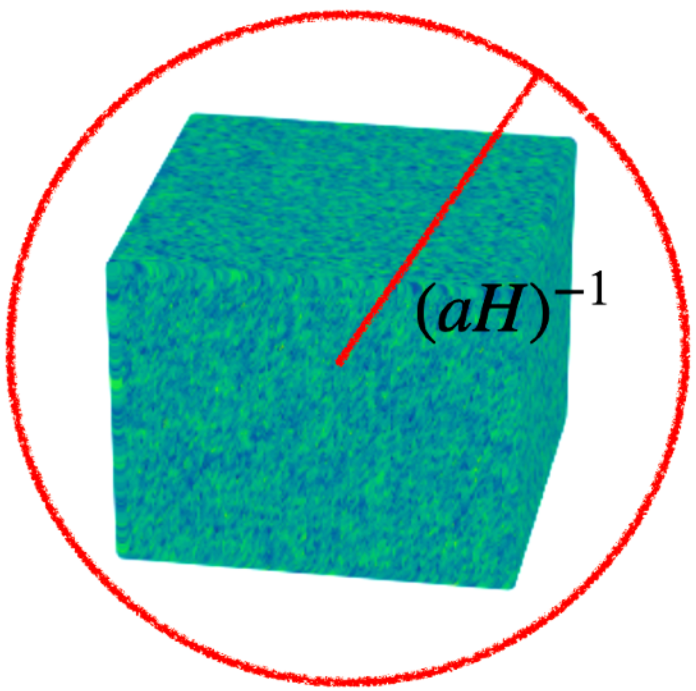
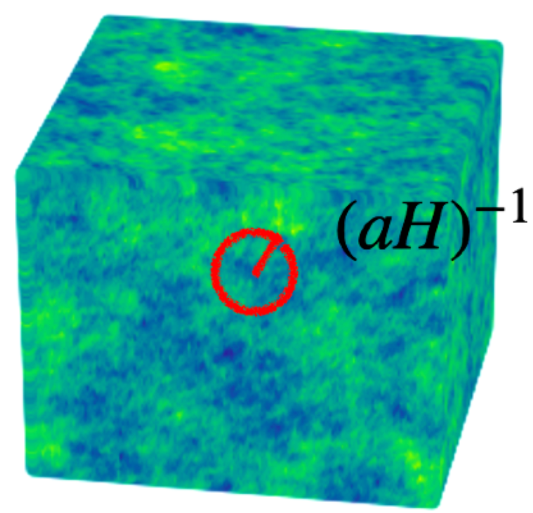
Lattice Simulations of Axion Inflation

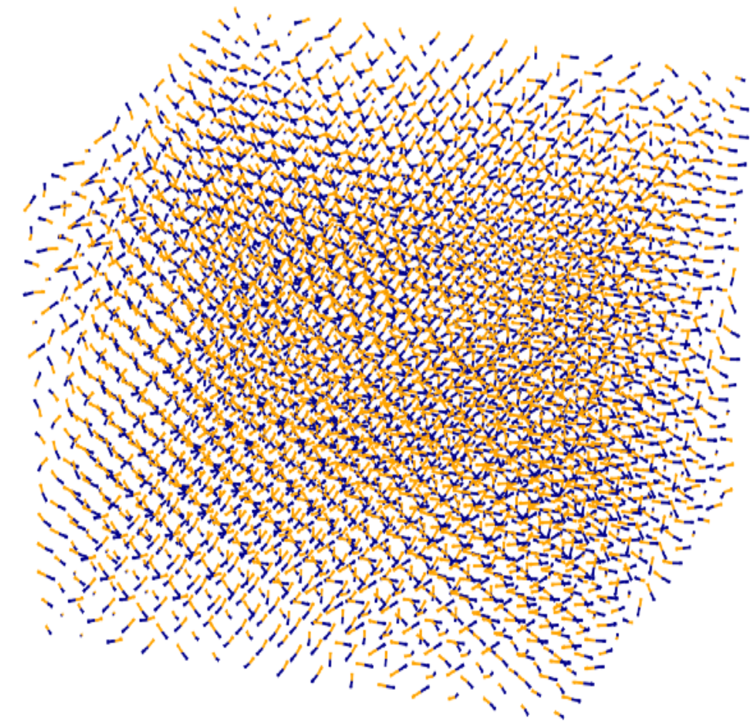
Angelo Caravano (LMU Munich)

Based on:

A. Caravano, E. Komatsu, K. D. Lozanov, J. Weller arXiv:2204.12874







@ Cosmology from home 2023

[**A. Guth**, Phys. Rev. D 23 (1981) 347.]
[**K. Sato**, Mon. Not. Roy. Astron. Soc. 195 (1981) 467.]
[**A.D. Linde**, Adv. Ser. Astrophys. Cosmol. 3 (1987) 149.]

Inflation (in 1 slide)

- Accelerated expansion of the early universe: $\ddot{a} > 0$ solves horizon and flatness problems

[**A. Guth**, Phys. Rev. D 23 (1981) 347.]
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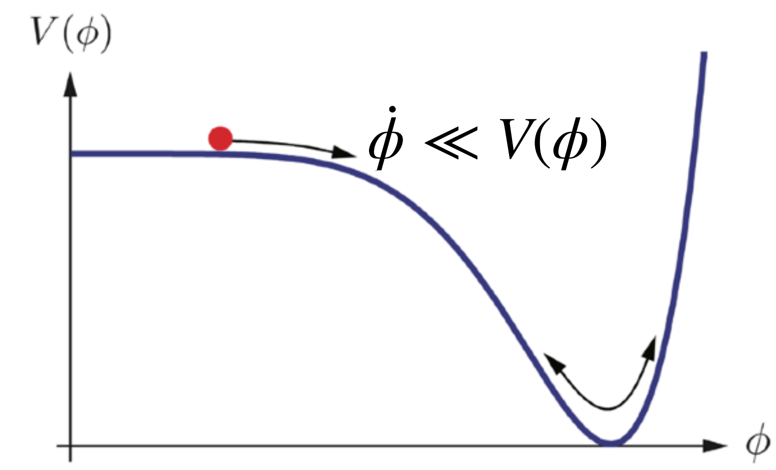
- Driven by a scalar field, the inflaton:

$$\phi = \phi(t)$$



Accelerated expansion if the potential is "flat"

Slow-roll condition: $\dot{\phi} \ll V(\phi) \implies a \sim e^{Ht}$

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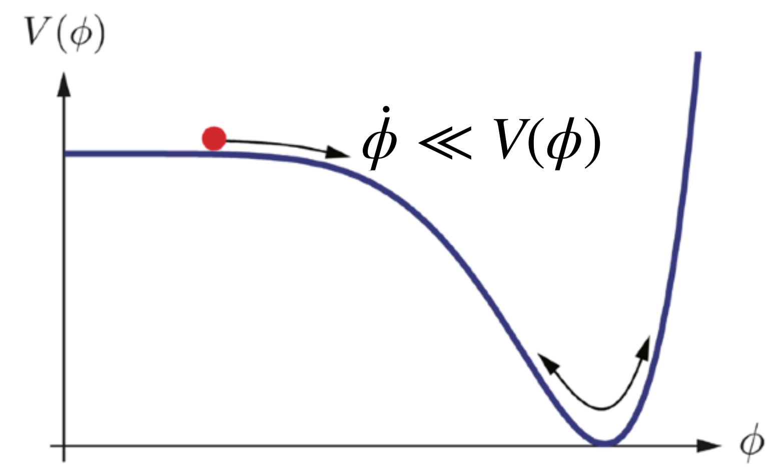
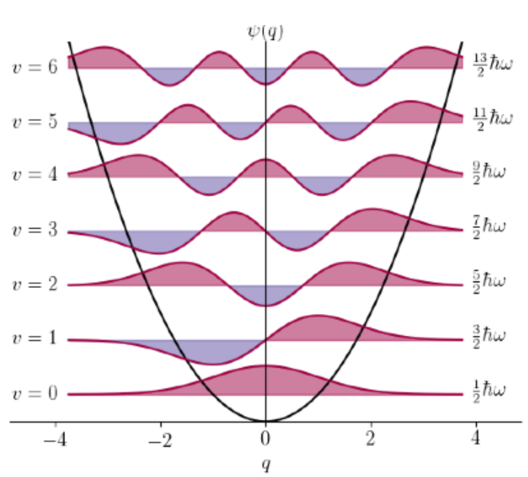
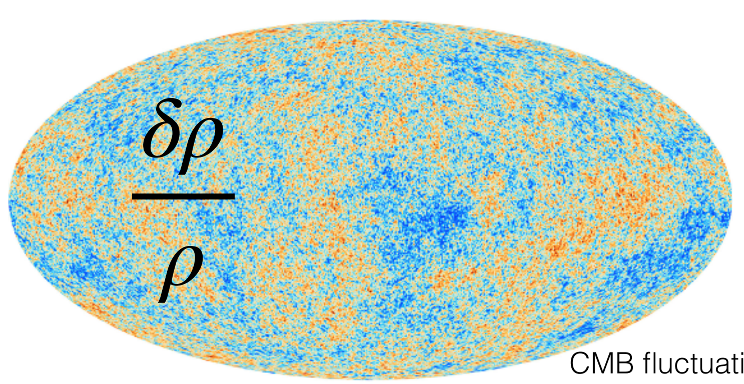


- Perturbations in the early universe as **quantum fluctuations**

$$\delta\hat{\phi}(\vec{k}, \tau)$$





CMB fluctuations from Planck

[**M. Anber, L. Sorbo** 0908.4089]

Axion-U(1) inflation

Adding an interaction between the inflation and a gauge field

$$\mathscr{L} \supset \phi F_{\mu\nu}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

[M. Anber, L. Sorbo 0908.4089]

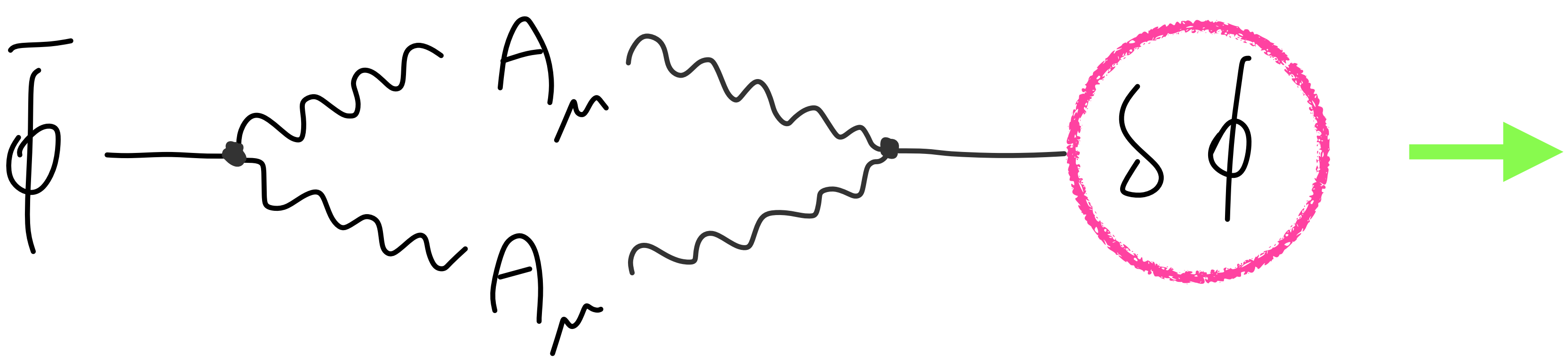
Axion-U(1) inflation

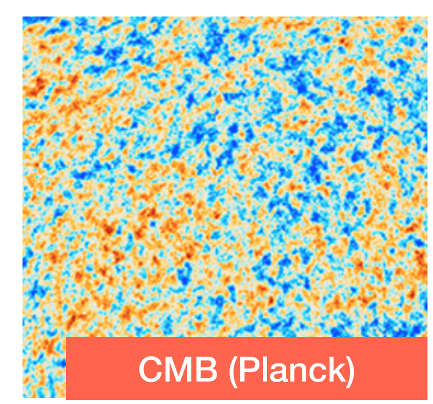
Interaction between the inflation and a gauge field $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

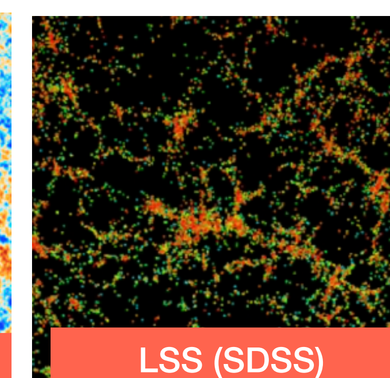
$$\mathscr{L} \supset \phi F_{\mu\nu} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$$

Observational consequences:

Production of gauge field particles → decay into inflaton perturbations







observable!

Axion-U(1) inflation

[**N. Barnaby, M. Peloso** 1011.1500]
[**M. Anber, L. Sorbo** 0908.4089]

Known results

- Power spectrum:

$$\mathscr{P}_\zeta(k) \simeq \mathscr{P}_{\rm vac} + \mathscr{P}_{\rm vac}^2 f_2(\xi) e^{4\pi\xi}$$

vacuum (single-field)

sourced

$$\mathscr{P}_{\rm vac} \simeq \frac{H^4}{(2\pi\dot{\phi})^2}$$

$$\xi = \frac{\alpha\dot{\phi}}{2fH}$$

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- Bispectrum:

$$f_{\rm NL}^{\rm (equil.)}(\xi) \simeq \frac{f_3(\xi)\mathscr{P}_{\rm vac}^3 e^{6\pi\xi}}{\mathscr{P}_\zeta^2}$$

Assuming constant ξ

Axion-U(1) inflation

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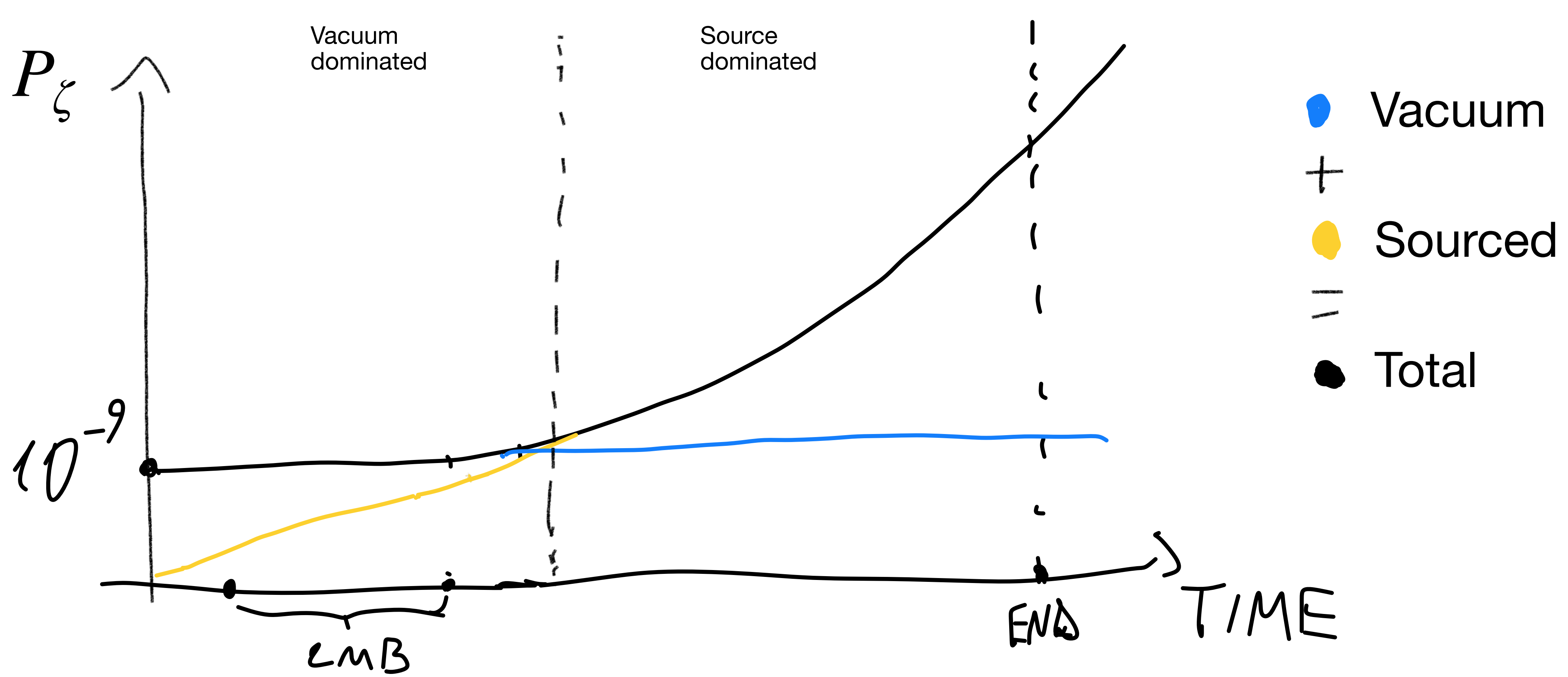
Scalar perturbations naturally grow on small scales



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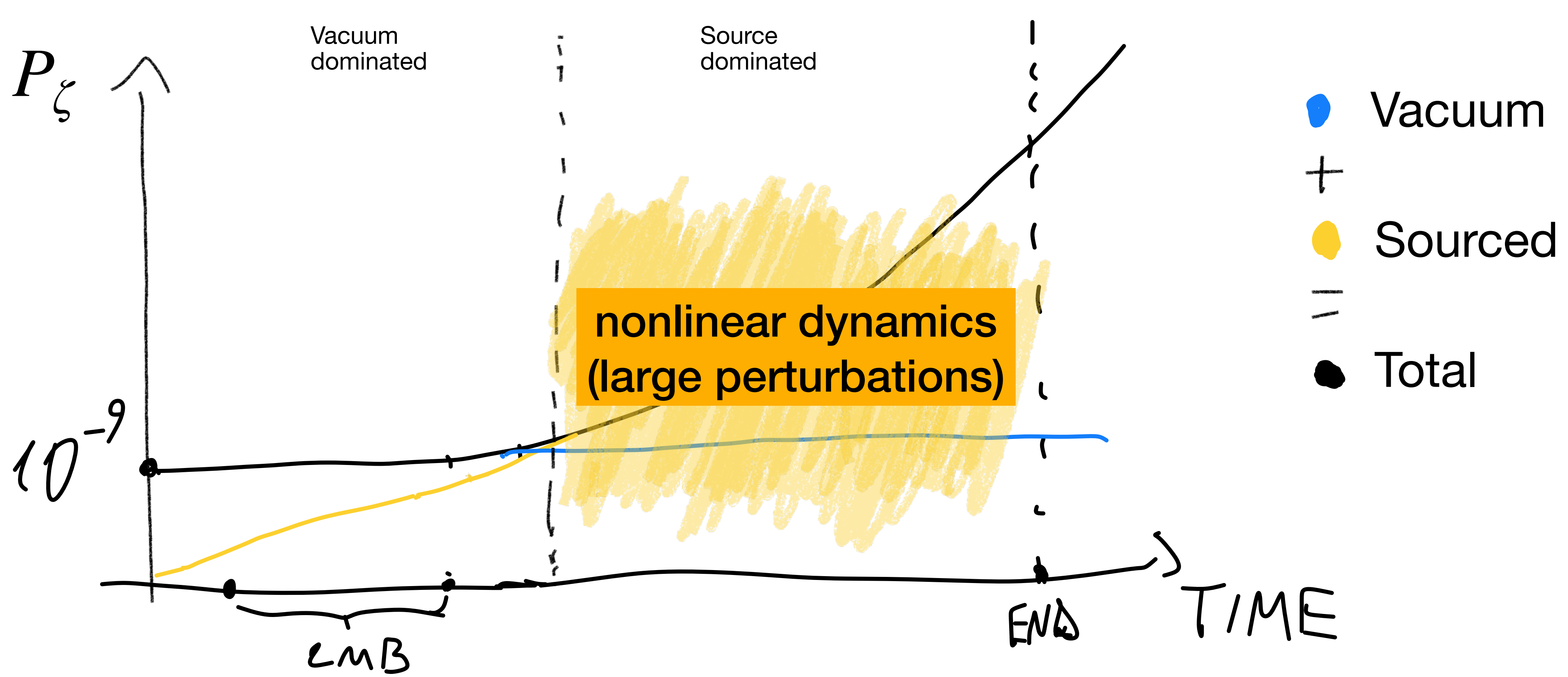

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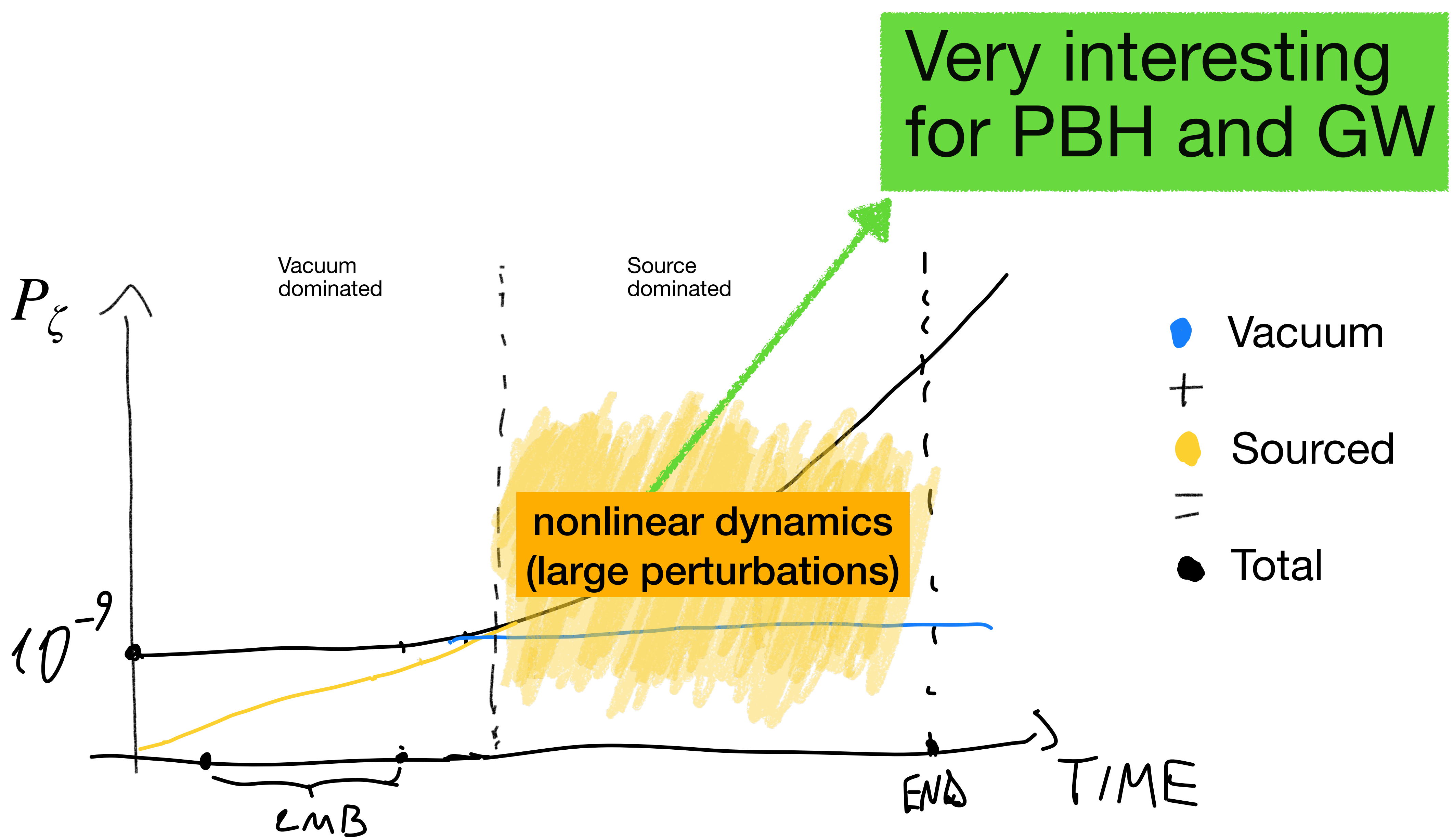

Axion-U(1) inflation

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More quantitatively:

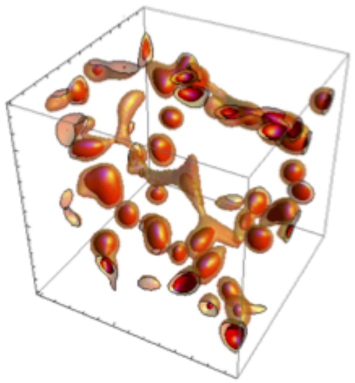
$$\partial_\tau^2 \bar{\phi} + 2\mathscr{H}\partial_\tau \bar{\phi} + a^2 V'(\bar{\phi}) = a^2 \frac{a}{f}\langle F_{\mu\nu}\tilde{F}^{\mu\nu}\rangle$$

If these terms become comparable → **backreaction**

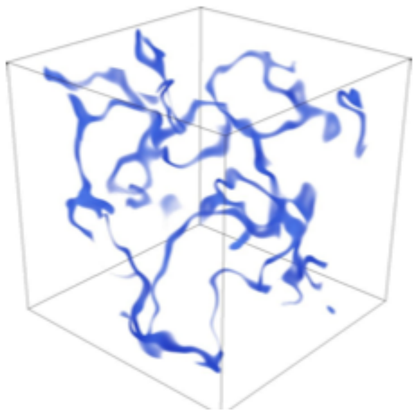
Some sort of extra friction, but not so simple (as we will see)

Lattice simulations

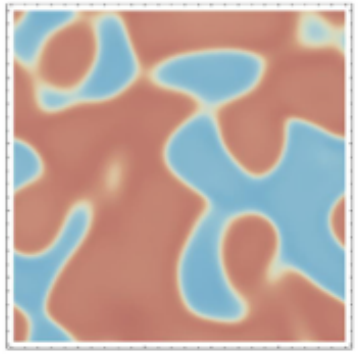
- Numerical tool to study non-linear cosmological phenomena.
- Typically associated with the reheating phase after inflation.



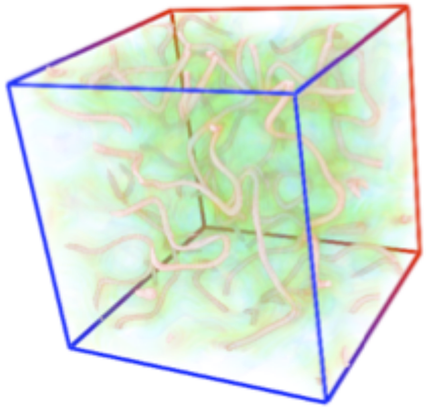
[**M. A. Amin, R. Easther, H. Finkel**, arXiv:1009.2505]



[**A. V. Frolov**, arXiv:1004.3559]



[**M. A. Amin, J. Fan, K. D. Lozanov, M. Reece**, arXiv:1802.00444]



[**J. Dufaux, D.G. Figueroa, J. Garcia-Bellido**, arXiv:1006.0217]

Lattice simulations

[**A. Caravano** 2209.13616 (PhD thesis)]

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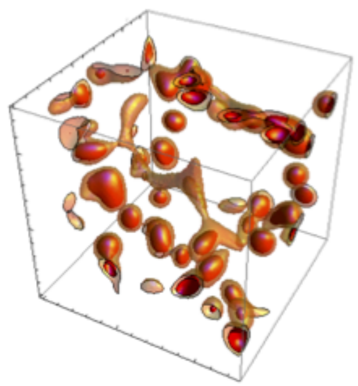

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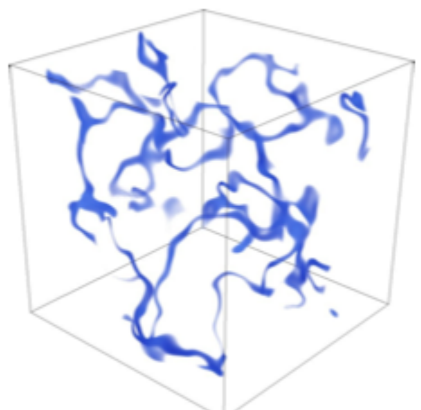

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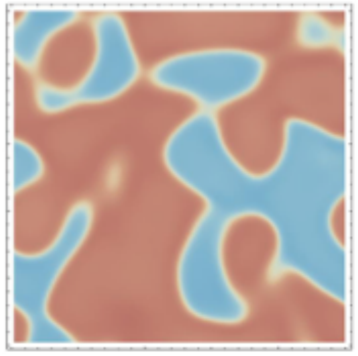

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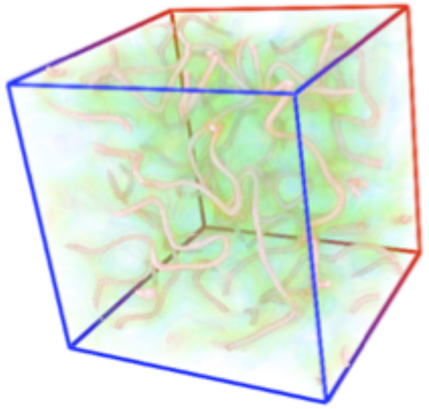


[**J. Dufaux, D.G. Figueroa, J. Garcia-Bellido**, arXiv:1006.0217]

Our goal:

Develop lattice techniques for inflation

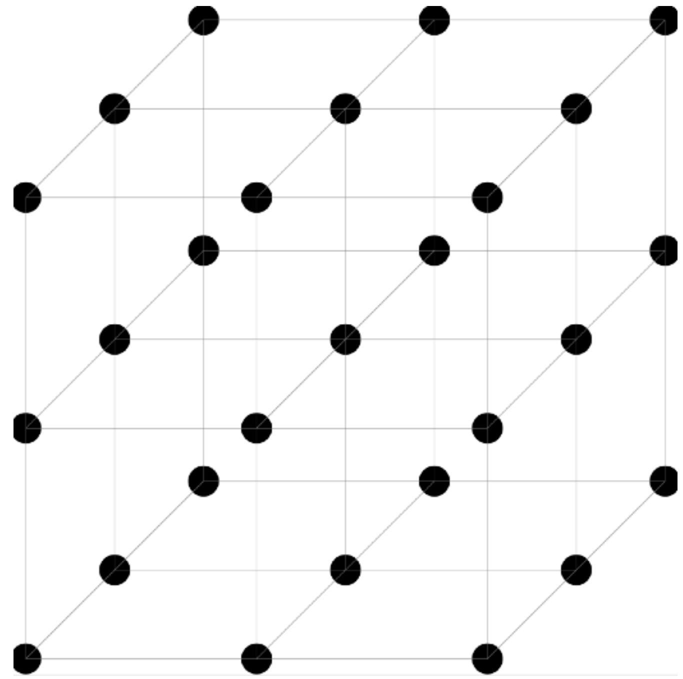
A. Caravano, E. Komatsu, K. D. Lozanov, J. Weller arXiv:2102.06378 arXiv:2110.10695 arXiv:2204.12874

In this talk: **focus on axion-U(1) model.**

Lattice approach

[**A. Caravano** 2209.13616 (PhD thesis)]

$\phi_i, \overrightarrow{A}_i$



Solve numerically for all lattice points:

$$\phi'' + 2H\phi' - \partial_j\partial_j\phi + a^2\frac{\partial V}{\partial \phi} = -a^2\frac{\alpha}{4f}F_{\mu\nu}\tilde{F}^{\mu\nu},$$

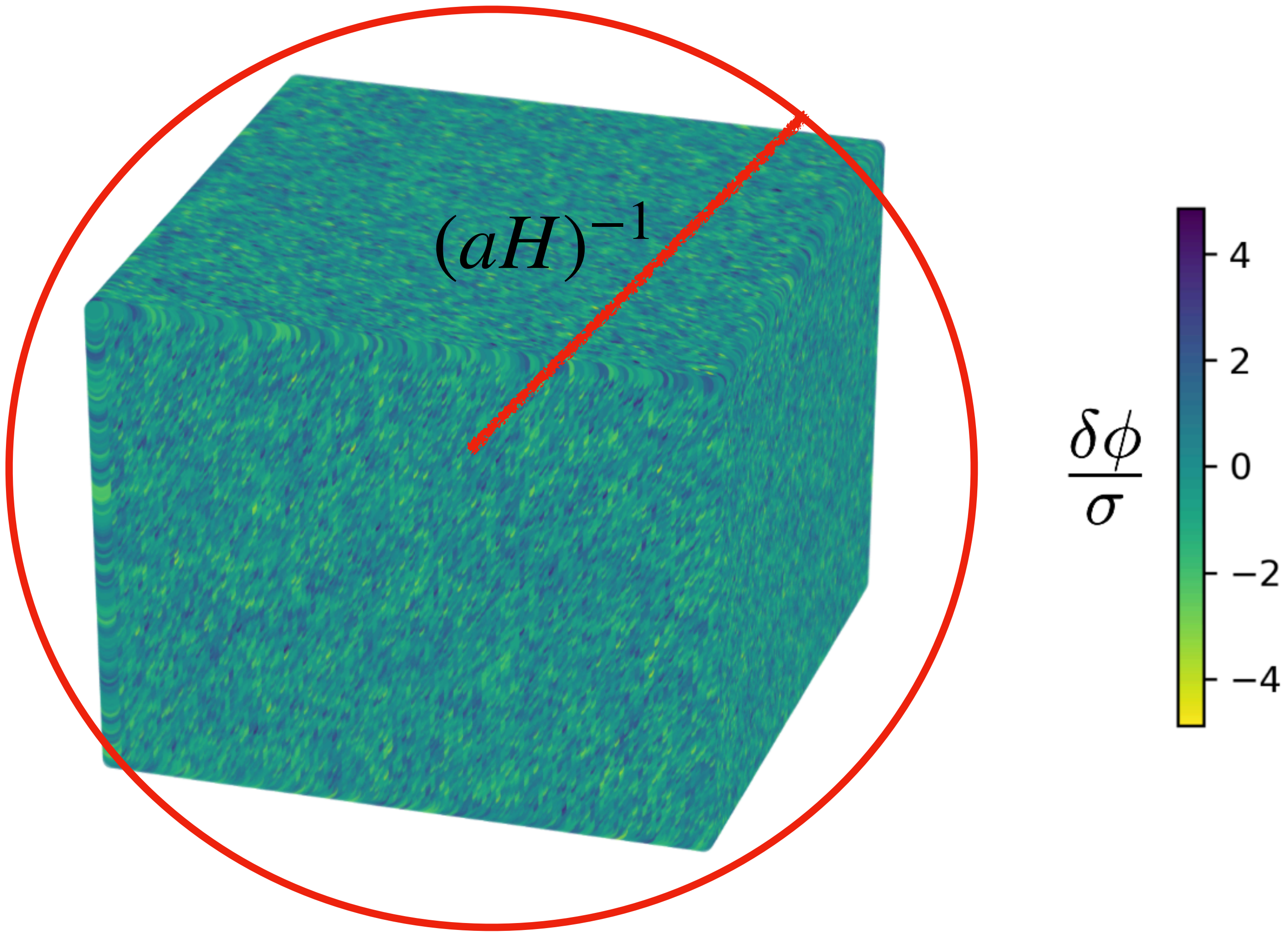
$$A_0'' - \partial_j\partial_j A_0 = \frac{\alpha}{f}\epsilon_{ijk}\partial_k\phi\partial_i A_j,$$

$$A_i'' - \partial_j\partial_j A_i = \frac{\alpha}{f}\epsilon_{ijk}\phi'\partial_j A_k - \frac{\alpha}{f}\epsilon_{ijk}\partial_j\phi(A_k' - \partial_k A_0)$$

Lattice approach

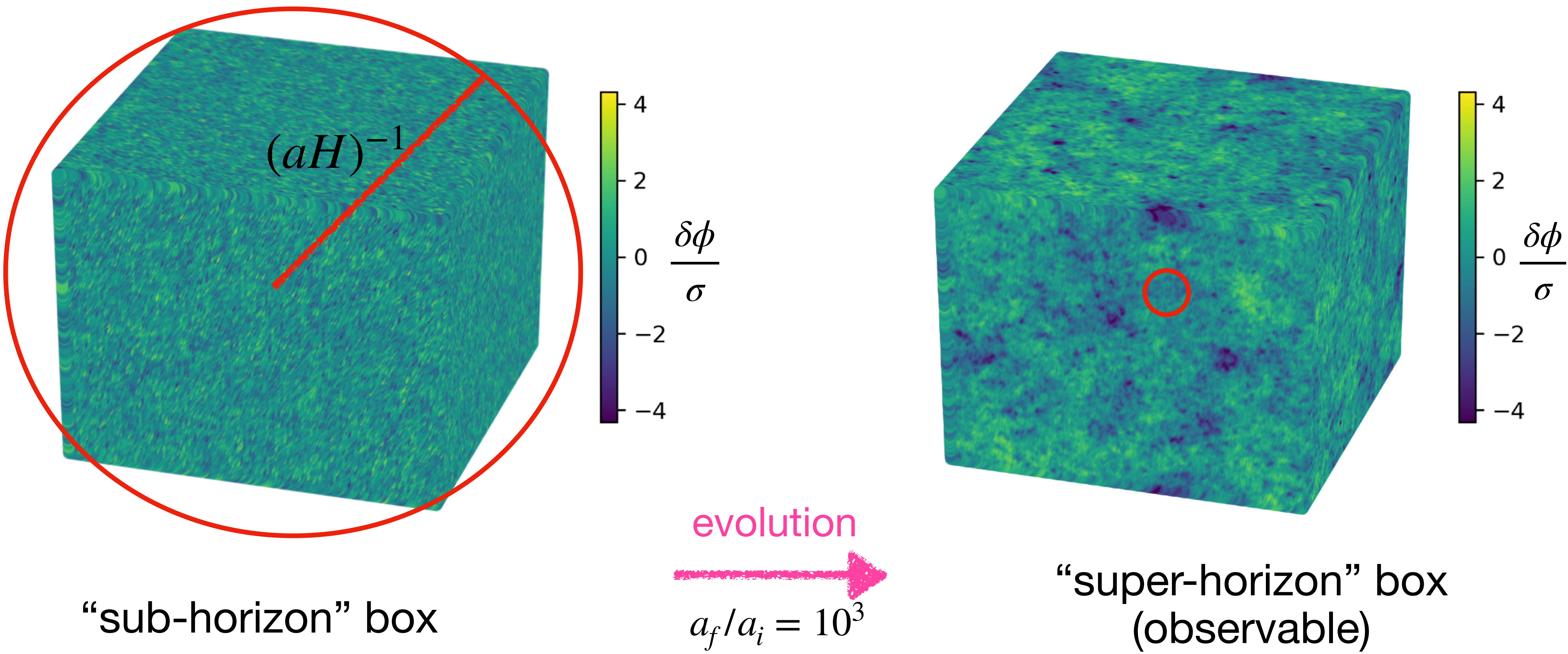
[**A. Caravano** 2209.13616 (PhD thesis)]

Start with a **sub-horizon** box



Lattice approach

[**A. Caravano** 2209.13616 (PhD thesis)]



"sub-horizon" box

"super-horizon" box
(observable)

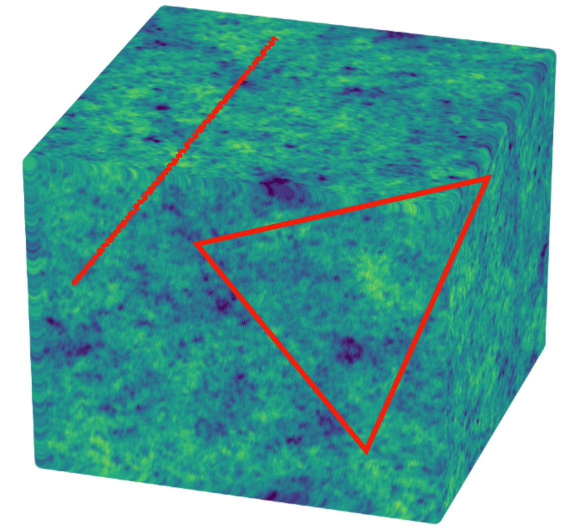
Results of the simulation:

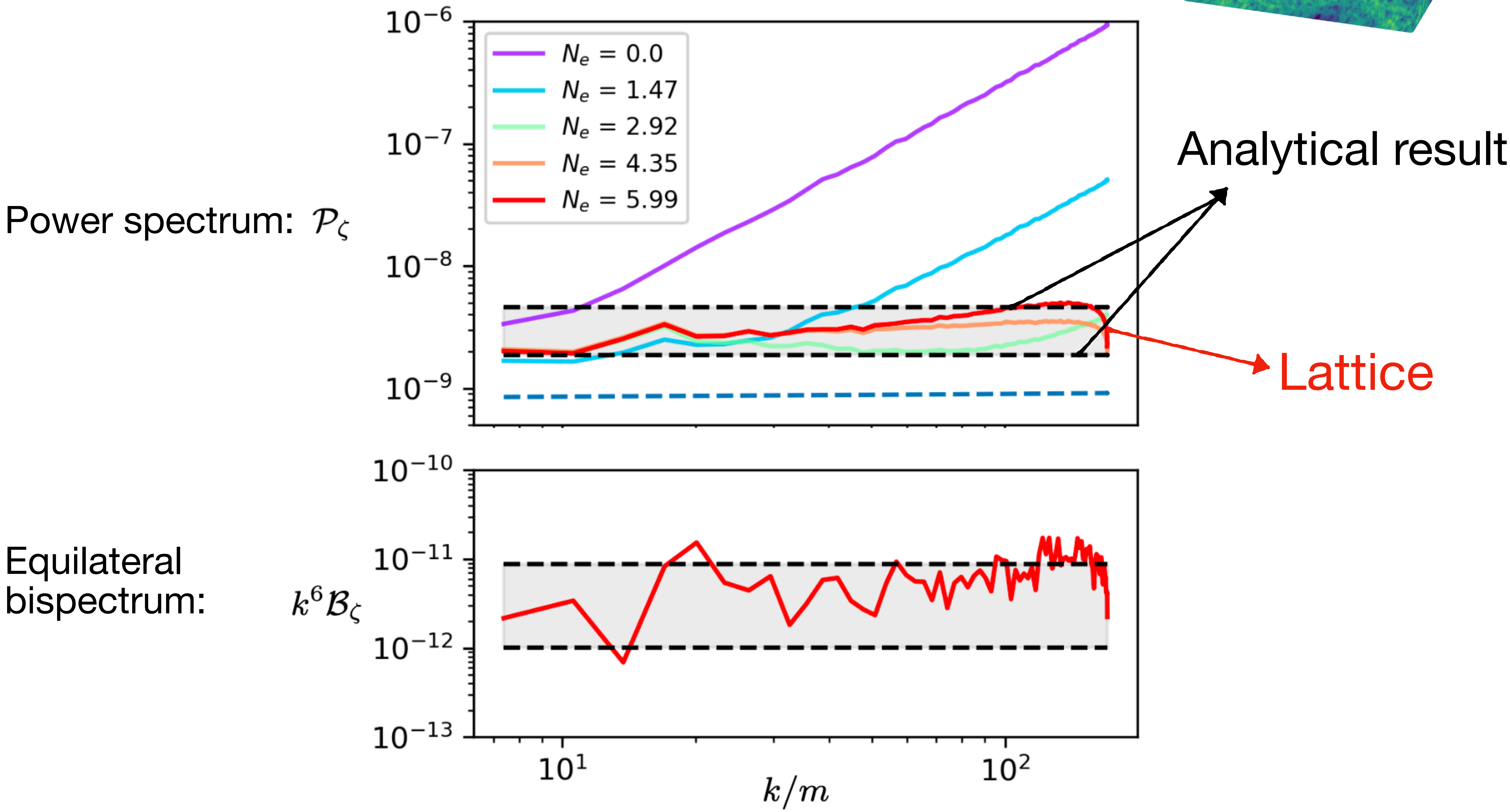
1. Large scales — small $\xi = \frac{\alpha\dot{\phi}}{2fH}$

2. Small scales — large $\xi = \frac{\alpha\dot{\phi}}{2fH}$

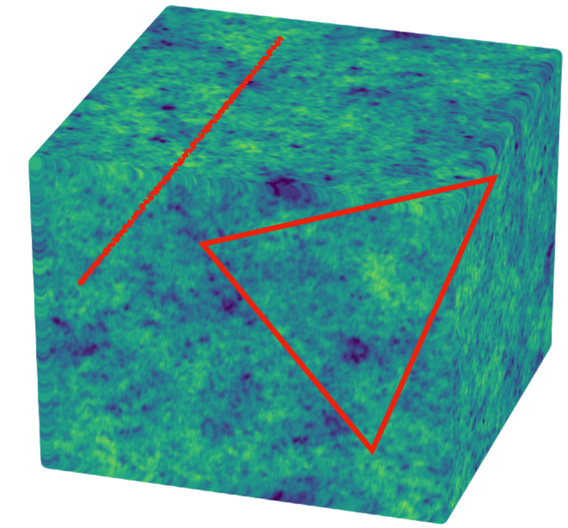
Linear regime (large scales)

Simulation confirms analytical results

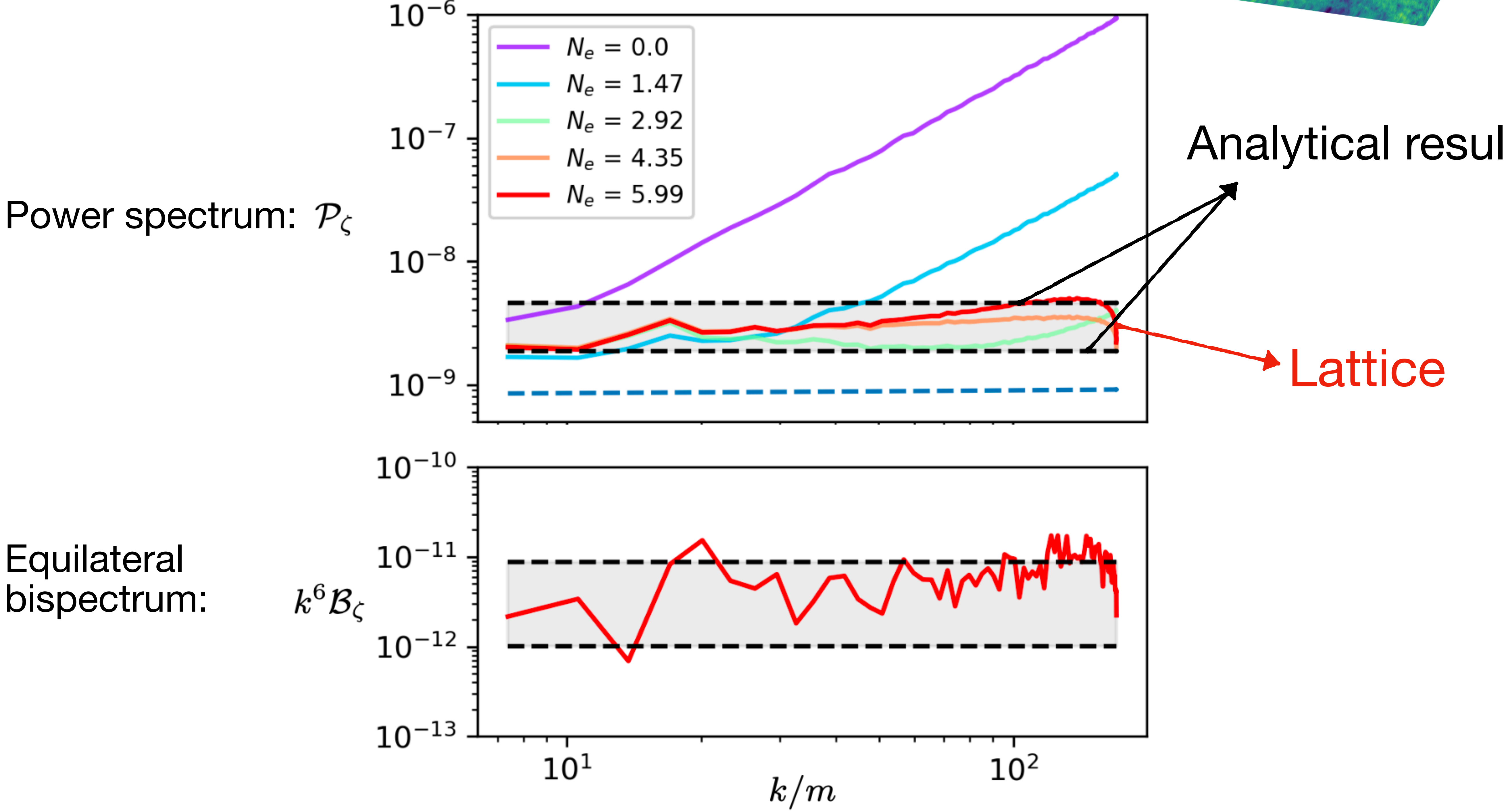




Linear regime (large scales)

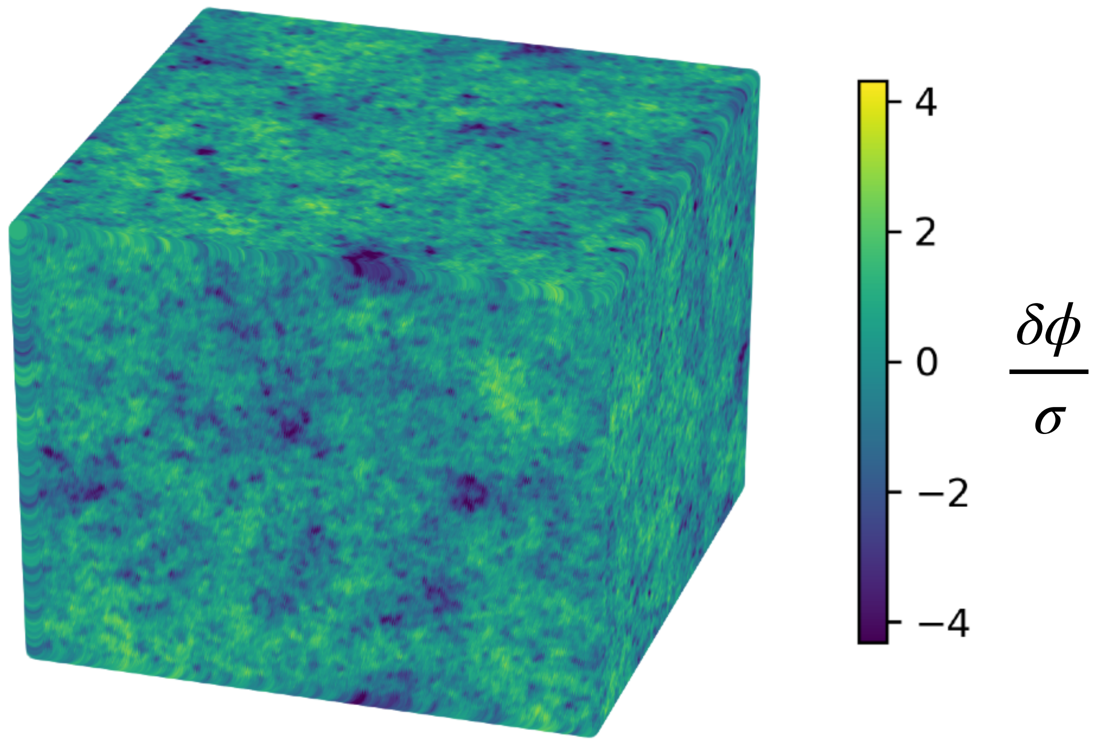


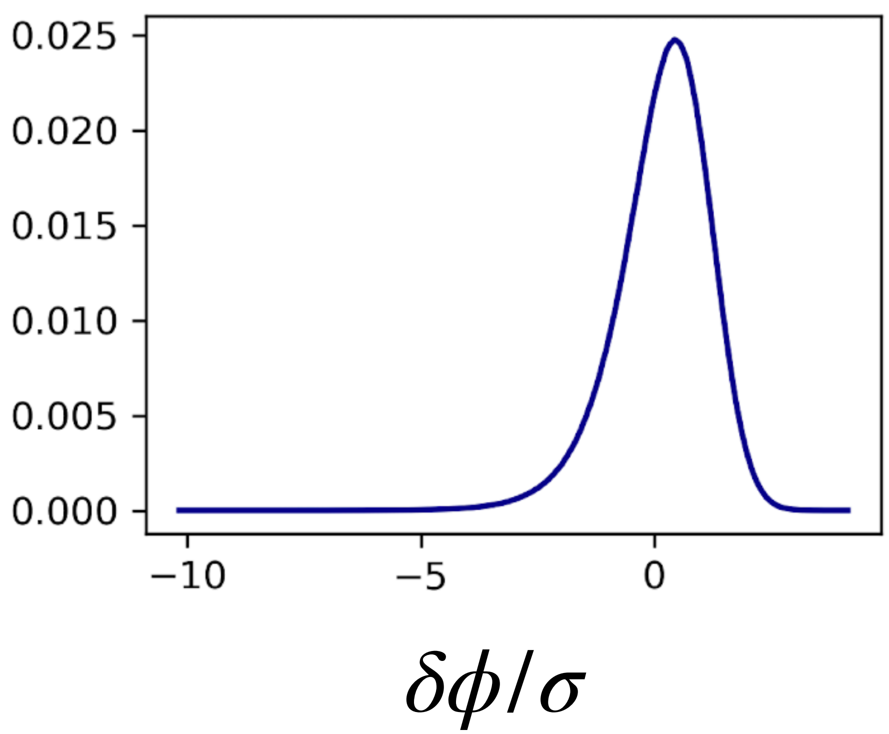
Simulation confirms analytical (**<u>very nontrivial result)</u>**

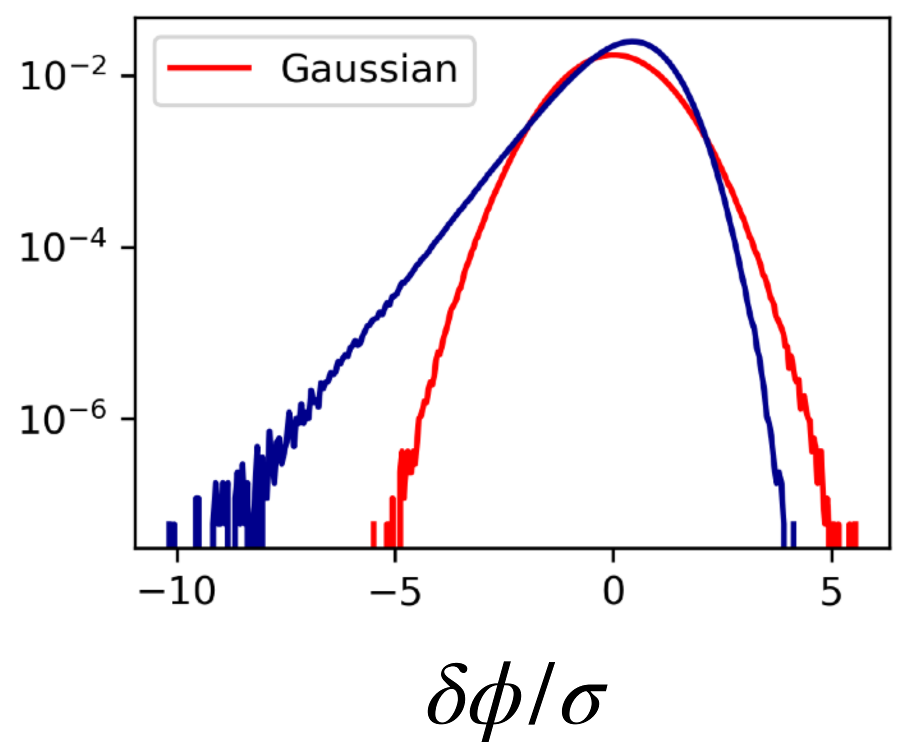


Linear regime (large scales)

Thanks to the lattice,
we know the full $\delta\phi(\mathbf{x})$ in real space!





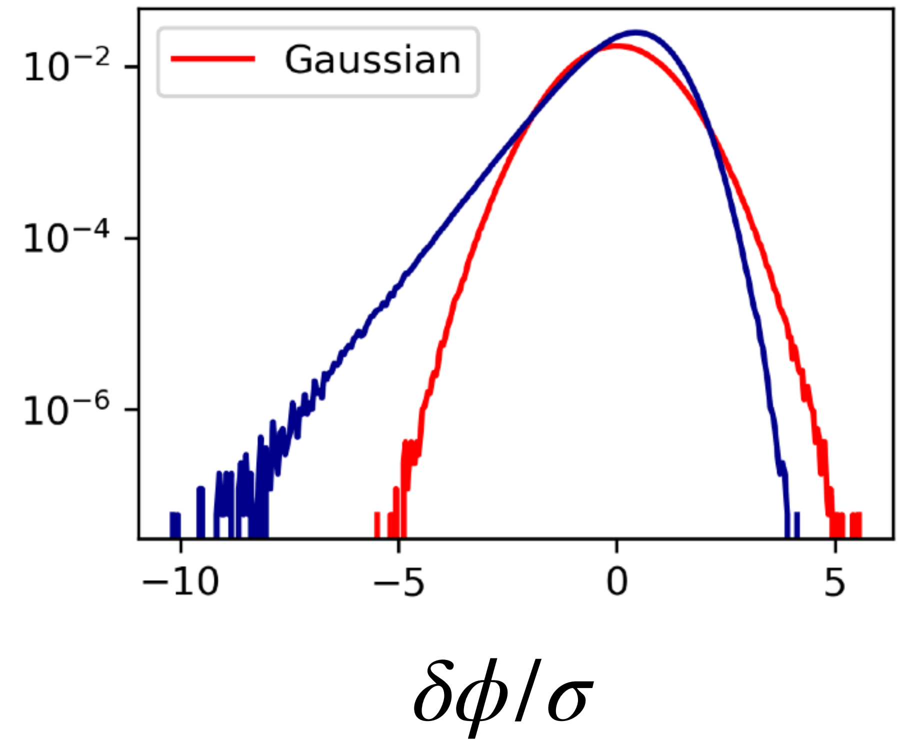


Linear regime (large scales)

Define cumulants:

$$\kappa_n = \frac{\langle \delta\phi^n \rangle_c}{\sigma^n}$$

κ_3 “skewness”, κ_4 “kurtosis”, etc.



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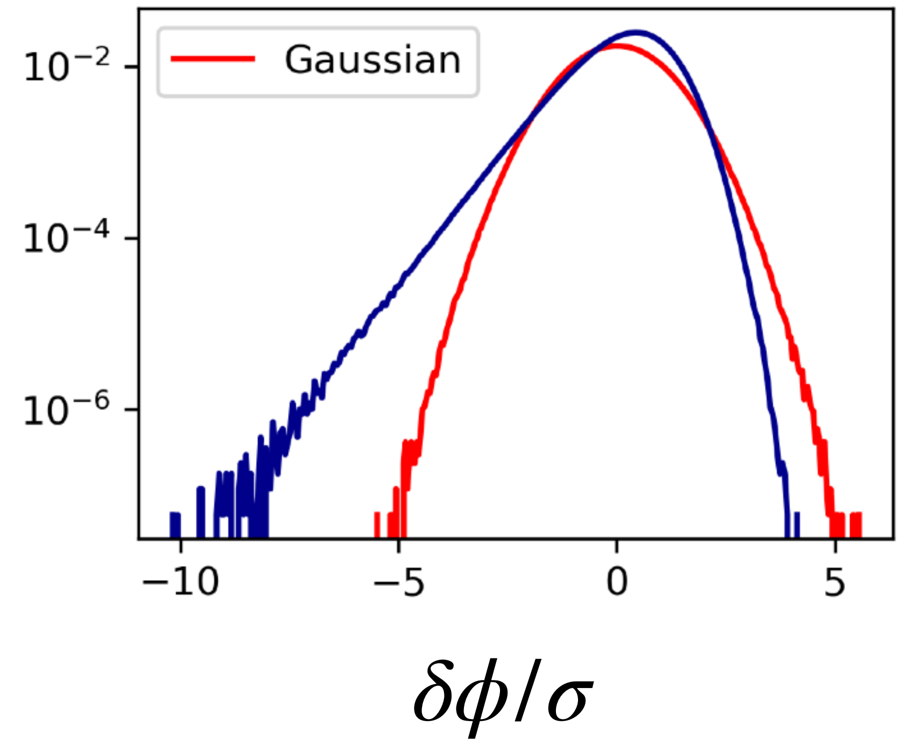

$$\ldots > \kappa_6 > \kappa_5 > \kappa_4 > \kappa_3$$

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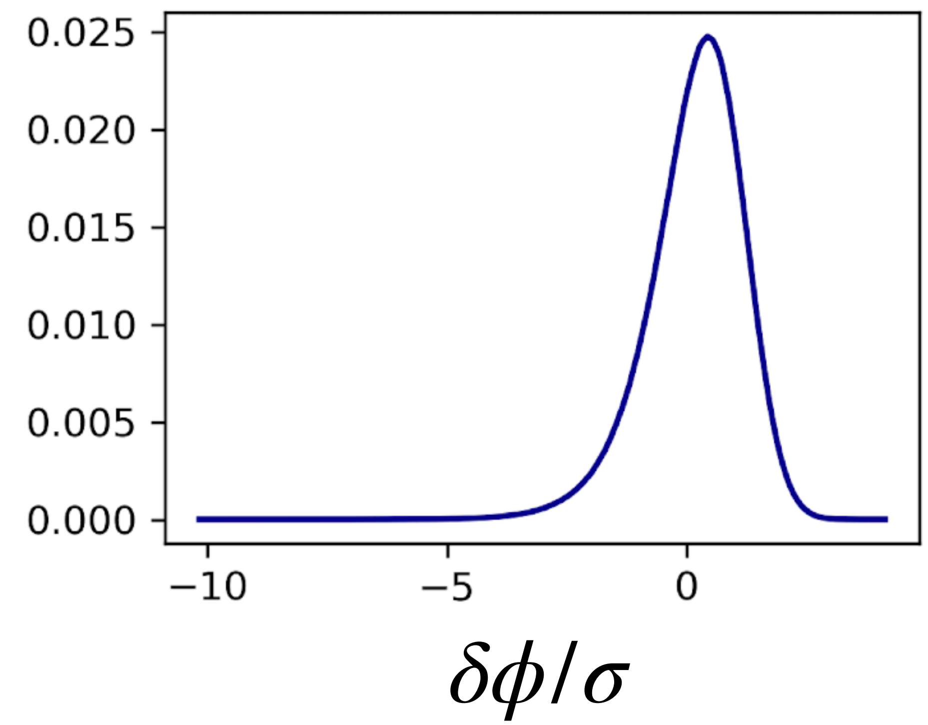
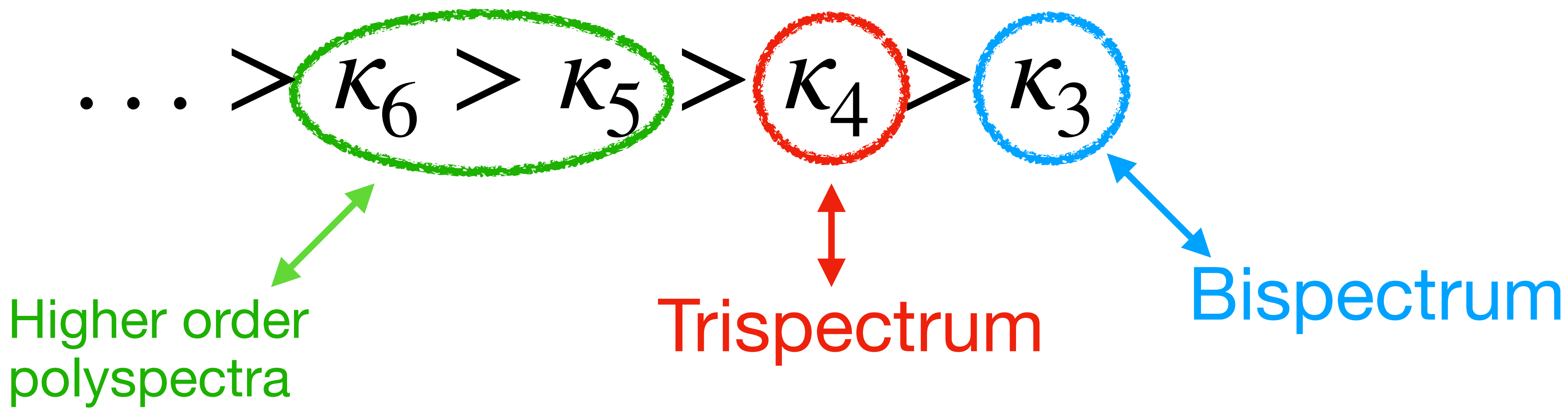




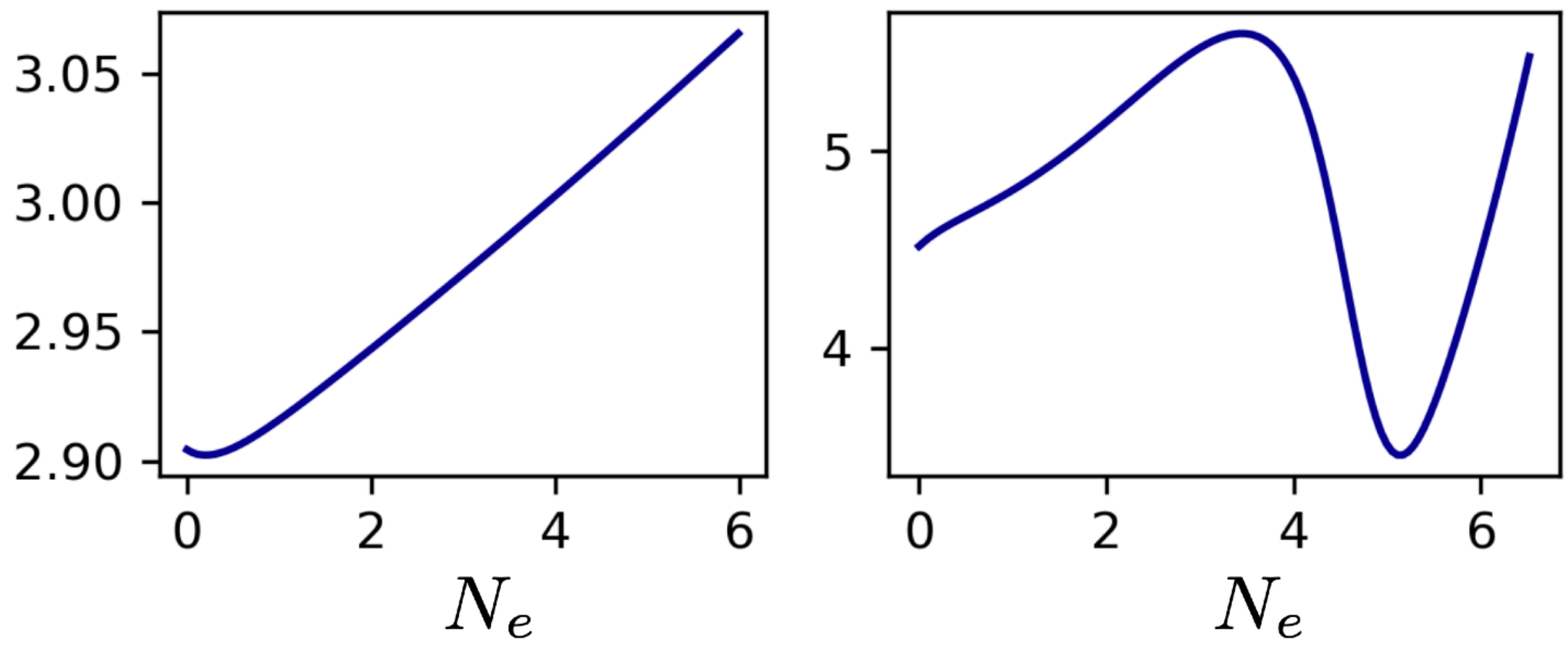
Strong backreaction (small scales)

Study transition linear $\longrightarrow$ nonlinear

$$\xi = \frac{\alpha \dot{\phi}}{2fH}$$

Linear
(no backreaction)

Linear-nonlinear transition
(strong backreaction)



e-folds number (time)

Strong backreaction (small scales)

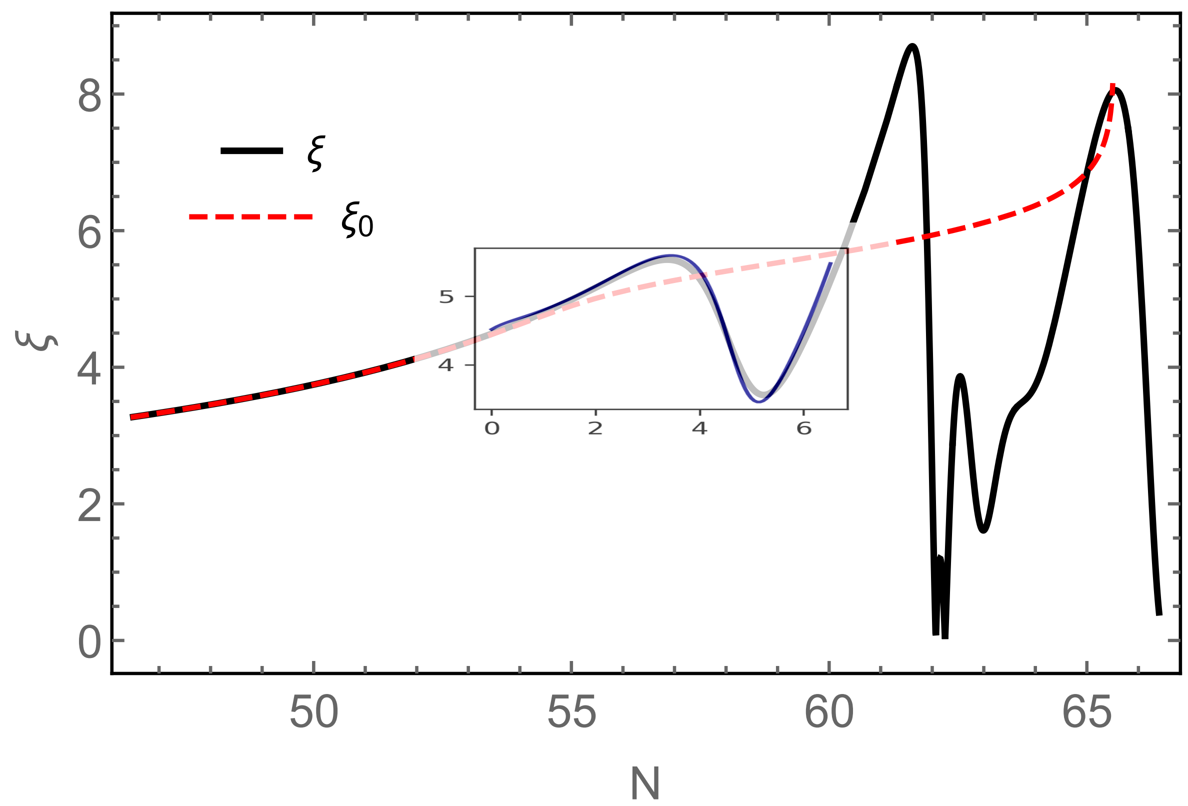


Figure from 2002.02952
[courtesy of V. Domcke]

Confirms the semi-analytical results of:

[**V. Domcke, V. Guidetti, Y. Welling, A. Westphal** arXiv:2002.02952]

[**E.V. Gorbar, K. Schmitz, O. O. Sobol, S. I. Vilchinskii** arXiv:2109.01651]

Strong backreaction (small scales)

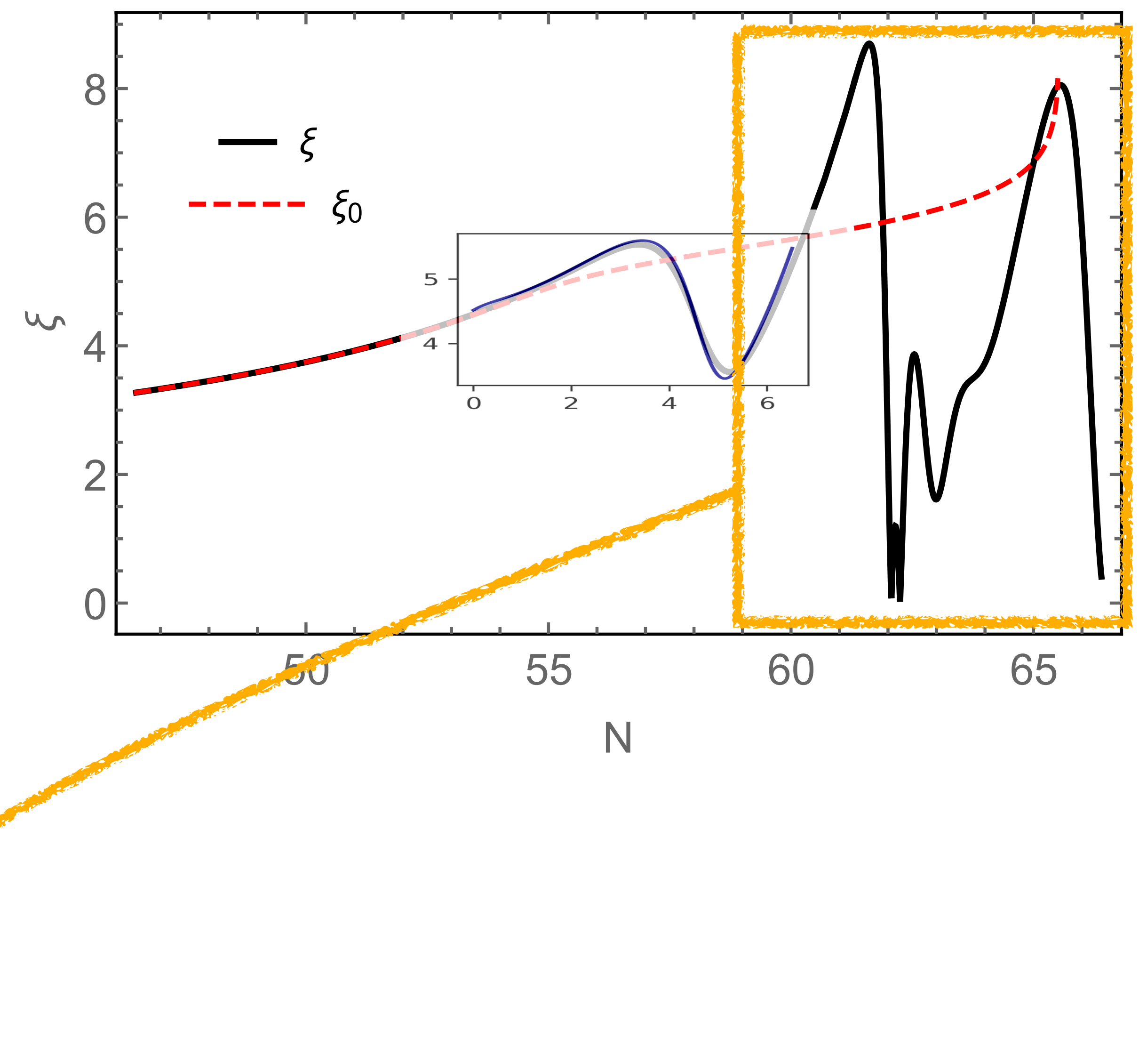


Figure from 2002.02952
[courtesy of V. Domcke]

What happens here is still under investigation. See e.g.:

D. Figueroa, J. Lizarraga, A. Urio, J. Urrestilla
2303.17436

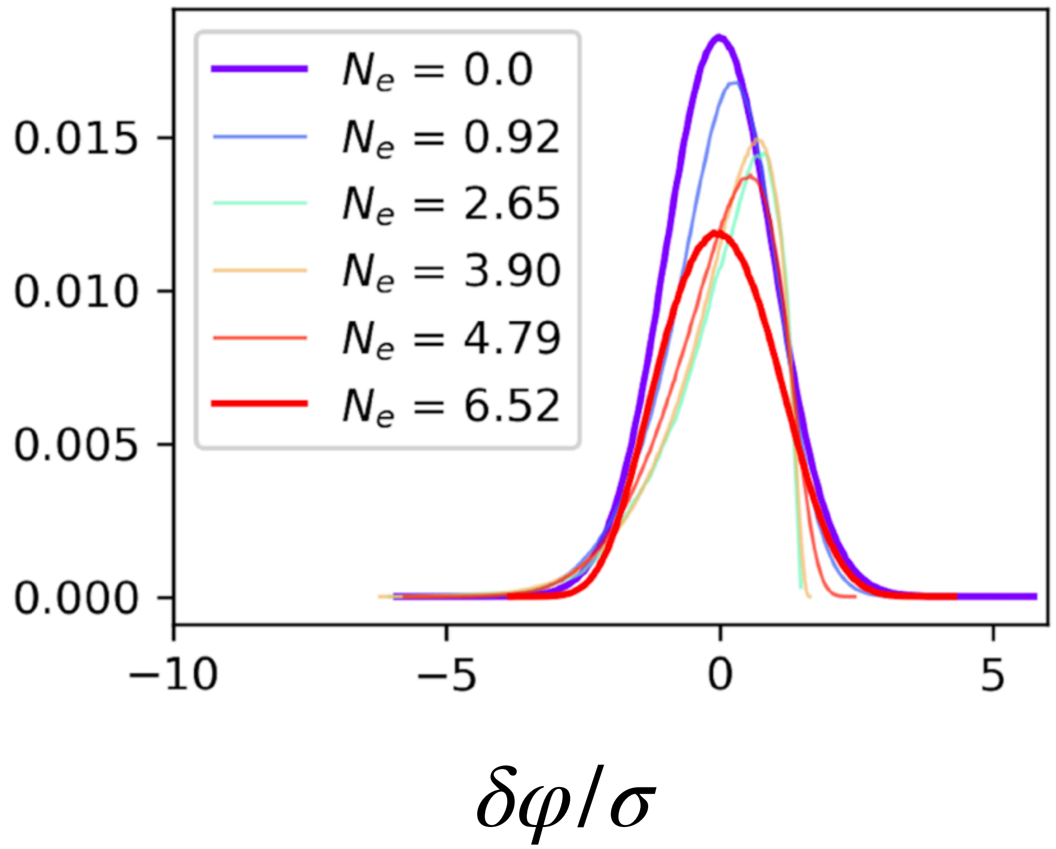
Lattice simulation

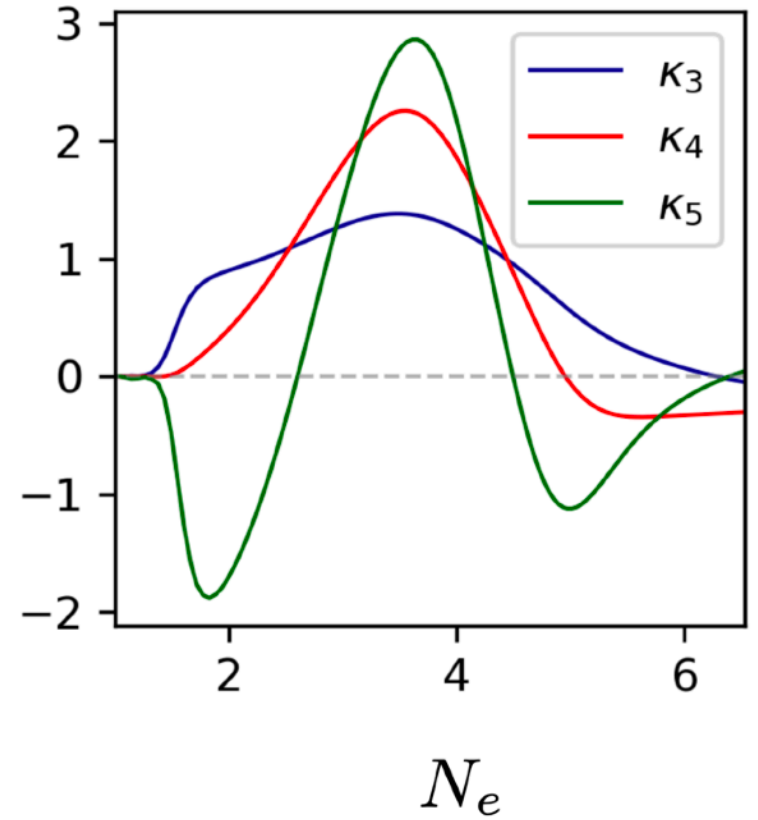
M. Peloso, L. Sorbo
2209.08131

Purely analytical

Strong backreaction (small scales)

Non-Gaussianity is **suppressed**
in the nonlinear regime!





Why? Central limit theorem

Suppression of non-Gaussianity

A. Linde, S. Mooij, E. Pajer, arXiv:1212.1693

J. Garcia-Bellido, M. Peloso, C. Unal, arXiv:1610.03763

etc…

Before our study, it was believed that:

Large $\xi \longrightarrow$ large non-Gaussianity

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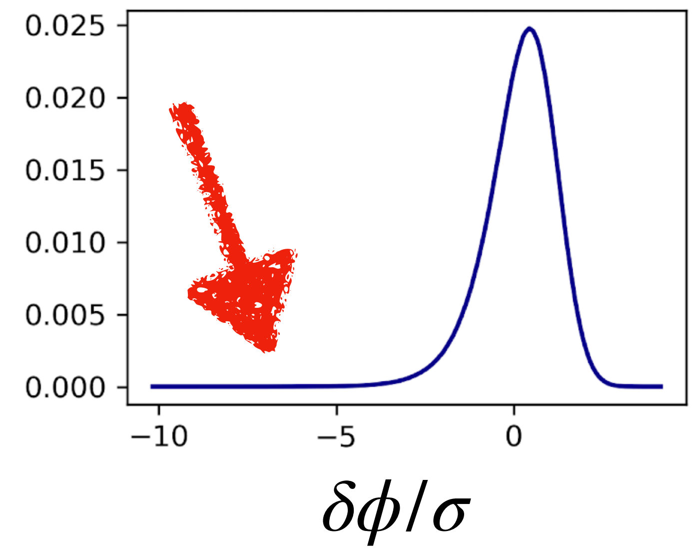
Very efficient production of Primordial BH



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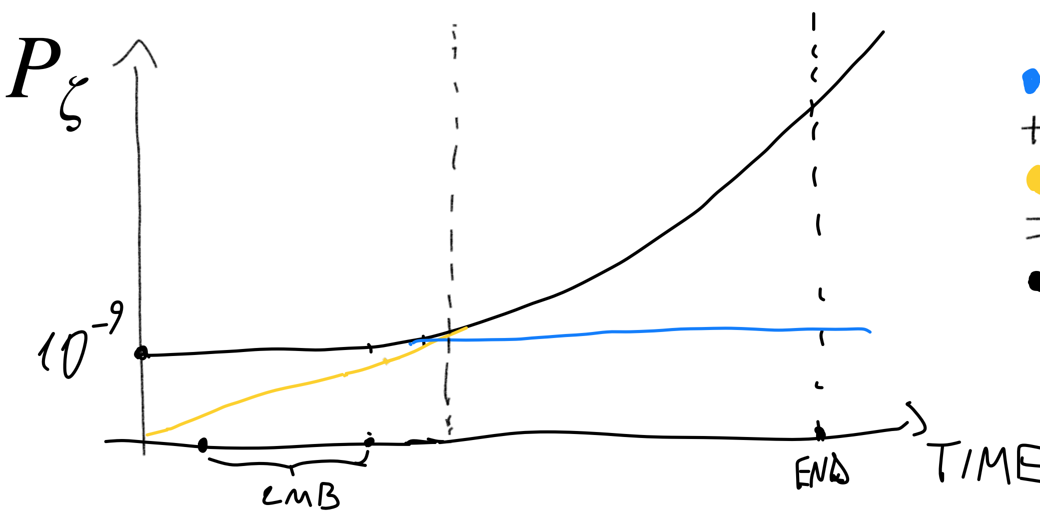
ξ has to remain small at all times



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No effects at "larger" scales (i.e. CMB, GW interferometers)

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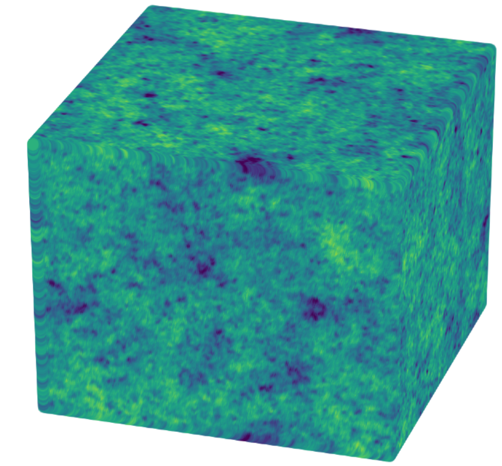
Very efficient production of Primordial BH

ξ has to remain

Needs to be revised

No effects at "larger" scales (i.e. CMB, GW interferometers)

Summary:



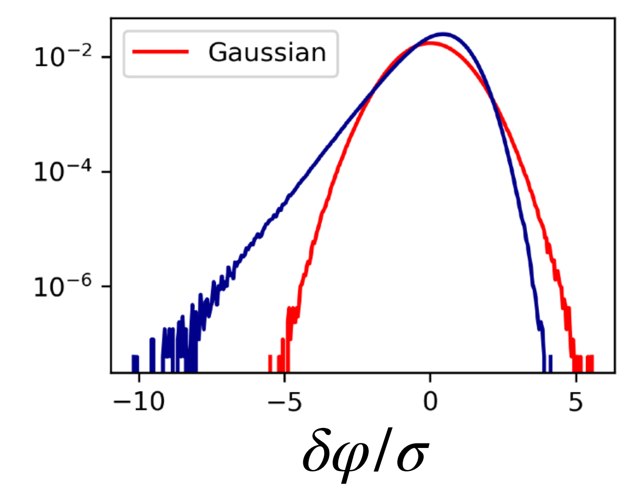
- First simulation of an axion-gauge model during inflation

We studied both:

- Linear regime (large scales):

Full characterisation of $\delta\phi$ and its non-Gaussianity

→ $\ldots > \kappa_6 > \kappa_5 > \kappa_4 > \kappa_3 > 1$



- Nonlinear regime (small scales):

Backreaction and its consequences on PBH and GW

→ Confirms nontrivial background dynamics

→ perturbations become Gaussian, due to nonlinearity