Matter Bounce Scenario in Extended Symmetric Teleparallel Gravity

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Outline of Presentation

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Introduction

• It is well known fact that our Universe is expanding

• Geometrically, the expansion rate along the spatial directions can be obtained through the scale factor $a(t)$ and the evolution of Hubble parameter is based on the scale factor as, $H = \dot{a}(t)/a(t)$. So, there are two possibilities:

i) The scale factor attains a value zero, that leads to the big bang singularity or the space time curvature singularity.

ii) The bouncing behaviour i.e. without attain the singularity, the evolution would increase again, which is an early Universe era. Since the scale factor never zero, the space time singularity would never occur. The bounce happens when H vanishes and $\dot{H} > 0$.

Introduction

• A quantum theory of gravity may avoid such a initial cosmological singularity, but we don't know what is the correct quantum theory of gravity. However, In the absence of fully accepted quantum gravity, bounce cosmology is the most promising one that allows a non-singular Universe.

• At the time of bounce, the following conditions has to be hold

$$a \neq 0 \qquad \dot{a} = 0 \qquad \ddot{a} > 0$$

• The extended symmetric teleparallel gravity, namely $f(Q)$ gravity is another geometrical modified theories of gravity that has been recently formulated using the non-metricity approach.[1]

• The matter bounce scenario motivated with the loop quantum cosmology (LQC)[2] in $f(Q)$ gravity would be investigated.

[1] J. B. Jimenez, L. Heisenberg, T. Koivisto, Phys. Rev. D, 98, 044048 (2018).

[2] A. Ashtekar, T. Pawlowski, P. Singh, Phys. Rev. D, **74**, 084003 (2006); M. Sami, P. Singh, S. Tsujikawa, Phys. Rev. D, **74**, 043514 (2006).

$f(Q)$ Gravity

The metric affine connection can be expressed in three independent components as [3],

$$\Gamma^{\alpha}_{\mu\nu} = \{^{\alpha}{}_{\mu\nu}\} + K^{\alpha}{}_{\mu\nu} + L^{\alpha}{}_{\mu\nu} \tag{1}$$

where the Levi-Civita Connection, Contortion and disformation tensor can be expressed as,

$$\begin{aligned} \{^{\alpha}{}_{\mu\nu}\} &\equiv \frac{1}{2} g^{\alpha\beta} \left(\partial_{\mu} g_{\beta\nu} + \partial_{\nu} g_{\beta\mu} - \partial_{\beta} g_{\mu\nu}\right) \\ K^{\alpha}{}_{\mu\nu} &\equiv \frac{1}{2} T^{\alpha}{}_{\mu\nu} + T_{(\mu}{}^{\alpha}{}_{\nu)}; \qquad T^{\alpha}{}_{\mu\nu} \equiv 2\Gamma^{\alpha}_{[\mu\nu]} \\ L^{\alpha}{}_{\mu\nu} &\equiv \frac{1}{2} Q^{\alpha}{}_{\mu\nu} - Q_{(\mu\nu)}{}^{\alpha}. \end{aligned} \tag{2}$$

The super potential of the model,

$$P^{\alpha}{}_{\mu\nu} = -\frac{1}{2} L^{\alpha}{}_{\mu\nu} + \frac{1}{4}\left(Q^{\alpha} - \tilde{Q}^{\alpha}\right) g_{\mu\nu} - \frac{1}{4}\delta^{\alpha}_{(\mu} Q_{\nu)}, \quad Q_{\alpha} = g^{\mu\nu} Q_{\alpha\mu\nu} \;\; \& \;\; \tilde{Q}_{\alpha} = g^{\mu\nu} Q_{\mu\alpha\nu} \tag{3}$$

The nonmetricity scalar,

$$Q = -Q_{\alpha\mu\nu} P^{\alpha\mu\nu} \tag{4}$$

[3] F. W. Hehl, J. D. McCrea, E. W. Mielke, Y. Neeman, Phys. Rep. 258, 1 (1995)

The Field Equations

The action of $f(Q)$ gravity,

$$S = \int d^4x\sqrt{-g}\left(-\frac{1}{2}f(Q) + \mathcal{L}_M\right), \tag{5}$$

The field equation of $f(Q)$ gravity,

$$\frac{2}{\sqrt{-g}}\nabla_\alpha\left(\sqrt{-g}f_Q P^\alpha_{\ \mu\nu}\right) + \frac{1}{2}g_{\mu\nu}f + f_Q\left(P_{\mu\alpha\beta}Q_\nu^{\ \alpha\beta} - 2Q_{\alpha\beta\mu}P^{\alpha\beta}_{\ \ \nu}\right) = T_{\mu\nu} \tag{6}$$

The energy-momentum tensor,

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta\sqrt{-g}\mathcal{L}_m}{\delta g^{\mu\nu}} \tag{7}$$

The homogeneous and isotropic FLRW space time,

$$ds^2 = -N^2(t)dt^2 + a^2(t)(dx^2 + dy^2 + dz^2), \tag{8}$$

The field equations,

$$6f_Q H^2 - \frac{1}{2}f = \rho \tag{9}$$

$$\left(12H^2 f_{QQ} + f_Q\right)\dot{H} = -\frac{1}{2}(\rho + p) \tag{10}$$

Matter Bounce Scenario in $f(Q)$ Gravity

As mentioned in (LQC)[4] theory Hubble squared term take the form

$$H^2 = \frac{\rho_m(\rho_c - \rho_m)}{3\rho_c} \tag{11}$$

The continuity equation and the energy density equations for the matter dominated case can be written as

$$\dot{\rho}_m = -3H\rho_m \qquad \text{and} \qquad \rho_m = \rho_{m0}a^{-3} \tag{12}$$

$$\rho_m = \frac{\rho_c}{\left(\frac{3}{4}\rho_c t^2 + 1\right)}, \qquad H(t) = \frac{2\rho_c t}{3\rho_c t^2 + 4}, \qquad a(t) = \left(\frac{3}{4}\rho_c t^2 + 1\right)^{\frac{1}{3}} \tag{13}$$

Using the relation between the e-folding parameter and the scale factor, $e^{-N} = \frac{a_0}{a}$,

$$H^2 = \frac{\rho_c}{3a_0^3}\left(e^{-3N} - \frac{e^{-6N}}{a_0^3}\right) \tag{14}$$

[4] A. Ashtekar, T. Pawlowski, P. Singh, Phys. Rev. D, **74**, 084003 (2006); M. Sami, P. Singh, S. Tsujikawa, Phys. Rev. D, **74**, 043514 (2006).

Matter Bounce Scenario in $f(Q)$ Gravity

We assume, $A = \frac{\rho_c}{3a_0^3}$ and $b = \frac{1}{a_0^3}$. Using the relation $Q = 6H^2$, the form of the e-folding parameter as,

$$N = -\frac{1}{3} Log\left(\frac{3A + \sqrt{9A^2 - 6AbQ}}{6Ab}\right) \tag{15}$$

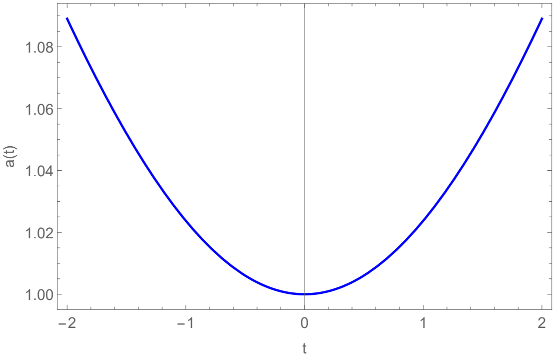
By setting $S_i = \rho_{i0} a_0^{-3(1+\omega_i)}$, the energy density becomes

$$\rho = \sum_i S_i \left(\frac{3A + \sqrt{9A^2 - 6AbQ}}{6Ab}\right)^{(1+\omega_i)} \tag{16}$$

On solving, we get

$$f(Q) = -\sqrt{\rho_c(\rho_c - 2Q)} - \sqrt{2\rho_c Q}\,\arcsin\left(\frac{\sqrt{2}\sqrt{Q}}{\sqrt{\rho_c}}\right) - \rho_c, \tag{17}$$

- The above form of $f(Q)$ produces the matter bounce evolution of the Universe.



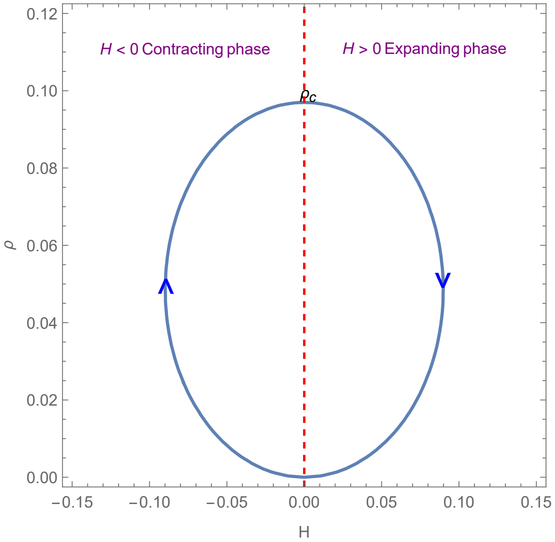


Figure: Scale factor vs cosmic time (left panel) and Energy density in Hubble parameter (right panel).

Conformal Transformation

Let us take the scalar field φ as independent variable and consider the action functional,

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left[Q - \mathcal{B}(\varphi)g^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi - 2\mathcal{V}(\varphi) + 2\lambda_\mu{}^{\beta\alpha\gamma}R^\mu{}_{\beta\alpha\gamma} + 2\lambda_\mu{}^{\alpha\beta}T^\mu{}_{\alpha\beta}\right] \tag{18}$$

The above action can be mapped to a scalar-tensor action by applying the following transformation of the metric

$$\tilde{g}_{\mu\nu} = e^{-\sqrt{2/3}\Omega(\varphi)}g_{\mu\nu} \tag{19}$$

If the $f(Q)$ model space-time is characterized by a FRW metric with η be the conformal time and $a(\eta)$ is the scale factor i.e.,

$$ds^2 = a^2(\eta)[-d\eta^2 + \delta_{\mu\nu}dx^\mu dx^\nu] \tag{20}$$

The associated scalar-tensor model's metric changes to

$$d\tilde{s}^2 = e^{-\sqrt{2/3}\Omega(\varphi)}a^2(\eta)[-d\eta^2 + \delta_{\mu\nu}dx^\mu dx^\nu] = a_s^2(\eta)[-d\eta^2 + \delta_{\mu\nu}dx^\mu dx^\nu] \tag{21}$$

One can get the field equations in the scalar tensor frame,

$$3H_s^2 = \frac{1}{2}\mathcal{B}\left(\frac{d\varphi}{dt_s}\right)^2 + \mathcal{V} \tag{22}$$

$$2\frac{dH_s}{dt_s} + 3H_s^2 = -\frac{1}{2}\mathcal{B}\left(\frac{d\varphi}{dt_s}\right)^2 + \mathcal{V} \tag{23}$$

Using the scalar field equation we get

$$\mathcal{B}\frac{d^2\varphi}{dt_s^2} + 3H_s\mathcal{B}\frac{d\varphi}{dt_s} + \frac{1}{2}\frac{d\mathcal{B}}{dt_s}\frac{d\varphi}{dt_s} + \frac{d\mathcal{V}}{d\varphi} = 0 \tag{24}$$

where $\frac{d}{dt_s} = \frac{1}{a_s(\eta)}\frac{d}{d\eta}$.

$$\epsilon_1 = -\frac{1}{H_s^2}\frac{dH_s}{dt_s}, \qquad \epsilon_2 = \frac{1}{H_s}\frac{d^2\varphi/dt_s^2}{d\varphi/dt_s}, \qquad \epsilon_4 = \frac{1}{2\mathcal{B}H_s}\frac{d\mathcal{B}}{dt_s} \tag{25}$$

$$n_s = 1 - 4\epsilon_1 - 2\epsilon_2 - 2\epsilon_4$$

$$r = \frac{8\mathcal{B}}{H_s^2}\left(\frac{d\varphi}{dt_s}\right)^2 = -\frac{16}{H_s^2}\frac{dH_s}{dt_s} = 16\epsilon_1 \tag{26}$$

Using the relation $\frac{dN_s}{dt_s} = H_s(N_s)$ the conformal time can be defined as follow,

$$\eta(N_s) = -\frac{Exp\left(\frac{1}{\beta}e^{-\beta N_f}\right)}{H_{s0}(1-e^{-\beta N_f})} e^{-(1-e^{-\beta N_f})N_s} \tag{27}$$

The conformal factor $\Omega(\varphi)$ can be written as

$$\Omega(\varphi(N_s)) = \sqrt{6}\; ln\left[e^{-N_s}\left(\frac{3}{4}\rho_c\eta^2(N_s)+1\right)^{\frac{1}{3}}\right] \tag{28}$$

It is simple to see that the conformally connected $f(Q)$ frame scale factor exhibits the following behaviour because of the aforesaid form of $\Omega(\varphi)$.

$$a(\eta) = \left(\frac{3}{4}\rho_c\eta^2+1\right)^{\frac{1}{3}} \tag{29}$$

considering approximation close to $\eta = 0$, the scale factor in terms of cosmic time turns out to be $a(t) = \left(\frac{3}{4}\rho_c t^2+1\right)^{\frac{1}{3}}$ in $f(Q)$ frame.

Using the relation $\mathcal{B}(\varphi) = (d\Omega/d\varphi)^2$ the spectral index can be defined as follows

$$n_s = 1 - 2e^{\beta(N_s-N_f)} - 2\frac{d^2\Omega}{dN_s^2}\left(\frac{d\Omega}{dN_s}\right)^{-1} + \frac{2}{\left(\frac{d^2\Omega}{dN_s^2} + (3 - e^{\beta(N_s-N_f)}\frac{d\Omega}{dN_s})\right)} \times \left[-3\frac{d\Omega}{dN_s}e^{\beta(N_s-N_f)} + \left(\frac{d^2\Omega}{dN_s^2} - \frac{d\Omega}{dN_s}e^{\beta(N_s-N_f)}\right)\left(6 - 3e^{\beta(N_s-N_f)} + \frac{d^2\Omega}{dN_s^2}\right) + \left(\frac{d^3\Omega}{dN_s^3} - \frac{d^2\Omega}{dN_s^2}e^{\beta(N_s-N_f)} - \beta\frac{d\Omega}{dN_s}e^{\beta(N_s-N_f)}\right)\right] \quad (30)$$

$$\Omega(N_s) = \sqrt{6}\left[-N_s + ln\left(\frac{3}{4}\rho_c\left(\frac{e^{\frac{1}{\beta}e^{-\beta N_f}-(1-e^{-\beta N_f})N_s}}{H_{s0}(1-e^{-\beta N_f})}\right)^2 + 1\right)^{\frac{1}{3}}\right] \quad (31)$$

For $\beta = 0.1$ and $N_T = 60$, the spectral index is consistent with Planck results. For scalar spectral index and the tensor to scalar ratio in the scalar-tensor frame values are $n_s = 0.9649 \pm 0.0042$ and $r < 0.064$ respectively from the Planck 2018 constraints.

Dynamical System Analysis

The general form of $f(Q)$ as $Q+\psi(Q)$ and accordingly,

$$3H^2 = \rho + \frac{\psi}{2} - Q\psi_Q \tag{32}$$

$$2\dot{H} + 3H^2 = -p - 2\dot{H}(2Q\psi_{QQ} + \psi_Q) + \left(\frac{\psi}{2} - Q\psi_Q\right) \tag{33}$$

The density parameters for the matter, radiation and dark energy phase are respectively denoted as, $\Omega_m = \frac{\rho_m}{3H^2}$, $\Omega_r = \frac{\rho_r}{3H^2}$ and $\Omega_{de} = \frac{\rho_{de}}{3H^2}$ with $\Omega_m + \Omega_r + \Omega_{de} = 1$. We consider the dimensionless variables,

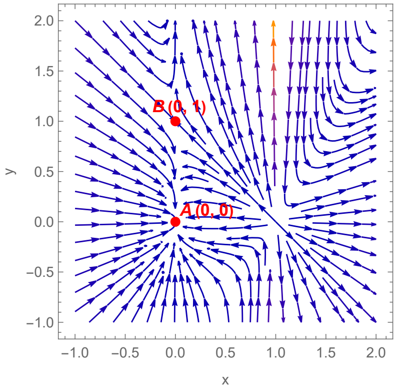
$$x = \frac{\psi - 2Q\psi_Q}{6H^2} \qquad y = \frac{\rho_r}{3H^2}. \tag{34}$$

The dimensionless variables can be represented as,

$$x' = x\left(3(x-1) - y\right) \tag{35}$$

$$y' = -\frac{y(x(-3x+y+4)+y-1)}{x-1} \tag{36}$$

Dynamical System Analysis



(x, y)	Ω_m	Ω_r	Ω_{de}	ω_{eff}	(q)	Eigenvalues	Stability
$A(0,0)$	1	0	0	0	1/2	{-3,-1}	Stable Node
$B(0,1)$	0	1	0	1/3	1	{-4,1}	Unstable

Scalar Perturbation

The first order perturbation in the FLRW background with the perturbation geometry functions $\delta(t)$ and matter functions $\delta_m(t)$ can be expressed as,

$$H(t) \to H_b(t)(1+\delta(t)), \qquad \rho(t) \to \rho_b(t)(1+\delta_m(t)) \tag{37}$$

The perturbation of the function $f(Q)$ and f_Q can be calculated as,

$$\delta f = f_Q \delta Q, \qquad \delta f_Q = f_{QQ} \delta Q, \tag{38}$$

Neglecting higher power of $\delta(t)$, the non-metricity scalar becomes,

$$Q = 6H^2 = 6H_b^2(1+\delta(t))^2 = 6H_b^2(1+2\delta(t)) \tag{39}$$

and subsequently

$$Q(2Qf_{QQ} + f_Q)\delta = \rho\delta_m, \tag{40}$$

Now, to obtain the analytical solution to the perturbation function, we consider the perturbation continuity equation as,

$$\dot{\delta}_m + 3H(1+\omega)\delta = 0 \tag{41}$$

Scalar Perturbations

From eqns. (40)-(41), the first order differential equation can be obtained,

$$\dot{\delta}_m + \frac{3H(1+\omega)\rho}{Q(2Qf_{QQ} + f_Q)}\delta_m = 0 \tag{42}$$

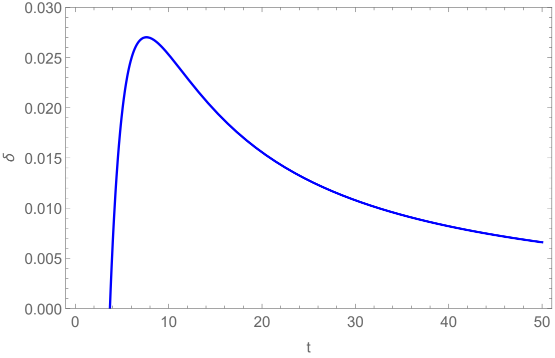
The simplified relation can be obtained,

$$\dot{\delta}_m - \frac{\dot{H}}{H}\delta_m = 0 \implies \delta_m = C_1 H \tag{43}$$

where C_1 is the integration constant. Subsequently from eqn. (41), we obtain

$$\delta = C_2 \frac{\dot{H}}{H} \tag{44}$$

where, $C_2 = -\frac{C_1}{3(1+\omega)}$. The evolution behavior of δ and δ_m are given in FIG.



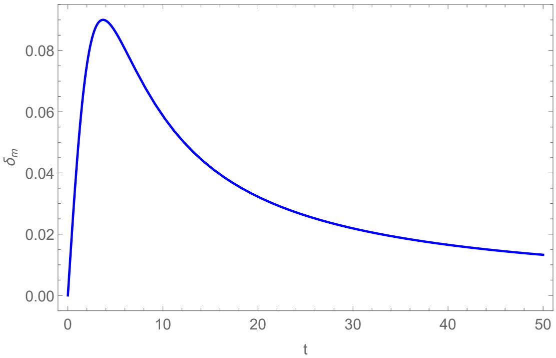


Figure: Evolution of perturbation $\delta(t)$ and $\delta_m(t)$ in cosmic time.

Conclusion

- The matter bounce scenario of the Universe has been rebuilt in extended symmetric teleparallel gravity using a special version of $f(Q)$.

- The scalar spectral index and tensor-to-scalar ratio in the scalar-tensor frame are $n_s = 0.9649 \pm 0.0042$ and $r < 0.064$, respectively, which constrain Planck 2018.

- Through the dynamical stability analysis, in the matter-dominated case the universe shows stable behavior.

- The evolution perturbations δ and δ_m in cosmic time approach zero at late times, confirming the stability of the model.

- Further study can be carried out on the reconstructed form of $f(Q)$, which may give some more results on the bouncing scenario.


Thank You
For Your
Attention