their model, $\alpha$ was small & negative, while no constraints were directly put on $\xi$.
- Similarly, a Higgs'-like inflation model found in arXiv:1701.03814 imposes $\xi \gg 1$ with no constraints on $\alpha$.

# METRIC FORMULATION RESULTS

| | | | Unitarity up to $M_P$ | $\lvert\alpha\rvert \to 0$ limit |
|---|---|---|---|---|
| $\lvert\alpha\rvert \ll 1$ | Large Background | $b^2 \to 1$ | Safe | Unsafe |
| | | $b^2 \to 1/2$ | Safe | Unsafe |
| | | $b^2 \to 0$ | Safe if $\frac{\alpha^2 \xi \bar{\phi}_1^2}{M_P^2} \leq 1$ | Safe |
| | Small Background | $\xi \gg 1$ | Unsafe | Unsafe |
| | | $0 < \xi < 1$ | Safe | Safe |
| $\lvert\alpha\rvert \to 1$ | Large Background | $b^2 \to 1, 1/2$ | NA | |
| | | $b^2 \to 0$ | Unsafe | |
| | Small Background | $\xi \gg 1$ | Safe | |
| | | $0 < \xi < 1$ | Safe | |
| $\lvert\alpha\rvert \gg 1$ | Large Background | $b^2 \to 1$ | NA | |
| | | $b^2 \to 1/2$ | Unsafe | |
| | | $b^2 \to 0$ | Unsafe | |
| | Small Background | $\xi \gg 1$ | NA | |
| | | $0 < \xi < 1$ | Unsafe | |

where $b^2 = \frac{6\xi x^2}{1 + 6\xi x^2}$

# PALATINI FORMULATION RESULTS

| | | | Unitarity up to $M_P$ | $\|\alpha\| \to 0$ limit |
|---|---|---|---|---|
| $\|\alpha\| \ll 1$ | Large Background | $\xi \gg 1$ | Safe | Safe |
| | | $0 < \xi < 1$ | Safe | Safe |
| | Small Background | $\xi \gg 1$ | Unsafe | Safe |
| | | $0 < \xi < 1$ | Safe | Safe |
| $\|\alpha\| \to 1$ | Large Background | $\xi \gg 1$ | Safe | |
| | | $0 < \xi < 1$ | Safe | |
| | Small Background | $\xi \gg 1$ | Unsafe | |
| | | $0 < \xi < 1$ | Safe | |
| $\|\alpha\| \gg 1$ | Large Background | $\xi \gg 1$ | Safe if $\|\alpha\| \leq \xi$ | |
| | | $0 < \xi < 1$ | Safe if $\frac{\|\alpha\| M_P^2}{\xi \bar{\phi}_1^2} \leq 1$ | |
| | Small Background | $\xi \gg 1$ | Unsafe | |
| | | $0 < \xi < 1$ | Unsafe | |

# CONCLUSIONS

- For the inflation scenario in arXiv:1705.07945, in the metric formulation, unitarity is preserved in inflation and reheating epochs for $0 < \xi < 1$. $\xi \to \dfrac{1}{6}$ is lucrative, but considering the safety of $\alpha \to 1$, the viable range is $\xi \ll 1$. In the Palatini formulation, the safe limit is again $0 < \xi < 1$.
- For a Higgs'-like inflation, we were unable to find any viable ranges based on the present analysis. This was primarily due to computational constraints for $|\alpha| \geq 1$.
- McDonald's assumption about the continuity of the scattering amplitude when working around a small or large background doesn't hold (also proved by Antoniadis et al. in arXiv:2106.09390 [hep-th]).

# THANK YOU